THE GEOMETRY OF THE ELECTROMAGNETIC SIX-VECTOR, THE ELECTROMAGNETIC ENERGY TENSOR, THE DIRAC EQUATIONS, AND THE HERTZIAN TENSOR

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Let S_3 denote the hyperplane at infinity in the tangent-space T_4 at any particular point (x^i) of the space-time V_4 of general relativity. Any vector X^i at (x^i) defines in S_3 a point of homogeneous coordinates X^i, while the metric tensor g_{ij} defines a quadric Q, namely the section by S_3 of the null hypercone of T_4. The six-vector F_{ij} of electromagnetic field-strength represents in S_3 a linear complex which has in general two lines in common with each regulus of Q, and so defines a skew quadrilateral upon the quadric. The vertices of this quadrilateral correspond to four null vectors of V_4 which define the principal directions both of F_{ij} and of the electromagnetic energy tensor E_{ij}. The latter defines in S_3 another quadric Q' which passes through the same skew quadrilateral. Q and Q' are apolar each way and are self-polar with respect to each other. (The above results have been published in *Proc. London Math. Soc.*, *41* (1936) 302—322.)

A skew quadrilateral upon a quadric likewise forms the geometrical background of Dirac's equations, the vertices of the quadrilateral corresponding in V_4 to four null vectors which are easily definable in terms of two-component spinors. The properties of this quadrilateral in relation to Dirac's equations are of considerable interest, and it can further be shown that *the whole Dirac theory can be expressed in a form not explicitly involving spinors*, but depending instead upon the four null vectors. This result is due in part to E. T. Whittaker.

The so-called Hertzian tensor gives rise to a similar geometrical configuration which has an interesting connection with the tensor formula for the potential of a charged particle in special relativity.