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SEVENTY FIVE YEARS OF THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION

ABSTRACT. The origins, history, work and aims of the International Commission on Mathematical Education are described.

This year ICMI[1] celebrates its seventy-fifth birthday. Its life has not always been easy – on two occasions it has of necessity lapsed into a coma – yet it has a proud and interesting history which reveals much about changing attitudes in mathematics education. Even more importantly it has a challenging present and future.

The idea of an International Commission to enquire into mathematical education was first suggested in 1905 by the American, David Eugene Smith, in *L'Enseignement Mathématique*, the revue founded in 1899 by Henri Fehr and Charles Laisant (Smith, 1905). A formal proposal was considered at the Fourth International Congress of Mathematicians held in Rome in April 1908 and it was resolved to establish the Commission internationale de l'enseignement mathématique (CIEM or, as its anglicized form now is, ICMI) (ICM, 1909). The first president was the great German mathematician, Felix Klein, and the first Secretary-General, Henri Fehr. Klein was to act as President until his death in 1925; Fehr had an even longer association with ICMI, for he served as Secretary until the outbreak of the Second World War. When ICMI was reconstituted in the 1950s, Fehr was made 'Président d'honneur' for life. Unfortunately, he died later that year aged 84.

The reasons for the formation of ICMI at that particular period are not hard to perceive. The educational systems of the major countries of Western Europe and North America had expanded during the early years of the century, new technologies set new demands, and innovators had attempted to carry out significant reforms of the (grammar) school mathematical curriculum. In Germany, Klein gave the lectures now known to us as *Elementary Mathematics from an Advanced Standpoint*, in France, a government decree of July 1905 invited 'teachers to follow a method entirely new in geometry', and in England, as a result of the efforts of John Perry and others, Euclid's rule came to an end (not the spirit of Euclid, which Dieudonné was later to deplore, but the use of his *Elements*). Perry, indeed, wanted far more than just the reform of geometry. He laid stress on making mathematics useful and on linking its teaching with that of science and engineering: he argued for 'utility' rather than 'rigour',

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laboratory-based experience rather than abstraction. His influence was world-wide, ranging from the U.S.A. to Japan, whilst German educators coined the term *Perryismus* (see, for example, Howson, 1982). The times then were not dissimilar to the 1960s and there was a call for an international exchange of experience and views.

The Commission met for the first time in Cologne in September 1908, each country being invited to send a small number of delegates (thus, for example, the U.S.A. was represented by Smith, W. Osgood and J. W. Young). The countries deemed to be 'participating' were

Austria	Greece	Norway	Sweden
Belgium	Holland	Portugal	Switzerland
Denmark	Hungary	Rumania	U.K.
France	Italy	Russia	U.S.A.
Germany	Japan	Spain	

In addition the following were listed as 'associated countries'

Argentina	Cape Colony (South Africa)	Mexico
Australia	Chile	Peru
Brazil	China	Serbia
Bulgaria	Egypt	Turkey
Canada	India	

(Of these thirty-three countries, Chile, Peru and Turkey are no longer members of ICMI.)

One of the first decisions of the Commission was to reject the terms of reference presented to it in Rome: to study programmes and methods of teaching to be found in the secondary schools of the various countries. To study secondary schools in isolation was, the Commission thought, impossible – ICMI had to consider the teaching of mathematics in all types of schools, including primary and vocational, and also in universities and other institutions of higher education.[2]

Smith had in his Rome address listed certain questions which at that time greatly concerned the American teacher. Seventy-five years on they have acquired a familiar ring:

What have been the results of attempting to remove the barrier between such topics as algebra and geometry, or to teach the two simultaneously (i.e., in the same grade), and are we prepared as yet to make any recommendation on this matter?

(10th grade Geometry, which E. H. Moore opposed in his AMS Presidential Address in 1902, is still a matter for debate in the U.S.A. In many countries,

e.g., the USSR, it is still thought advantageous to separate 'Algebra' and 'Geometry' within the school mathematics course, each with its own time-table allocation and textbooks.)

What is the safe minimum of Euclidean geometry, the calculus and mechanics?

What position should the secondary schools take with respect to the nature of applications and the relations of applied to pure mathematics?

What should be the relative nature of the courses in the secondary schools for those who do not intend to proceed to the universities, and for those who do intend to do so? (ICM, 1909, pp. 465–477).

That these and other questions posed by Smith are still with us is not necessarily a cause for pessimism, for it is in the nature of mathematics education that questions such as the last must continuously be asked. Changing social and mathematical contexts ensure that any 'solution' can only be a temporary one.

It was to the study of these and similar questions that the founding fathers (for women, as the table of ICMI officers appended shows, have never been represented on its major committee) turned. Several international meetings were held between that at Cologne in 1908 and the ICM at Cambridge in 1912. The Commission met in Karlsruhe, Basle, Brussels, Milan and, so as to be near its President, the Harz mountains.

At some of these meetings general topics were discussed. At others, for example that in Milan in 1911, attention was concentrated on specific questions – in that particular case on 'What mathematics should be taught to those students studying the physical and natural sciences?' 'What is the place of rigour in mathematics teaching?' and 'How can the teaching of the different branches of mathematics best be integrated?' Once again, we have three questions equally relevant today.

The main work of the Commission at this time was, however, the preparation of a vast survey of teaching practices in member countries. Each participating country appointed a sub-commission to prepare national reports, often in many volumes, and the result was outstanding both in terms of quantity and quality.[3] Thus, for example, the French report ran to five volumes and that of the U.S.A. to eleven. The British contributed only two volumes, but the first of these had over 600 pages! Certainly, nothing on the same scale had been attempted before, or has been attempted since. Moreover, not only did countries comment on their own systems,[4] but, for example, as part of the German contribution G. Wolff (who attended ICME2 at Exeter in 1972 and ICME3 at Karlsruhe in 1976) wrote a fascinating account of secondary education in England, *Der Mathematische Unterricht der Höheren Knabenschulen*

Englands (Wolff, 1915), which still remains a model of a successful comparative case study. That its delayed publication should have taken place in 1915 when the two countries were locked in battle is just one further bewildering and poignant fact to be recorded from those years.

In addition to receiving the various reports, the 1912 Congress paid special attention to two other problems on which it had instigated surveys, 'The mathematical training of the physicist' and 'Intuition and experiment in mathematical teaching in the secondary schools'.

Encouraged by this extremely fruitful four years' work, the Commission chose two further topics to be studied prior to a meeting to be held in 1914 in Paris. One was the position occupied by calculus in the secondary schools, the second 'The place of mathematics in higher technical education', and the speakers included Klein and Emile Borel. The questions which the Sub-Commission on Calculus put to the member countries were extremely well devised and many present-day lecturers to first-year university students would still welcome information on, for example, what proportion of schools consider functions of several variables, discuss the remainder term in Taylor's Theorem, introduce the integral as the limit of a summation or as a primitive function, call attention to non-differentiable functions, and deal with irrationals logically and systematically (which is *not* the same as rigorously).[5]

In six years, then, ICMI had covered an enormous amount of ground; as H. F. Baker (1926) wrote in his obituary of Klein, it 'had amassed an amount of information beyond belief'. Alas, the First World War was to bring these extremely productive years to an end. The ICM planned for Stockholm in 1916 was cancelled and when the ICMs were resumed in 1920 (at Strasbourg) little time was devoted to consideration of pedagogical matters. It was not until the 1928 ICM in Bologna that ICMI was formally reconstituted, although in the intervening years Fehr attempted from Geneva to keep the sub-commissions active and in touch. As we have already observed some nations still published reports during the war years, notably Germany and the United States, before it, too, entered the conflict. The latter, indeed, produced a series of four volumes which compared mathematics curricula and teacher-training programmes as they were described in the various national reports produced for ICMI in 1912.

In many ways it is frightening to contemplate the work which the Commission successfully completed in those early years, to see how it so quickly identified key questions and started to collect data concerning them, and to realise how little progress may appear to have been made in the seventy years since. I have already referred to '10th grade geometry'; one can also think, say, of the slow rate at which calculus has penetrated U.S. High Schools. Yet, when

considered in more detail, one sees that the Commission usually took what we should think of today as a one-sided view of mathematics education. It was very concerned with subject matter and to a lesser degree with how that content was presented. The psychological problems of learning mathematics had yet to be given any deep consideration (although several of those involved in the Commission's work openly supported the Herbartian theories of psychology then current). One address in those early days, however, did attempt to set mathematics education in a wider context, that given by Federigo Enriques in Milan on 'Mathématiques et Théorie de la connaissance' (Enriques, 1911). It was also the case, as Baker pointed out, that the Commission was more successful in amassing information than in analysing and interpreting it – he expressed the hope that one day the material collected 'may be utilized'.

When the Commission was re-established at Bologna in 1928, the officers were chosen from those who had been active in ICMI prior to the First World War. The choice of Smith as President was interesting in so far as he, unlike other Presidents, was not highly distinguished as a mathematician, being, in fact, the holder of a chair in mathematics education (at Teachers College, Columbia University).[6] Guido Castelnuovo and Hadamard resumed their places on the Central Committee, now as Vice-Presidents. In 1932 Hadamard succeeded Smith as President and remained in that office until the outbreak of the Second World War caused ICMI to relapse into its second 'coma'. Although Castelnuovo retired from the Committee in 1932, his daughter, Emma, has ensured that the name has continued to feature prominently in ICMI affairs.

The years between the two World Wars were not, however, particularly momentous ones for the Commission. The re-establishment of ICMI coincided with a world-wide depression and there was little interest shown in educational innovation and expansion. Such changes as took place were usually low key – educational systems grew slowly, Gestalt psychology became established but, like the writings of the young Swiss psychologist, Piaget, failed to have any great influence on classroom practice. Smith had retired from Columbia in 1922, and the two Vice-Presidents of ICMI were both well into their sixties. The war had apparently resulted in a 'missing generation' and the ICMI activities wore a tired look.[7] At Bologna it was agreed that a survey on the training of secondary school mathematics teachers would be carried out prior to the ICM to be held in Zurich in 1932. The survey was planned with the thoroughness which distinguished the pre-1914 work. There was, for example, a section devoted to the professional training of teachers which contained questions such as: did they receive university courses in the methodology of teaching

mathematics and if so from what type of person (a university professor or a secondary-school teacher); were they obliged to follow general courses in pedagogy and psychology, and if so how useful were such courses thought; what opportunities were they given to develop practical (laboratory) skills, how much teaching practice was provided and at what stage of their training; how was teaching practice supervised and assessed?[8] Reports were presented at Zurich from fourteen countries (including the newly-created Czechoslovakia, Poland and Yugoslavia). There it was agreed that the topic for the 1936 Oslo ICM should be 'Present trends in the teaching of mathematics'. On that occasion twelve countries presented reports including Japan, the only non-European country other than the U.S.A. to participate actively in the inter-war ICMI activities (which would appear to have been largely confined to the two surveys described). As Fehr (1952) wrote, such surveys attempted to provide a purely objective view of the current state of mathematics teaching; the Commission did not wish to impose any international uniformity, but only to illuminate new trends and to help facilitate progress – a principle to which ICMI still adheres.

It was not until 1952 that the Commission resumed its activities. Now, however, it became attached to the newly created International Mathematical Union (IMU) as a sub-commission of this latter body. It was at the first general assembly of the IMU in Rome, 1952, that the decision was taken to reconstitute ICMI and a special committee of five (including Henri Fehr) was appointed to draw up a plan of work in preparation for the ICM to be held in Amsterdam in 1954 (Behnke, 1954). Additional members were then co-opted to form an ICMI Executive Committee under the Presidency of Albert Châtelet. Twenty-seven countries established National Committees for Mathematics communicating with IMU and each of these committees had a national sub-commission which was assigned the task of collaborating with ICMI. The countries were:

Argentina	Finland	Italy	Peru
Australia	France	Japan	Spain
Austria	Germany	Malaya-Singapore	Sweden
Belgium	Great Britain	Mexico	Switzerland
Canada	Greece	Netherlands	U.S.A.
Cuba	India	Norway	Yugoslavia
Denmark	Israel	Pakistan	

The new Executive Committee met for the first time in January 1954 when it was agreed that reports would be made at the Amsterdam ICM on two issues:

(1) The part of mathematics and the mathematician in contemporary life;

(2) Mathematical instruction for students between the ages of 16 and 21.

A special book exhibition was arranged in connection with the second theme.

The choice of the first theme was in marked contrast to much that had gone before: attention was now directed not so much at current practice, but at the changing social and scientific contexts within which mathematics curricula had to be designed. The war had, for good or ill, disclosed what Behnke described as 'new possibilities, new opportunities for applying mathematics'. Biometry, econometry and cybernetics were new areas which offered 'the possibilities of work and activities for the mathematician'. The ICMI inquiry, it was urged, should 'not neglect any aspects of the problems being either of intellectual, scientific, social kind or of the nature of practical application'.

At the 1954 ICM the General Assembly of IMU agreed terms of reference for ICMI including procedures for the election of its officers: at that time the IMU assembly elected the President and ten 'free' members of the Commission, whereas the two Vice-Presidents and the Secretary, who, with the President, formed the ICMI 'Bureau', and three further executive committee members were elected by the 'free' members and the national representatives. A meeting to elect officers and to determine a plan of work for the next three years was held at Geneva in July, 1955 (Desforge, 1955).

It was proposed that the Commission should study three themes:

(i) a continuation of that prepared for Amsterdam on the contemporary role of mathematics and mathematicians;

(ii) mathematical teaching up to the age of fifteen years (thus complementing the second Amsterdam theme);

(iii) the scientific basis of mathematics in secondary school teaching and the scientific training of secondary school mathematics teachers;

and report on these at the 1958 Edinburgh ICM.

Kurepa who had introduced the first theme at Amsterdam recalled the interest it created there and mentioned the possibility of UNESCO cooperating to produce a film on the subject. This last was a novel suggestion, but one which did not materialise. Behnke, who spoke to the second proposal stressed the need carefully to listen to psychologists and didacticians when discussing mathematics teaching up to the age of fifteen. Whilst his views received support, it was urged that the mathematician's voice should also be heard equally on this issue. The debate was more significant in retrospect than it might have appeared at the time, for very soon a whole wave of curriculum reform was to break – what has now become known as the 'new math'. The collaboration between mathematicians and psychologists which the 1955 meeting thought

essential did not take place in the majority of countries. Expectations and hopes went unfulfilled; the mathematicians blamed the educationists and the educationists blamed the mathematicians.

The third theme was thought ill-balanced and it was agreed to drop the reference to teacher training.

At this point in the debate there was a new contributor; one who was to play an increasingly important role in ICMI affairs in the following two decades and who without doubt was to have the greatest influence on ICMI's development – Hans Freudenthal.

Freudenthal was worried by the very general nature of the themes suggested: there was the risk that national sub-commissions would respond to them with descriptions of organisation and administration, rather than with accounts of a scientific nature. It was the duty of the ICMI to promote research and study of this latter scientific kind and one way in which the danger of which he warned might be averted was to suggest limited and well-defined subjects for inquiry, for example,

- the need for a preliminary, intuitive approach in the teaching of geometry;
- the help which psychology could bring to the early stages of mathematics teaching and learning;
- the importance of the teaching of geometry;
- logic and mathematics teaching (Desforge, *op. cit.*, p. 200).

It was, he argued, essential that the themes chosen should foster work in depth and should encourage schoolteachers to call upon their experiences.

Freudenthal's arguments were well received, but at that time led only to the substitution of the proposed theme (1) by:

- a comparative study of the methods used in introducing geometry.

The Geneva meeting, then, although having little immediate impact upon the way in which ICMI proceeded, was of great importance in sowing the seeds of future changes. The traditional 'survey' came under attack for not promoting that work in depth – research and study – which ICMI, it was argued, should foster. (It is surprising how infrequently the word 'research' occurred in ICMI writings before that date.) That didacticians and psychologists might have valuable contributions to make to mathematics education was also argued more strongly than hitherto.

Another significant event at the meeting was the election of Ram Behari to the executive committee, the first member to come from outside Western Europe and North America. Akizuki, from Japan, narrowly failed to be elected on that occasion, but he was to join the committee four years later together with A. D. Alexandrov (the first member from the USSR). ICMI was now becoming more truly international and this fact was emphasised when it was

officially represented by Marshall Stone at a meeting held in Bombay in 1956 – probably the first major international conference on the teaching of mathematics and science to be held in Asia (Tata Institute, 1956).

Now, however, significant changes and innovations began to take place in mathematics education in several countries. The nations of Europe were beginning to enjoy economic prosperity and educational systems were expanding quickly. Many colonies of France and the United Kingdom were becoming (or had, like India, become) independent. With independence came the desire to 'catch up quickly' with their past rulers, and education was seen as a vital factor in national development. Within mathematics education enormous changes were taking place. The effect of Bourbaki on university curricula was enormous, Birkhoff and Maclane's *A Survey of Modern Algebra* (1941) which the *American Mathematical Monthly* had declined to review when it was first published, on the grounds that it was 'advanced' work, now became a much-used introductory undergraduate text. Calls were made for a radical revision of school mathematics courses and, in the U.S.A., a timely aid to fund-raising was provided in 1957 by the first flight of the Soviet Sputnik. This resulted in the launching of the nationally-financed School Mathematics Study Group, an offshoot of two conferences of mathematicians drawn from the major U.S. universities. Elsewhere, particularly in Belgium, radical changes in school mathematics were being proposed. In 1959 the first of the Arlon seminars took place, on the subject of topology in schools; later seminars were to consider such topics as analysis and vector spaces. In November of that same year came the most influential of all the seminars, that held at Royaumont under the aegis of the Organisation for European Economic Cooperation. The Royaumont seminar, then, concerned only a minority of those countries which belonged to ICMI. Nevertheless, it was chaired by the then ICMI President, Stone, and its two Vice-Presidents, Behnke and Kurepa, and the three Western European members of its relevant Executive Committees – Frostman, Maxwell and Piene were present at the meeting. Nowadays, Royaumont is usually remembered for Dieudonné's polemical talk and there is no doubt that this was to have an enormous effect on the reforms and experiments which were to follow. Yet those who have not taken the Royaumont report down from their bookshelves for some years might be surprised by the relatively mild (and perceptive) tone of many of the contributions and by the useful survey of contemporary practice in participating countries which it provides (OEEC, 1961a).

Royaumont was followed in 1960 by an OEEC seminar in Yugoslavia at which Stone, Kurepa, Artin and Choquet amongst others (including teacher trainers and schoolteachers) helped prepare guidelines on how new material

might be introduced to the grammar school (lycée, gymnasium) curriculum (OEEC, 1961b).

Not surprisingly, therefore, 'modern mathematics' occupied a prominent part in the discussions held at the 1962 ICM in Stockholm. There the three topics chosen for discussion were:

(1) Which subjects in modern mathematics and which applications of modern mathematics can find a place in programmes of secondary school instruction?

(2) Connections between arithmetic and algebra in the mathematical instruction of children up to the age of 15.

(3) Education of the teachers for the various levels of mathematical instruction.

Stockholm was to inaugurate a most important stage in ICMI's history; one in which, with the help of UNESCO and other bodies, it was to initiate a number of noteworthy activities. Work began on Volume 1 of *New Trends in Mathematics Teaching* (UNESCO, 1967), consisting of articles presented to various congresses, short accounts of other international meetings and details of curriculum projects and journals devoted to mathematics education. It is tempting to quote at length from some of the papers, but there is little to be gained from doing this – provided, of course, that nowadays educators and mathematicians have learned the lessons of the sixties: that hopes cannot always be translated into practice, that more can be taught than can be learned, that motivations for teaching do not automatically translate into motivations, for learning, ...

The number of international meetings increased rapidly. Within the academic year 1964–5, ICMI was involved in four major colloquia: at Frascati, Italy on 'Mathematics on entry to university; the present situation and that which is desirable'; at Utrecht, Netherlands on 'Modern trends in secondary school mathematics teaching'; at Dakar, Senegal, on 'The teaching of mathematics in relation to that of other sciences' (the first ICMI-sponsored meeting to be held in Africa); and at Echternach (Luxemburg) on 'The influence of mathematical research on teaching'.[9]

Perhaps, one quotation from a description of the Utrecht meeting (UNESCO, 1967, p. 378) merits particular attention:

'The crucial point of the colloquium was the confrontation of the ideas provoked by the lectures and the interventions of Professor A. Wittenberg, who stressed particularly the necessity of a precise pedagogical conception of the reform. He also underlined the dangers connected with a formal modernization which does not have an adequate basis in a clear consciousness of the objectives, the means of getting results, and a clear conception of education in general'.

Regrettably, mathematics education was soon to lose Wittenberg and with him one of the few knowledgeable, constructive and serious critics of the contemporary reforms.

Yet much more was happening within ICMI than the setting out of suggestions for what 'should be taught in secondary schools: Theory of sets, Relations, Group Theory, Integers, Complex Numbers, Trigonometry, Vector Space, Calculus, Differential Equations with constant coefficients, Linear and homographic groups, Analytical geometry . . . [to] be taught with all possible exactness and rigour . . . ' (Dakar Conference Report, UNESCO, 1967, p. 71). There were those who drew specific attention to the need to generate mathematical activity amongst pupils – not merely to catalogue the mathematics to be taught (for example, Cser, 1967). This interest was reflected in the choice of the first topic for discussion at the 1966 ICM in Moscow: 'The development of mathematical activity in pupils. The role of problems in this development'. The other two topics had titles more in keeping with previous surveys: 'The use of the axiomatic method in secondary school teaching' and 'The mathematical training of university physicists – is there a need for a separate course or not?'

During this period there were also changes in the way ICMI functioned and even more drastic proposals for what might happen. First it was agreed that a country which was not yet ready or equipped to take its place as a member of the International Mathematical Union might nevertheless become a member of ICMI. The first two countries to take advantage of this were Luxemburg and Senegal. This possibility still exists and indeed during May, 1983, Costa Rica and Mozambique were welcomed as new members of ICMI. Secondly, the Inter-American Committee for Mathematical Education which, with ICMI support, had organised its first Congress in Bogota in 1961 (IACME, 1962) asked in 1965 to be affiliated officially to ICMI and to be recognised as a regional committee; a recognition which was granted with effect from July, 1965 (Lichnerowicz, 1966). Together IACME and ICMI planned, with UNESCO's aid, to mount a congress in Lima, Peru in 1966. It was noted, however, that the financial resources generally available to ICMI were insufficient to meet its needs – 'it is one of the most thankless tasks of the excutive committee and its president to search for financial support from outside organisations'. Moreover, the future tasks which ICMI faced could not be accomplished with its structure as it then was (and still remains). Across the world there was a constant demand for information and the exchange of knowledge; this need could only be met through the establishment of a permanent secretariat, adequately financed and led by a competent specialist who devoted a major part of his time to ICMI duties. (Lichnerowicz, *op. cit.*, p. 138).

The call for a permanent secretariat was repeated by the new executive committee when it took office in 1967 (Delessert, 1967). This was but one of the many suggestions put forward at a meeting of ICMI held in Utrecht under its new President, Hans Freudenthal. Some, offered by the various sub-commissions, were quickly dropped, for example, that ICMI should write pilot texts for use in schools; some, such as the need for ICMI to identify the importance of the computer at all educational levels, are still being acted upon – and because of improved technologies will demand continual review; others, such as the organisation of international exchanges of mathematics teachers, remain good ideas which ICMI's structure prevents it from putting into practice.

Two suggestions were, however, to be accepted with marked effect. Freudenthal returned to the attack on the traditional ICMI reports presented at the ICMs (Delessert, *op. cit.*, p. 245) – "the national reports were generally of little significance". What was required, he argued, was a congress devoted solely to mathematics education, held in a different year to the ICM, at which invited talks could be given and opportunities for personal contributions presented. The idea was accepted and Maurice Glaymann proposed that the first such congress should be held in 1969 in Lyons, France.

André Revuz raised the problem of *L'Enseignement Mathématique* which had always been ICMI's official journal and which seemed on too high a level for secondary-school teachers. The need for other publications, possibly in several languages, was discussed, and a committee was established to consider the problem. In the event ICMI took no further action, but in the following year, Freudenthal himself launched the new international periodical *Educational Studies in Mathematics* and in the years immediately following it was this journal rather than *L'Enseignement Mathématique* which had the stronger links with ICMI.

The first volume of *Educational Studies in Mathematics* indeed consisted of the papers presented at an ICMI sponsored colloquium held at Utrecht in 1967 under the title 'How to teach mathematics so as to be useful' (ICMI, 1968a). *ESM* also published the recommendations of another important ICMI-sponsored meeting held that year in Lausanne on 'The coordination of the teaching of mathematics and physics' (ICMI, 1968b). Volume 2 was again largely devoted to ICMI-related matters: a report (edited by Freudenthal) on 'Mathematical Contests in Secondary Education (Olympiads)' (ICMI, 1969a) and the papers presented at the First International Congress on Mathematical Education held in August, 1969 at Lyons (ICMI, 1969b).

The Lyons congress was a landmark in ICMI's history. Over six hundred mathematics educators from forty-two countries met in an unprecedented fashion.[10] The lack of precedents was to tell against the effectiveness of the

congress, for its format did not encourage discussion and active participation. The meeting was built around twenty one-hour plenary lectures supplemented by a number of short (15-minute) contributions by congress members – the latter usually delivered to a handful of people sitting forlornly in a vast congress hall. Last minute attempts to arrange discussions met with mixed success – the members flocked to them in greater numbers than the rooms could accommodate! The discussions also revealed to its full extent what still remains a major problem for those organising international meetings on mathematics education – that of language. Simultaneous translation facilities can be made available for plenary sessions, but their cost is such that it can never be feasible to offer them in any but one hall. Yet mathematics education makes enormous linguistic demands of the speaker and hearer. We lack the international terminology and vocabulary of the mathematician and cannot resort to readily recognised and comprehended symbolism. So much depends on the precise and varied use of language – nuances that are not properly understood can so easily become trite, shallow statements when they are the victims of inadequate translation.

Lyons also had importance in that – perhaps unwittingly – it ushered in a period in which the national sub-commissions were asked to do less and emphasis within ICMI shifted from them to individuals. This, like the vast majority of all changes, produced both good and bad effects. Professionalism ultimately hinges on the performance of individuals and it must be admitted that not all national committees were representative or active. Yet the author remembers that his first contributions to ICMI were made in connection with the formulation of a response to the 1966 ICM 'topics'. Even though I did not visit the congress I was able, through the National Sub-Commission, to participate in discussions and in framing a national statement. The sub-commissions therefore were able to involve another stratum of educators in the work of ICMI – which were not represented at the ICM and cannot always be present at ICMEs.

An attempt actively to involve sub-commissions was made at the ICME which followed Lyons, that at Exeter in 1972 (ICMI, 1973) which attracted almost 1400 members from 76 countries. There emphasis switched from the 'set' lecture – there were only seven of these – to forty or so working groups and to national presentations. Seventeen national sub-commissions accepted the invitation to mount presentations; some, for example, those of the hosts, the United Kingdom, and of the U.S.A., formed almost mini-conferences in themselves, whilst others were more modest 'exhibitions'. Exeter, then, had the air of a 'World Expo' of mathematics education. Obvious consequences were that the congress was somewhat diffuse, members were faced with a multitude

of alternatives at every session only one of which they could attend, and that some activities and presentations lacked support. A reaction inevitably followed at ICME3 but by that time other important developments had taken place within ICMI.

The first of these was the appearance in October 1972 of *ICMI Bulletin No. 1*, an immediate response to requests made at ICME2. Since then the Bulletin has established itself as a useful means of communication even if its frequency of appearance has at times been somewhat erratic. *Bulletin No. 1* listed a number of forthcoming ICMI symposia: in Luxemburg, Hungary, Poland, Kenya, Japan, Denmark (later moved to Federal Germany) and India. Of these, that in Kenya deserves particular mention. The theme of the conference, which was very generously subsidised by UNESCO and the British Council, was 'Interactions between Linguistics and Mathematical Education'. It was a theme which clearly had – and still has – considerable meaning for African countries, many of which teach mathematics in English or French rather than the students' mother tongues. However, what so distinguished the seminar was the meticulous manner in which it was planned and prepared, the way in which relevant specialists from outside mathematics education were involved, its duration (eleven days), and the spread of its membership. All factors which, alas, are closely governed by the amount of funding available! Where it disappointed was in not making its papers and findings available in a more readily accessible form.[11]

UNESCO's assistance was also to play a major part in shaping ICME3 in 1976 at Karlsruhe, Federal Germany. As I have already hinted, ICME3 differed in many ways from its immediate predecessor. Again, little emphasis was laid on full plenary sessions, but on this occasion the congress was built around thirteen sections covering most aspects of education, each corresponding to a chapter in what was to be published as *New Trends in Mathematics Teaching Vol. 4* (UNESCO, 1979). The section themes provided a useful framework around which to consider mathematics education and indeed a somewhat similar design is to be used at ICME5. However, at Karlsruhe the aim of producing a book, perhaps, loomed too large and many members (the number now having grown to 1831 (ICME, 1977)) felt that they had insufficient opportunity to contribute. Once again there was to be a swing of the pendulum, and the Berkeley ICME of 1980 offered over 400 speakers drawn from 100 different countries to a membership of more than 2000 (Steen and Albers, 1981), as well as a variety of discussion groups and poster sessions.

Some of the working groups established at Exeter continued to meet at Karlsruhe and this led to another interesting development, for it was agreed that the International Group for the Psychology of Mathematical Education (PME)

and the International Study Group on the Relations between the History and Pedagogy of Mathematics[12] should become independent groups affiliated to ICMI. PME mounted its own congress in Utrecht, Holland in 1977, the year following Karlsruhe, and has since met annually at a wide variety of venues (see Burton, 1983).

The vast differences in the forms which the ICMEs have taken could be attributed to the national characteristics of the hosts. This, however, whilst not entirely to be dismissed is perhaps too facile an explanation. We have, in fact, witnessed four attempts – and plans for a fifth are now well-advanced – to deal with extremely difficult problems inherent in the nature of mathematics education. First, it is essential that standards of professionalism are established within our discipline and on that account ICMEs must be showcases in which work at an approved level is displayed. They cannot be allowed to become truly 'representative' until the general standards of research and thought have been raised. However, mathematics educators, unlike mathematicians, cannot speak universal truths. The context – social and mathematical – within which they work individually is usually such that their results and thoughts are not readily transferable. There is a need for translation, for mediation and for adaptation. Such actions can only take place as a result of questioning and discussion. ICMEs then must offer opportunities for interaction and the exchange of knowledge and ideas. There is a need to encourage participation, allied to that of establishing professional standards. It is in the weights that have been assigned to these two objectives and the procedures designed to attain them, that the ICMEs have shown the greatest measures of divergence.

Such thoughts, however, lead one more to the consideration of ICMI's future than its past. As we have seen, in its first 75 years ICMI has attained much, and, of course, there are many achievements – for example, the survey on 'Change in Mathematics Education since the late 1950's – Ideas and Realisation' (ICMI, 1978)[13] – which I have neglected to mention. The two decades from 1960 to 1980 were years which saw considerable activity – indeed, it was a major problem to keep track of the many activities taking place and reports which were generated. Now there is less public money available for symposia and travel: there is a need, therefore, to utilise those resources which we have, finance, information, goodwill and man-power, with particular care.

The status of mathematics education has fluctuated considerably, too, in those decades. During the 1960s governments welcomed the pleas of educators for 'mathematics for all'. In the 1970s disillusionment set in and the 'back to basics' movement began: much was heard of 'minimal competencies'. We are

now in a relatively quiescent period so far as governmental pressures on mathematics educators are concerned. Yet vast changes are taking place in society and in technology which demand responses from us. It is with such considerations in mind that the Executive Committee of ICMI is seeking to establish a number of 'studies' of topics of international interest and concern. The first four of these relate to the impact of the computer on mathematics and on its teaching in higher education; on current knowledge of cognition and of how teachers of mathematics might respond to this; on probable changes in education and the part within it that mathematics occupies which are likely to result from changes in society; and on mathematics as a service subject in higher education. The aim of such studies is not merely to give surveys of what is best in current practice or most up-to-date knowledge, but to provide frameworks within which national and regional discussion can take place. We hope that local discussions and work will supply input for, and benefit from the output of, these studies. They will provide, therefore, both opportunities for participation and also for the establishment and reinforcement of professional standards within a discipline which each year becomes more demanding and more important.

NOTES

* The views expressed in this article are those of the author and not necessarily those of the ICMI Executive Committee or its National Representatives.

[1] Purists might argue that it is CIEM (Commission internationale de l'enseignement mathématique) which celebrates its 75th birthday, for the initials ICMI are of relatively recent origin. Before the Second World War the Commission's English title was always the International Commission on the Teaching of Mathematics, in the 1950s it was first known as the International Mathematical Instruction Commission (IMIC); only in 1954 did it become the International Commission on Mathematical Instruction (ICMI).

[2] For details of the early history of ICMI see the *Proceedings* of the ICMs held in Rome and Cambridge (ICM, 1909; ICM, 1913), in particular Fehr's article 'La Commission internationale de l'Enseignement mathématique de 1908 à 1912' to be found in ICM (1913, pp. 591–597). Other accounts are also to be found in national journals, e.g., Godfrey (1912), and, of course, in *L'Enseignement Mathématique*, 1908–1915, Series 1, Vols. 10–17.

[3] A full list of publications of the Commission and of the National Sub-Commissions during the years 1908–1920 can be found in *L'Enseignement Mathématique*, 1920, Series 1, Vol. 21, pp. 319–339.

[4] We note that Finland and Poland, though at that time part of Tsarist Russia, prepared separate volumes which were included as part of the Russian contribution.

[5] See, for example, Godfrey (1913) and *L'Enseignement Mathématique*, 1914, Series 1, Vol. 16.

[6] The dominance of mathematicians (*vis à vis* mathematics educators) on ICMI committees has been a frequent cause of disquiet and has led to several changes in the methods for electing ICMI's officers.

APPENDIX: OFFICERS OF ICMI[14]

Date	President	Vice-President	Secretary	Executive Committee
1908	F. Klein	G. Greenhill	H. Fehr	
1912	F. Klein	G. Greenhill D. E. Smith	H. Fehr	G. Castelnuovo, E. Czuber, J. Hadamard
1928	D. E. Smith	G. Castelnuovo J. Hadamard	H. Fehr	W. Lietzmann
1932 (and 1936)	J. Hadamard	P. Heegaard W. Lietzmann G. Scorza	H. Fehr	E. H. Neville
1952				H. Behnke, A. Châtelet, H. Fehr, R. L. Jeffery, G. Kurepa
1954	A. Châtelet	G. Kurepa S. MacLane	H. Behnke	A. F. Andersen, G. Ascoli, E. W. Beth, R. L. Jeffery, E. A. Maxwell
1955–58	H. Behnke	G. Kurepa M. H. Stone	J. Desforge	Ram Behari, E. A. Maxwell, K. Piene
1959–62	M. H. Stone	H. Behnke G. Kurepa	G. Walusinski	Y. Akizuki, A. D. Alexandrov, O. Frostman
1963–66	A. Lichnerowicz	S. Straszewicz E. Moise	A. Delessert	Y. Akizuki, H. Behnke, H. Freudenthal
1967–70	H. Freudenthal	E. Moise S. L. Sobolev	A. Delessert	H. Behnke, B. Thwaites, A. Revuz
1971–74	M. J. Lighthill	S. Iyanaga J. Suranyi	E. A. Maxwell	H. Freudenthal, H. O. Pollak, S. L. Sobolev
1975–78	S. Iyanaga	B. Christiansen H. G. Steiner	Y. Kawada	E. G. Begle, L. D. Kudrjavcev, M. J. Lighthill
1979–82	H. Whitney	B. Christiansen U. D'Ambrosio	P. J. Hilton	S. H. Erlwanger, B. H. Neumann, Z. Semadini
1983–86	J.-P. Kahane	B. Christiansen Z. Semadini	A. G. Howson	B. F. Nebres, M. F. Newman, H. O. Pollak

[7] Readers who know Hadamard's *Psychology of Invention in the Mathematical Field* may be surprised that ICMI did not lay more stress on the psychological aspects of learning. However, Hadamard's interest in psychology really grew from a meeting held in Paris in 1937 and his book was not published until 1945 when he was eighty (almost fifty years after his famous proof of the prime number theorem).

[8] The questionnaire is reprinted in *L'Enseignement Mathématique*, 1931, Series 1, Vol. 30, pp. 291–296; the results in Volumes 31 and 32. Articles by Fehr setting out the goals for the Commission for the periods 1929–1932 and 1933–1936 are to be found in Volumes 28 and 30 respectively. A list of publications by the Commission and its Sub-Commissions in the inter-war years can be found in Volume 39, pp. 166–168.

[9] Details of these meetings can be found in UNESCO (1967).

[10] *ICMI Bulletin No. 5* (1975) contains an interesting analysis by J. P. Becker of the participants at ICMEs 1 and 2.

[11] A brief report (UNESCO, 1975) was made available through UNESCO as were a limited number of the working papers prepared for the symposium.

[12] See Freudenthal (1978) for the inaugural address given at the Utrecht meeting of PME. Papers given at the meeting are also to be found in Volume 9 of *Ed. Stud. Math.*, as is (pp. 94–95) a description of the establishment and aims of the study group on the history of mathematics.

[13] It could be argued that this interesting exercise suffered from the same weaknesses as earlier ICMI surveys in that having collected the data and presented it in a readable form, ICMI left the analysis of the reports to individual and/or chance. Thus, for example, by an unfortunate lack of liaison, the opportunity was not taken to form a working group at the Berkeley ICME to consider the reports and their implications for curriculum development worldwide.

[14] While compiling this appendix I was greatly helped by the bibliographic references in de Rham (1976).

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