Use of Mathematical Software for Teaching and Learning Mathematics

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ABSTRACT

The computer algebra systems (CAS) such as Mathematica, Maple, MuPAD, MathCAD, Derive, Maxima have potential to facilitate an active approach to learning, to allow students to become involved in discovery and to consolidate their own knowledge, thus developing conceptual and geometrical understanding and a deeper approach to learning. Emergence of such mathematical tools and its ability to deal with most of the undergraduate mathematics cannot be ignored by mathematics educators. Use of Computer Algebra Systems in mathematics teaching is in its infancy in India.

The main idea of this paper is to give introduction to computer algebra systems, its advantages and disadvantages in mathematics teaching. We include our experiment and experiences in Mumbai University, India, where an attempt was made to include CAS-based practicals at the final year under graduate mathematics course. However this experiment did not really work. We look at some of the reasons due to which this experiment did not work and the lessons we learned from this experiment. We also mention some of the challenges one faces in the deployment of CAS in teaching mathematics and some steps to be taken to overcome these challenges in India. Some of our experiences may also be useful to mathematics educators from other developing countries, which lack the necessary infrastructure and technical expertise to implement these ideas.

We believe that mathematics teaching can be made much more interesting, inventive and exploratory using CAS. We include a small module developed using a MuPAD Pro to support our claim. The role of teachers is very important in order to make the effective use of available mathematical tools.

1. INTRODUCTION

Few working in mathematics education today would be unaware of the growth in recent years of computer technologies for teaching, learning and research in mathematics. Calculating technology in mathematics has evolved from four-function calculators to scientific calculators to graphing calculators and now to computers with computer algebra system software. The use of CAS in education is still relatively rare but the growing body of research and the interest suggests that its extended use is imminent.

The underlying concepts and proofs of many mathematical concepts involve difficult and abstract ideas that present a mountainous obstacle to many students. Computer algebra systems offer both an opportunity and a challenge to present new approaches that assist students and teachers to develop better understanding of the concepts. They can be used to change the emphasis of learning and teaching of mathematical concepts away from techniques and routine symbolic manipulation towards higher-level cognitive skills that focus on concepts and problem solving. Two of the key indicators of deep learning and conceptual understanding are the ability to transfer knowledge learned in one task to another task and the ability to move between different representations of mathematical objects. Computer algebra systems are multiple representation systems and they have the ability to facilitate graphical, algebraic and numerical approaches to a most of the mathematical concepts. Most of the CAS also provide a high-level programming language which helps the users to prepare their own set of library files to suit their needs. CAS thus allow learners to discover rules, to make and test conjectures and to explore the relationship between different representations of functions and other mathematical objects using a blend of visual, symbolic and computational approaches. Students enjoy the power and versatility of computer algebra and are encouraged to become reflective, deep learners.

While use of CAS in many countries in teaching and learning mathematics have made a significant impact at University level, in India the progress and awareness of these technology has been really very slow. Mostly, it has been confined among the researchers and handful of university and college teachers in well established research institutes, IIT's and University Departments. In this article we look at the advantages and disadvantages of using these tools in teaching mathematics at undergraduate and postgraduate levels. We also look at some of the challenges and hurdles in using these tools in India and how to overcome them. We present a Mumbai chapter on use of these technology where an attempt was made to implement these tools at under graduate level partially. However, this has not really made an impact because of several hurdles.

2. HISTORICAL PERSPECTIVE

Computer algebra systems began to appear in the early 1970s, and evolved out of research into artificial intelligence. Pioneering work was conducted by the Nobel laureate Martin Veltman, who designed a program for symbolic mathematics, especially High Energy Physics in 1963. The first popular systems were Reduce, Derive, and Macsyma which are still commercially available. A free version of Macsyma called Maxima is actively being maintained. The current market leaders are Maple, Mathematica, MatLab, SciLab and MuPAD. These are commonly used by mathematicians, scientists, and engineers. Some computer algebra systems focus on a specific area of application; these are typically developed in academia and are free.

Here is a list of some of the most popular free and commercial mathematical software. More informations on these can be found on their respective websites.

Software	*Year of Start*	*Utility*
Mathematica*	1998	General purpose CAS
Maple*	1985	General purpose CAS
MuPAD*	1993	General purpose CAS
MatLab*	Late 1970	General purpose CAS
MathCAD*	1985	General purpose CAS
Magma*	1993	Arithmetic Geometry, Number Theory
SciLab	1994	General purpose CAS
Maxima	1998	General purpose CAS
YACAS	1999	General Purpose CAS
SAGE	2005	Algebra and Geometry Experimentation
Macaulay2	1995	Commutative Algebra, Algebraic Geometry
GAP	1986	Group Theory, Discrete Math
GP/PARI	1985	Number Theory
Kash/Kant	2005	Algebraic Number Theory
Octave	1993	Numerical computations, Matlab-like
Singular	1997	Commutative Algebra, Algebraic Geometry
CoCoA	1995	Polynomial Calculation
Gnuplot	1986	Plotting software
Dynamic Solver	2002	Differential Equation
R	1993	Statistics

Here star (*) ones are commercial software and remaining are free software. Note that the above list is not complete still and there may be many more mathematical software.

3. INTRODUCTION TO CAS

Computer algebra systems (CAS) are special kind of mathematical applications providing users means for doing symbolic, algebraic and graphical manipulations with computers. This means that instead of only counting with numbers, computer algebra systems can also manipulate symbols and, when possible, carry out complex calculations exactly. These systems can be roughly divided into two main categories: special purpose systems and general purpose systems. Special purpose systems usually deal with some specialized branch of mathematics, viz. dynamical solver for differential equations, singular for algebra and algebraic geometry, KASH for algebraic number theory, gap for group theory, magma for number theory, CoCoA, Macauly2 for commutative algebra/algebraic geometry, Octave for numerical computations etc. General purpose system, on the other hand, usually try to cover as many mathematical areas as possible. This generality makes general purpose systems ideal for open learning environments in most cases.

Most CAS allow the user to write sequential programs for complex tasks, and have all features of high-level programming languages. CAS also have most of the features of numerical systems for visualization of 2D and 3D-plots, numerical computations and animations. It is therefore an ideal tool for directing learning towards multiple-linked representations of mathematical concepts. Through carefully designed activities students can investigate the links between different representations of objects, recognize their common properties and begin to construct their personal structures of mathematical knowledge. Student activities have to be designed with very detailed cognitive steps in mind. Appropriate teacher intervention will usually be required to ensure that the students follow through the required learning stages, in particular, the reflective thinking.

A typical student approach to problem solving is to find a suitable worked out example to mimic and then carry out the computation. Clearly this strategy is limited by the extent of the students' memory bank of similar problems and inhibits flexible thinking. A better approach is to consider alternatives, experiment, conjecture and test, then analyze the results. A computer algebra system can be a major factor in developing an exploratory approach to learning mathematics and, in particular, investigating problems from multiple representational perspectives. Using CAS to produce graphs, carry out calculus operations or perform repetitive calculations, students can be encouraged to make and test conjectures, to consider alternative solutions and to tackle open-ended problems. Removing the burden of manipulation and computation allows students to spend the more time on these other activities. This approach can make the study of mathematics more enjoyable, more relevant and more rewarding to it. At present most of their time is spent practicing routine skills. Perhaps it is not surprising that students view mathematics as a collection of formulae (to be memorized) and to do maths is to compute. If more routine computation is done on a computer more time is available for concentrating on concepts, motivation, applications and investigations.

With the traditional undergraduate curriculum, students do not often regard themselves as active participants in mathematical exploration. Rather they are passive recipients of a body of knowledge, comprising definitions, rules and algorithms. Computers offer a number of didactic advantages that can be exploited to promote a more active approach to learning. Students can become involved in the discovery and understanding process, no longer viewing mathematics as simply receiving and remembering algorithms and formulae. The power of computer algebra goes beyond routine computation. It has the potential to facilitate an active approach to learning, allowing students to become involved in discovery and constructing their own knowledge, thus developing conceptual understanding and a deeper approach to learning.

We include a sample output using MuPAD Pro 3.1, which explains the geometricmeaning of Lagrange multipliers to solve constrained optimization problem.

Example 1. Use the method of Lagrange Multipliers to maximize/minimize

$y - x^2$ ubjected to $y^2 + x^2 = 2$

For convenience let $f(x,y) = y - x^2$ and $g(x,y) = 2x^2 + y^2 - 2$. Geometrically, the maximum/minimum of the above problem occur where ever the gradient of $f(x,y) = y - x^2$ and gradient of $g(x,y) = 2x^2 + y^2 - 2$ are parallel. This is same as, the level curves of f and b have common tangents at these points.

Using MuPAD animation, we can show that there are four points on the ellipse $g(x,y) = 0$ at which this happens.

>>f:=(x,y)-> y-x^2//to define the function f;

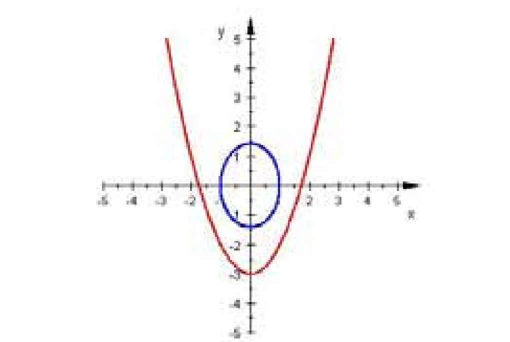
>>g:=(x,y)->2*x^2+y^2-2//to define the function g

>>pf:=plot::Implicit2d(f(x,y)=c,x=-5..5,y=-5..5,
c=-3..3,Color=RGB::Red,Frames=100,LineWidth=0.5)

>>pg:=plot::Implicit2d(g(x,y)=0,x=-3..3,y=-3..3,Color=RGB::Blue,Line Width=0.75)

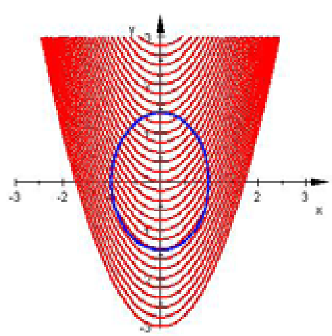
>>plot(pf,pg,Scaling=Constrained);

The output is shown in the figure below

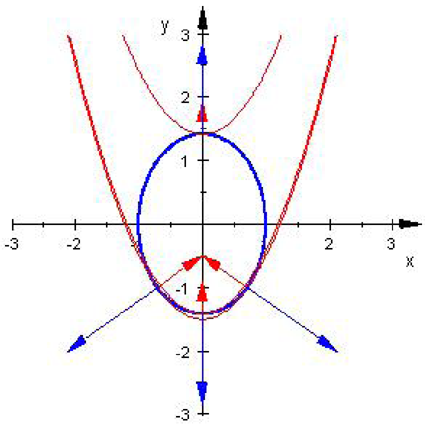


When we animate the graph we see that there are four points at which the level curves of f and g have common tangents. This is shown in the next figure.

>>plot(plot::Implicit2d(f(x,y)=c/5,x=-3..3,y=- 3..3,Color=RGB::Red,LineWidth=0.5,
VisibleAfter = c)c=-15..15,pg,Scaling=Constrained)



Now we can plot (using MuPAD) the gradient at the point at which the level curves f and g have common tangent. Look at the Figure below. We are suppressing the MuPAD codes which produced this figure.



All the steps to solve the above problem can be performed using MuPAD and it can be shown that there are four points and at which gradient f and g are parallel. We are not including the MuPAD codes for the analytic solution of the above problem, as we wanted to bring out the geometric behind the problem.

4. ADVANTAGES OF USING CAS

1. Helps develop visual/geometrical understanding.
2. CAS can help to increase the value of the knowledge and degree of interest of students.
3. Can explore concepts before "hand skills" to do so are available.
4. Can explore realistic problems.
5. CAS help to increase student motivation and improve students attitudes towards Mathematics.
6. Due to the potential interactivity of these tools, students are able to attain a higher level of abstraction in mathematical problem-solving something which clearly represents a significant didactic accomplishment.
7. Allows students to concentrate on problem formulation and solution analysis.

8. Easy to give math demos and advanced mathematical ideas can be introduced very easily and concretely.
9. Users having knowledge of some programming language (C, C++, Pascal, Fortran) have greater advantage and can prepare their own library function which are suited to their needs.
10. It will help teachers to develop innovative, challenging and exploratory teaching modules.
11. Researchers do not need to spend more time on tedious computations rather they can spend more time in analyzing and the computation part can be easily be done using these tools.
12. When CAS are not used, the teacher tends to be the sole center of attention whereas, when they are used, there is an observable increase in student participation, autonomous activity and interaction among students, hereby making the process of acquiring and constructing mathematical knowledge more student-centred.

13..Enhances job opportunities for students.

14..People from other disciplines not having sound mathematical knowledge can very easily solve mathematical problems which they come across.

The benefits of using CAS in mathematics teaching is enormous and almost every conference on technology for mathematics advocates this. For more detailed discussion one can refer to ((Albano G., Desiderio M. 2002) , (Artigue M. 2001)}, , (Bertemes J. 2006), (Bohm J. CAME 2007), (Mackie D. 1996), (Majewski M., 2004), Westermann T. 2000), (Yearwood, J.U. A. , 1996) etc.)

5. DRAWBACKS OF USING CAS

In spite of so many benefits of using CAS there are some drawback, that is why many people advocate against its use and raises some concerns. Through our experiences and discussion with teachers and students, we are listing some of the drawbacks of using CAS.

1. Students tend to use CAS blindly and they do not bother about thevalidity of answer obtained through CAS.
2. Most often students try to use CAS as an advanced calculator and refuse to learn concepts.

3. Decline of students' paper-and-pen skills.
4. Difficulties in evaluation of a course taught using CAS.
5. Greater time needed for class preparation.
6. Lack of familiarity with the computer and CAS.
7. Fear of making syntactical errors in class.
8. Lack of administrative recognition of increasing teaching load.
9. CAS syntax can be an unreasonable burden on students.
10. The course can be victimized by equipment failure or inadequate equipment.
11. Students' algebraic manipulation skills will deteriorate if they are allowed to rely on computer algebra but that these skills are an essential foundation for mathematics.
12. CAS at time can produce meaningless expressions.
13. Using CAS can potentially prevent students from making the proper connections between the techniques used and their mental approach to Mathematics.

6. CHALLENGES AND HOW TO OVERCOME THOSE

There are several challenges if we want to implement CAS-based mathematics teaching in India. However, these challenges can be over come. We list some of the major challenges which we/will come across in order to employment of such tools in mathematics teaching.

6.1 Challenges and Difficulties

1. Availability of computers in the laboratory and to teachers and students is still a distant dream.
2. Most of the CAS are too costly and hence not affordable to college students and teachers.
3. Classrooms are not equipped with relevant hardwares which is required to integrate teaching using CAS.
4. Teachers are not having proper computer literacy and knowledge of CAS.
5. Many teachers are not willing to move from traditional teaching style to CAS-based teaching wherever necessary.
6. Unavailability of innovative and exploratory teaching modules.
7. Courses are not designed properly. It does not give space, time and opportunity of exploring the subject using CAS.

6.2 Overcoming these challenges and difficulties

1. All colleges/institutes to have proper computer labs and to give students enough opportunities to explore.
2. Use of free mathematical software like Scilab, Maxima, octave etc. to be encouraged.
3. Development of similar software may be initiated and encouraged.
4. Classrooms should be equipped with relevant hardwares.
5. A series of teacher-training programmes throughout the country may be initiated in order to make them aware of such tools.
6. Innovative teaching modules and projects be prepared which make students and teachers realize that these tools are not merely advanced calculators but can be used to solve a very complex problems and help them to experiment and explore (one of the vital aspects of learning).
7. Courses may be redesigned to encourage the use of CAS and also provide time for its use.
8. Students must be allowed sufficient time to learn the language and features of CAS before using it to enhance their learning.
9. In recent years most of the students have knowledge of some programming language which will be very useful in order to experiment and explore not just existing inbuilt function in CAS but can create their own need based functions. This also fosters creativity.
10. CAS should not be used as a black-box in the beginning of introduction of a mathematical concept. Till the topic is not learnt properly, CAS should be used as a white-box. Once the topic is thoroughly learnt then it can be used as a black-box. Black-box/White-box principle (Buchberger B.,1990) is very useful for developing innovative teaching modules using CAS.

We believe that the most appropriate approach involves using programming and CAS together to allow students to create the specific necessary functions that will allow them to solve the problems involved in the subject matter under study.

7. CAS AND TEACHERS

It goes without saying that the classroom teacher is the key to the successful introduction of new methods and new technologies. Of course, it is possible for the student to come across these independently in the case of CAS. With the increasing speed of technological development, it is crucial that teachers keep themselves informed so that they are in a position to make valid judgments and adapt their teaching accordingly.

Teachers, of course, have a crucial role in students learning (with or without CAS). Integrating CAS into teaching changes many aspects of classroom practice which teachers will make on the basis of their prior teaching styles and their beliefs about mathematics and how it should be taught. While using CAS to solve problems, students sometime make silly mistakes which produces a totally irrelevant output.

Teacher support and appropriate intervention is crucial to correct such mistakes. Judging the right amount of help at the right time is a skill acquired through experience. Computer algebra system use in mathematics teaching and learning is in its infancy. Nevertheless there are many teachers and educationalists who have integrated CAS into their teaching or conducted research into student understanding with CAS or who have led curriculum/assessment projects involving CAS use.

8. USE OF CAS-- A MUMBAI CHAPTER

Use of computer algebra systems (CAS) at the University of Mumbai was initiated in late 1990's by means of workshops integrated with refresher courses for degree and engineering college teachers at the Department of Mathematics, University of Mumbai. Initially teachers who attended refresher courses were made aware of some of the mathematical tools mainly mathematica, WinPlot and MuPAD for teaching mathematics. Because those days only few computers were available in computer lab, occasionally they were given hands-on practices in groups. It was in the year 2003, a three days workshop on use of MuPAD 2.5 Lite and other related free mathematical software was held for Mumbai University degree college teachers teaching mathematics, keeping in mind to encourage the use of some of mathematical tools in mathematics teaching at college level. Teaching modules for few mathematical topics in analysis, multi-variable calculus, linear algebra were prepared to help the participants. In the beginning there were some concerns that many teachers may oppose this move, however after attending the workshops all were very happy and very keen to use them. About 100 teachers participated in this workshop very enthusiastically. The participants were also given hands-practices in different groups. All the teachers were very happy to see the kind of innovations and motivations that can be inculcated in teaching mathematics using these tools. Board of studies of Mathematics of Mumbai University then recommended that it will be compulsory for the final year students of mathematics to include printouts of solutions of two problems using MuPAD or any other mathematical software in each of the four papers in their syllabus. The main idea behind this endeavor was to expose the teachers and the students to some of these tools which will help in understanding and visualizing many mathematical concepts.

However, we believe that this has not worked properly. There are number of reasons behind this:

1. Most of colleges did not have required ambiance for teacher to integrate the CAS with their teaching.
2. College computer laboratory was also not available for this purpose in most of the colleges.
3. Most of the teachers themselves did not have access to computers at their college and their residence.
4. Teachers did not take interest in exploring and experimenting with these tools themselves and did not encourage their students to experiment these tools.
5. There were no follow-up workshops any further.
6. MuPAD Lite 2.5 is no longer freely available.

Due to the above difficulties in most of the cases students were just reproducing same solutions again and again and the original idea in our opinion got defeated. Few workshops in Mumbai at University Institute of Chemical Technology (2005), Indian Institute of Technology (2006 and 2007) were held to make the teachers, research scholars aware of the some of these technologies however there are not enough.

With the insights provided by this experience, we can improve the strategy/methodology of deployment of CAS in Mathematics teaching and move forward. We believe that the situation now has improved considerably. Most of the colleges do have relevant hardware, good computer laboratory where students and teachers can experiment, explore and discover using some of the CAS. Thus, if proper guidance is provided, these technology can make an impact and mathematics learning can become much more interesting and enjoyable.

9. WHERE DO WE STAND?

Many foreign universities have fully integrated CAS into mathematics teaching for several university degrees, to the extent that their use is no longer considered to be novel or innovative, but rather something common place in such courses. CAS-based mathematics teaching at the undergraduate and postgraduate level has not been explored in India much. Therefore, there is a lot of scope for improvements. In recent years many government funding agencies have provided financial support to setup computer laboratories in colleges and to acquire useful software. Therefore, we believe that the situation now is far more conducive than what it was few years back, in order to make these tools as a part of our curriculum and make teaching and learning process much more interesting, insightful and make students involved.

The authors had opportunity to interact with many young college teachers and students of undergraduate and postgraduate level of various universities who were very enthusiastic to learn these tools and incorporate them into their teaching. This makes the implementation of CAS-based mathematics teaching and learning much easier. What we need is to create proper awareness of these tools among the teachers by holding workshops and training programmes at various places. One of the good things about all these tools is that they have a very good inbuilt documentation, tutorials which make the learning much easier. Already tonnes of tutorials, lessons are available on the web which can be used for self-learning.

10. CONCLUSIONS

A computer algebra system is a tool not a self-contained learning package or encyclopaedia of mathematical knowledge. It is the way in which it is presented to and used by students that determines its ability to influence learning. Much emphasis these days is placed on student-centered learning and less on the teaching but teaching and learning are equally important. It is necessary to first understand the learning process and then design teaching and learning activities to achieve these. Only then will students become deep learners.

Our accumulated experience reveals that CAS are computer tools which are easy to use and useful in both pure and applied mathematics courses. Use of CAS in the teaching of Mathematics should be channelized to maximize the opportunities offered by CAS technologies. Optimal use should be aimed at improving student motivation, autonomy and achieving participatory and student-centered learning. One powerful idea involves combining CAS resources with the flexibility of a programming language.

There are many implications of using computers in the teaching and learning of mathematics at university. As students often point out to us it is very exciting, enjoyable and productive to use computers in class. They are keen to use computers, so the environment becomes more conducive for learning. Students' natural curiosity can be utilized to its fullest potential because they are keen to explore and discover.

Irrespective of the software packages used, it is important to remember that the software should support the learning and curriculum and can not substitute good teaching. Traditional teaching methods must be supported with modern tools for problem-solving. It does not imply a reduction in the standard of education or of necessary subjects, but it is vital that the curriculum is carefully considered and that passive teaching is replaced in favour of new methods which promote active participation of students.

In order to make the CAS based mathematics teaching reality, we must take some of the following measures:

1. To develop methodology for teaching mathematics with CAS.
2. To develop strategies to implement teaching methodologies.
3. To produce innovative teaching modules using CAS.
4. To organize regular workshops, training programmes for mathematics teachers.
5. To redesign the course curriculum.
6. A lot of research is needed to understand the students attitude and psychology of learning mathematics using CAS.

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