DG 21: Current problems and challenges in lower secondary mathematics education

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1. Introduction

This DG had two goals: (i) to facilitate an informed and constructive discussion about the pressing issues associated with the teaching and learning of mathematics at the lower secondary level (students from ± 12 years to ± 15 years of age), and (ii) to provide a forum for participants to share any policies, strategies and/or approaches that are useful in addressing these issues.

Contributors were asked to address the following questions: What are the most important current problems and challenges pertaining to the teaching and learning of mathematics at this level? What issues impact the mathematics learning experiences of students? What mathematics is seen to be critical or essential at this level of schooling? What is the role of language, technology, and culture in the provision of and access to quality mathematics instruction? What dilemmas confront teachers of mathematics at this level? What are the implications for pre-service education, professional development, school organisation, and curriculum resources? How should these problems, challenges, issues, and dilemmas be addressed? What has worked or is working, where and why?

Eight contributions were received and published on the ICME11.

Banerjee, R.	Assessing the curriculum reforms in India: The case of integers and algebra for beginning middle school students
Ellerton, N. & Clements, McK.	La Villita: The return trip from Normal to Infinity—Supporting a problem-based mathematics program
Grugeon-Allys, B.	An example formative assessment to identify specific starting points for teaching algebra
Ingram, N.	Affective issues in mathematics engagement
Knudson-Martin, J.	Impact of diversity and school structure on achievement
Siemon, D.	From additive to multiplicative thinking—A major challenge in lower secondary mathematics education
Vale, C.	Diversity and differentiated curriculum
Yuwen, Li	Stimulating creativity: Open mind questions and creativity education in mathematics

In planning the three sessions, contributions were grouped under three themes: (i) the impact of culture, minority status, and school structure on student access and achievement; (ii) the use of formative assessment to support more targeted teaching; and (iii) the impact of curriculum reform on students' mathematics learning. While separated for the purpose of structuring the programme and clarifying key points, the contributions all acknowledged the enormous range in student ability and shared a common concern for students left behind or marginalised at this level of schooling. As a consequence, the sessions were linked. These issues are briefly summarised below under the themes which initially prompted them.

1. Factors affecting access and achievement

Two contributions reported work with underachieving or disadvantaged school populations in North America (Ellerton & Clements, Knudson-Mack). The third reported on the need to recognise affective issues in mathematics learning and the teacher's critical role in

helping students build ‘mathematical identities’ (Ingram). A number of overlapping challenges were identified and discussed.

1.1. Dealing with difference

A major issue that resonated with many was the issue of how to deal with diversity at this level of schooling. Knudson-Mack reported on the negative impacts of assigning students to particular classes on the basis of their performance in mathematics, that is, ‘streaming’ or ‘tracking’. He also reported that Latino students were disproportionately represented in the lowest streams of non-College preparation tracks and fell further behind with little opportunity to access the knowledge and skills needed for further courses. This prompted discussion on school organisation to support the learning needs of all students, particularly those from non English speaking backgrounds (NESB) who were often mis-assigned to ‘remedial’ classes in mathematics. Knudson-Mack asked how other countries, school systems catered for students identified as ‘other’ and posed the question, ‘Should we expect these students to adjust to the curriculum or should schools adjust the curriculum to these students?’

Ellerton and Clements were invited to review the effectiveness of a problem-based mathematics programme introduced by a school in a disadvantaged area of Chicago as a means of improving low academic performance and poor high school completion rates.

> The term “problem-based” was interpreted by the school as providing a set of culturally and locally relevant experiences that would enable the students to learn the key areas of the mathematics curriculum. (Ellerton & Clements, p. 2)

The programme was largely developed by the two mathematics teachers with a view to increasing student engagement in mathematics and developing a broader range of competencies. However, while there was evidence that the students demonstrated gains in problem-solving skills and enjoyed the more culturally appropriate problem situations, concerns remained about their acquisition of basic skills of the kind needed for the state-wide testing.

In the discussion that followed, the concept of ‘complex instruction’ (for example, Boaler, 2006; Cohen et al, 1999) was suggested as a way forward. Mixed-ability, cooperative group work on conceptually challenging and intellectually rich learning tasks were felt to offer the best means of ensuring all students could engage with the learning materials and make some progress. Another strategy was the use of formative assessment to identify starting points for more targeted teaching approaches. This issue was taken up in the second session.

1.2. Normative testing

State or system-wide mathematics testing generated considerable debate. All three papers referred to the debilitating effects on students’ self-esteem and confidence. Ellerton and Clements, discussing a problem-based approach to learning mathematics, expressed concern about the gap between the skills and attitudes learnt as a consequence of this programme and the skills that are routinely tested on systemic tests of performance. Questions were raised about the messages these forms of assessment conveyed about what was valued in mathematics education.

Such forms of assessment have an impact on the curriculum and pedagogical decisions made by teachers and schools. Tests provided a de facto standard of what was valued and so courage was needed to divert from the mandated curriculum or selected text or prescribed pace of teaching. A greater awareness is needed of the impact of the work of Jo Boaler and her colleagues which has demonstrated that ‘reform’ approaches which cater more appropriately for individual learning needs have been shown to produce better results on these forms of tests

1.3. Meeting the learning needs of adolescent learners

A major challenge at this level of schooling is the issue of adolescent learners, in particular their self-concept and the importance of group belonging. Ingram’s paper drew on the work of Sfard and Prusak (2005) in relation to a student’s actual and designated identities and pointed out the teacher’s critical role in helping students’ build identities as capable learners in a group of capable learners. A useful strategy she pointed to was assigned seating plans, as

students at this level were unlikely to move away from interfering peers of their own volition for fear of social ridicule or rejection. This strategy is consistent with the notion of 'complex instruction' and well-constructed cooperative learning experiences.

Targeting teaching to individual needs in ways that did not isolate them socially was suggested, such as offering additional workshops on particular topics which students could choose to participate in. Cross-age tutoring was suggested as a means of engaging learners with key ideas or strategies from an earlier level or multi-age, mixed ability organisations.

2. Use of formative assessment to inform teaching

The three contributions in this session were focussed on addressing the enormous range in student ability at this level of schooling. Vale made the point that while streaming and/or tracking were largely discredited strategies, teachers often lacked the confidence and skills to deal with the expanded range of learning needs in mixed-ability classrooms. The remaining contributions reported on the use of formative assessment tools designed to identify student learning needs in terms of multiplicative thinking (Siemon) and the transition from arithmetic to algebraic thinking (Grugeon), two areas of mathematics curricula at this level most responsible for the very large range in student ability. The issues discussed are summarised below.

2.1. Constructing quality formative assessments

Siemon and Grugeon presented data to support of the view that students' capacity to engage flexibly with an extended range of concepts and strategies for multiplication and division and their ability to work meaningfully with symbols and generalisations were critical success factors at this level of schooling.

Drawing on a large-scale research project, Siemon illustrated the rich tasks and scoring rubrics used to establish a Learning and Assessment Framework for Multiplicative Thinking (see Siemon et al, 2006). Grugeon constructed a "multidimensional model of students' expected algebraic competence" (p. 1) from research findings. In both cases, the notion of a hypothesised learning trajectory (Simon, 1995) informed the development of the diagnostic test items and rich tasks. In Grugeon's case, this resulted in an adaptive, computer-based diagnostic tool (see http://pepite.univ-lemans.fr) to evaluate student's algebraic competence and situate

> the student along four dimensions: (1) relationship between arithmetic and algebra, approached through the meaning of letters (unknown, variable, generalized number, abbreviation or label) and the status of equality sign, (2) algebraic calculus, (3) translation between various representations (graphical, geometrical, algebraic, natural language) and (4) type of justifications (proof by example, proof by algebra, proof by explanation, proof by incorrect rule) (Grugeon, p. 1).

She reported that the length of this test was a limitation and invited participants to suggest aspects that are more critical than others in determining students' algebraic competence.

For Siemon and her team this involved the development of rich tasks designed to evaluate key aspects of multiplicative thinking which allowed all students to make a start and revealed students' capacity to reason mathematically. Rasch modelling was used to identify eight discrete categories of multiplicative thinking. Where teachers were supported to use the Learning Assessment Framework to better target their teaching, student performance improved, particularly for those students at the greatest risk of 'falling behind'.

A number of issues were raised in discussion. One was the relationship between engagement and learning. While it was agreed that not much learning could take place if students were disengaged, it was also acknowledged that students could be engaged and interested but not deepening their understanding of important mathematics concepts and skills. This prompted discussion about suitable tasks (for example, http://connectedmath.msu.edu/) and the teacher knowledge needed to use such tasks. A number of participants felt that engagement at any price was worth it in terms of changing students' perceptions about the nature of mathematics teaching and learning as this was seen as a first step in re-orienting students to

learn mathematics. Another issue was the importance of identifying student learning needs in relation to the 'big ideas' at this level. According to Siemon, this requires assessment techniques that expose students' thinking and provide information about what each student knows already and what might be within his/her grasp with some support from teachers and/or peers. However, it also needs teachers to understand what different student responses might mean, and to have access to some practical ideas and strategies to address the learning needs identified.

While there was a comment that there was a danger in viewing progress in mathematics too much in terms of a 'one-way street', it was agreed the research-based frameworks of the types presented which incorporated rich descriptions and allowed for parallel journeys offered a useful way forward. Two useful notions arose in the context of this discussion, namely, the idea of 'learning landscapes' and 'cognitive geography', which emerged as a means of moving away from a one-size-fits-all approach to curriculum.

2.1. Differentiating instruction

This was seen to be a major issue at this level. The group returned to the discussions about complex instruction and the difficulties this imposed for teachers. Vale described and illustrated the notion of a 'tiered curriculum' as one means by which different learning needs and styles might be accommodated purposefully in mixed-ability settings.

> In a tiered curriculum the teacher prepares a collection of activities that are appropriate for the diversity of student need and hierarchical in terms of level of complexity … the use of digital resources is an important feature of such curricula as these resources may provide alternate learning approaches and various representations and models of mathematics (Vale, p. 2).

A critical feature of this approach was team teaching in an open plan environment including 20 internet-connected computers and a range of learning spaces (a conversation pit, some desks in rows to support instruction, and some tables organised to support collaborative work). The space accommodated three classes at once and lessons included a short explicit teaching segment to the whole group, a choice of activities including ICT-based tasks, and voluntary 'clinics' which students could choose that reviewed key knowledge and skills in an explicit manner. While this approach had some limitations, the teachers felt it worked well to address diverse learning needs.

3. Impact of curriculum reform

Attempts to reform mathematics education at this level were the subject of the final session. Two contributions were heard, one from Banerjee on an evaluation of a curriculum reform initiative in India, and the other from Yuwen Li on the use of open-ended questions and the promotion of creativity in Taiwanese classrooms. Again the issue of transferability was raised, with the issue of teacher education and professional development to support more inclusive, reform-based approaches to teaching and learning mathematics at this level.

3.1. Transferability of reform approaches

Both Banerjee and Yuwen Li expressed concern that the students in their respective countries were not being well enough prepared for the abstractions introduced in lower secondary schools. Although a new curriculum had been introduced India with an emphasis on the use of concrete representations, Rahki noted that beneath this veneer, little had changed.

> The treatment of the manipulation of the symbolic expressions … has largely remained the same as earlier textbooks, with some attempts to model integer operations using number line and two coloured buttons, but not always convincingly explained (in fact quite confusing!) and followed by statements of rules of manipulation (p. 2).

An assumption underpinning the introduction of the new materials appeared to be that students would come to understand the more abstract mathematics if their experience of mathematics was more enjoyable. Rahki questioned the validity of this approach on the basis of her large scale study which demonstrated that the use of referents without negotiating their meaning and use more generally did not transfer to improved performance in the lower

secondary level. Rather than delay the mathematics, Rahki argued for a greater focus on the generalisations which would support later reasoning and the use of algebra as a tool for problem solving rather than an end itself.

Concerned with the mechanical, exam-oriented approach to mathematics in the middle years, Yuwen Li reported on the value of open-ended questions to build students' creativity and problem solving skills in Taiwanese primary schools. This contribution prompted discussion on how a reform-based curriculum might be used to improve performance on standardised tests. It was agreed that problem-based, negotiated approaches, which focussed on the mathematics in a meaningful way, were preferable to more narrow approaches which emphasised skills and memorising rules, it was acknowledged that this presented a significant challenge for teachers.

3.2 Teacher knowledge, professional learning

Teacher knowledge was recognised as a major issue in improving the quality of the mathematics teaching and learning at this level. Thinking about how teachers might teach as opposed to what teachers believe they have to teach was seen as a useful starting point although it was recognised that there was a danger in teachers adopting the 'clothes of reform' without considering the implications of reform for their own views about mathematics and the teaching and learning of mathematics. Equipping teachers with the resources to identify and respond to the learning needs of students at this level of schooling was recognised as a critical area of need but this had to be supported by quality professional learning delivered over time and in the context of on-going relationships involving practitioners, university staff, and system personnel.

In concluding this report we honour the work of Dr Ken Rowe, a noted Australian educator who tragically died in the Black Saturday bush fires in Australia in February 2009. Ken wrote extensively about the importance of building an evidence base for effective teaching and supporting teacher professional learning. For example, in a submission to the National Numeracy Review in 2007, Rowe noted that

> because teachers are the most valuable resource available to schools, it is vital that teachers be equipped with evidence-based teaching practices that are demonstrably effective in meeting the developmental and learning needs of all students – regardless of students' socio-cultural and socioeconomic backgrounds, and whether or not they experience learning difficulties. (Rowe, 2007, p. 2)

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