TSG 13: Research and development in the teaching and learning of probability

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For the first time in ICME congresses there were separate TSGs on probability and on statistics. This is important because trends in research and curriculum development too often narrow the fields of probability and statistics towards to data handling.

The importance of research in probability teaching

Probability and statistics have only recently been introduced into the school curricula in many countries. While the role of application-oriented statistics is undisputed, discussion about the role of probability is more ambivalent. Reduction of probability to the classical conception, mainly based on combinatorics, or its perception as a mathematical discipline are sometimes used as arguments for abandoning it. However, there are good reasons for a strong role for probability within a stochastics curricula:

1. Misconceptions about probability affect people's decision-making in important situations, such as medical tests, jury verdicts, investment, and assessment.
2. Probability is essential for understanding any inferential procedure within statistics.
3. Probability offers a tool for modelling and "creating" reality. The concepts of risk and reliability are closely related to and dependent upon probability.

The challenge is to teach probability to enhance understanding. The focus has to be on creating approaches that are more accessible and motivating. Additionally, the frequentist and subjectivist views of probability and connections of probability to practical applications should be taken into account. Simulation is one such strategy, as is visualisation of abstract concepts; there are more. The use of technology helps to reduce the calculations and enables a focus on concepts. The world of personal attitudes and intuitions is another source of success or failure in teaching probability. The papers in TSG 13 are organised in the following overlapping themes: individuals' corner; impact of technology; teachers' corner; conditional probability and Bayes' theorem; and fundamental ideas.

Individuals' Corner

This theme was "Individual understanding and misconceptions—Intuitive concepts about probability". There has been a move away from misconceptions, which may be changed, towards pre-conceptions, which should be refined in teaching.

The social situation of the class with the children interacting in their discussion is a feature of Rolka's (Germany, joint work with S. Prediger) investigation with 12 year olds. The common struggle for a joint strategy seems to generate a better understanding of the value of the strategy agreed upon in a simple game of chance. D. Abrahamson (US) undertook a case study of a single 11-year old child with an in-depth interview after a teaching phase in class where the binomial urn experiment was replaced step by step by a computer environment. As a unifying element he uses "four blocks", which change their appearance from "spoons" to scoop samples to the building elements of a combination tower of all possibilities, which finally build up the histogram of repeated samples.

Empirical studies by F. Chiesi & C. Primi (Italy) and L. Zapata (Colombia/US) dealt with heuristics in the tradition of Kahneman and Tversky. The Italian study dealt with the development of "negative" and "positive recency" with age. The study shows an increase in the

normative solution first (from age 9 to 11), but then this drops down (age 25) while positive or negative heuristics develop in a diametrically opposed manner. Further in-depth investigations might clarify whether such development can be confirmed. At the outset of L. Zapata's investigations there are the well-known tasks on the conjunction fallacy and tasks related to the law of large and small numbers. New in Zapata's study are single interviews with teachers. Her target was to clarify whether more experienced teachers are better in coping with pertinent learning difficulties?

One result of an investigation of K. Lysø (Norway) with student teachers is the documentation of an inclination to reformulate tasks, about which not much is known up to now: the students quite often reformulated two-stage experiments into one-stage tasks resulting in a wrong answer—but they could not see why their reconstruction was misleading.

S. Anastasiadou (Greece) used a battery of simple items and the method of similarity diagrams to detect a widespread lack of skill in changing between different representations of a task or a notion. Such lack of conceptual flexibility sheds a new light on the impact of variety in teaching as students tend to learn each new representation as a completely new concept.

Impact of Technology

There was no systematic evaluation of the possibilities and limitations of new media in the group. However, presenters freely used various kinds of software in a substantial way. Spreadsheets (EXCEL), Fathom, or Tinkerplot were used for efficient calculations but also for illustrating key ideas (concept of distribution, law of large numbers), for example, by S. Inzunsa (Mexico) or R. Peard (Australia).

New media indirectly form the backbone of the elaboration of D. Pratt (UK) on shaping the experience of naïve probabilists. By intentional sequences of the program ChanceMaker, he supplies challenging experiences to learners in order to shape their intuitions and strategies. Pratt locates new challenges for designers of software and teachers using this software. In a fusion of control over the initial parameters (via randomness) and representations of results (for example, histograms for the data), he seeks to prepare insights into randomness, which should widen and refine intuitive notions. Software offers more efficient, graphically orientated possibilities to supply (in fast motion), and order, experience with randomness. His essay could have been attributed to the theme Fundamental Ideas: How is it possible to form such basic ideas—with the support of new media? According to Pratt, a new world of up-to-date unknown intuitions might emerge, which would affect both concepts and their understanding.

Teachers' Corner

This theme covered pre-service and in-service education as well as teachers' conceptions of teaching and of probabilistic notions. Some of the following contributions would also fit the theme of Fundamental Ideas.

K. Lysø (Norway) discussed the merits of an early phase of teaching elementary probability to students, which is similar to empirical research. The choice of the (simple) items does not really matter; the success of the method depends mainly on how the items are dealt with after the written test. It may have a lasting effect on discussion of which reconstruction of the task would make sense and therefore lead to a sensible solution, even if it deviates from the normative solution. L. Zapata (Columbia/US) tried to derive meta-knowledge for teachers from her in-depth interviews. Novice teachers exhibit the same intuitive misconceptions as their students and thus are not really able to help them. Possibly this result is another argument for including a didactical course on the subject in teacher education at university and not to restrict it to a mathematical course.

J. Watson (Australia, together with S. Ireland) reported on in-depth interviews with 12-year olds on relations between empirical and theoretical probabilities. The preceding course covered coin-tossing experiments first physically performed, then simulated by Tinkerplot.

While the software improved students' success, some questions remain. Can the computer really generate randomness? How can students learn to read pre-fabricated diagrams? How can we ensure that children have sufficient experience in proportional thinking?

V. Kataoka (Brazil) described experiments from interactive in-service workshops, for example, break a stick randomly into three pieces then we try to form a triangle of the pieces. The obvious discrepancy between the theory (predicting 25% or 19% success) and the data (success rates of 75% are not rare) is a shock for our usual interpretation of probability as relative frequencies. There may be a great potential in the cognitive dissonance. Clearing the cloud and the tension are eagerly awaited in the class.

An interesting extra-curricular activity was presented by H. Trevethan (Mexico). The concept of science fairs envisages that students collaborate on a project with the aim of presenting it to a wider public at the fair. Changing the role from a passive learner to one who actively explains and is responsible for what goes on is of lasting effect for learners. This authentic transfer of responsibility might also be taken up in teaching.

Conditional probability and Bayes' theorem

Conditional probability and Bayesian inference are important ingredients of university teaching, including in non-mathematical courses. These concepts are important but difficult ingredients of curricula. Thus far, many different types of related errors have been investigated in isolation.

C. Batanero and C. Diaz (Spain) developed a questionnaire and analysed the data on university students by factor analysis. In general, they found a significant decrease in misconceptions at higher mathematical levels. For interrelations between misconceptions, however, the result is less optimistic as these seem to be isolated. Types of misconceptions still have to be repeatedly put to the fore in teaching in order to extend students' experience.

P. Huerta (Spain) developed a theory of conditional problems. From the derived structure for ternary problems, it was seen that out of 20 different types of problems only one subclass has been used in existing research, which is a severe limitation. He develops a graph theoretic representation of all problems, which allows for visual solution of problems. As a side effect, the number of steps in this solution may serve to conceptualise and measure the grade of difficulty of the particular problem at hand.

L. Martignon and S. Krauss (Germany) presented a class experiment on conditional probability and related reasoning with 10-year olds. They start with Wason cards (with a letter on one and a number on the other side). Which cards do you necessarily have to turn around in order to check whether the following rule holds? "If one side of a card exhibits a vowel, its other side must exhibit an odd number." While in a logical context children solve the task badly, they improve in a statistical embedding. The researchers move on to represent the cards by tinker cubes (with two colours). Their pilot projects show that such media favour learning steps in proportional thinking linked to probabilities right from the beginning.

The pitfalls of the interpretation of results from statistical tests or confidence intervals are well known. These originate from the reduction of the interpretation of probability to situations which may be repeated independently. On this issue there has been a debate not only in the foundations of statistics but also in the didactical community. Taking this as a starter, Ö. Vanscsó (Hungary) developed a parallel course in classical and Bayesian statistics with the motto "You will understand a theory much better if you contrast it to another—especially to one which is quite different". Accordingly he works on a conception for teaching both schools of statistics without favouring one of them.

Fundamental Ideas

Within this theme, the organisers intended to attract papers devoted to probabilistic ideas like random variable, distribution, expectation, to the convergence of relative frequencies, to the central limit theorem, or to revising probabilities, Bayes' theorem, and independence.

R. Peard (Australia) returned to the roots of motivating concepts by problems and questions from games of chance. He did not argue with the potential to explain the notions by the context in which they emerged, rather he advocated the application argument as games of chance have developed into an important business sector. It is therefore necessary for students to be familiar with this business and clarify where their chances really lie.

R. Kapadia (UK) presented tasks from national tests and concludes from the poor achievement of students that teaching has not really improved over twenty years. This may be rooted to recent curricular trends as data handling is favoured at the cost of probability. The fundamental idea of judging probabilities and risks subjectively, affected and supported by qualitative and objective information, has still not found a sustainable form of teaching.

The panel discussion on Fundamental Ideas focused on key ideas of probability in teaching. R. Kapadia (UK) enriched the discussion with a list of behavioural attitudes which should be addressed explicitly in teaching, for example, people have difficulties judging very small or very high probabilities especially if these are connected to adverse consequences. Y. Wu presented a panoramic view of the Chinese approach; the similarity to Western approaches indicates cultural invariants in the concepts of probability. M. Borovcnik (Austria) outlined some peculiarities of stochastic thinking, making it different from other approaches: causal re-interpretations and emotionally laden private criteria signify personal thinking. We are not so open to rational views when we are faced with uncertainty.

A perspective for the future

The experiment with the split of probability and statistics into two TSGs at ICME was successful. It allowed for a more convenient focus of the pertinent presentations and discussions. It showed that, against the international trend in curricula, there is substantial interest by researchers in probability issues. This also holds for the joint study of ICME and IASE where a panel discussion about a vital role for probability within curricula led to a lively discussion on the role of probability within curricula and educational research.

After the congress, many TSG members joined a project on the electronic publishing of research. Electronic resources provide many new facilities which incorporate multilingual approaches, interactivity, graphical support of abstract ideas and results, as well as authentic material backing up the conclusions of empirical investigations, animations and programmes to build up didactic sequences.

Links and references

Borovcnik, M. & Kapadia, R. (2009) Special issue on "Research and Developments in Probability Education". *International Electronic Journal of Mathematics Education (IEJME), 4, (3)*. Online: www.iejme.com/032009/main.htm

TSG 13 on "Probability" at ICME 11. Online: http://tsg.icme11.org/tsg/show/14, also at www.stat.auckland.ac.nz/~iase/publications.php?show=icme11

TSG 14 on "Statistics" at ICME 11. Online: tsg.icme11.org/tsg/show/15