TSG 15: Research and development in the teaching and learning of discrete mathematics

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Introduction

Discrete mathematics occupies a variable place in mathematics education: in some countries, only a very small number of discrete mathematics concepts are taught, except those related to combinatorics and the basics of number theory. In a few other countries, for example in Hungary, there has been a long tradition of introducing graph theory in secondary schools. Discrete mathematics can be introduced, either as a mathematical theory, or as a set of tools to solve problems (a graph is a basic and intrinsic modelling tool). We collectively assessed and analysed the state-of-art of curricula in discrete mathematics. This led to two fundamental questions: Why and how can we introduce discrete mathematics in schools? How can discrete mathematics contribute to students acquiring fundamental skills involved in defining, modelling and proving, at various levels of knowledge? The individual contributions to the TSG are summarised below.

Historical projects in discrete mathematics and computer science

Jerry Lodder, Janet Barnett, Guram Bezhanishvili, Hing Leung, David Pengelley, Desh Ranjan.

The pedagogical technique of teaching mathematics from primary historical sources is discussed for courses in finite mathematics and algorithmic thought. Use of the historical record provides context, motivation, and direction to the subject matter, along with the human side of mathematical discovery. Two particular sources are described in detail, namely Blaise Pascal's *Treatise on the Arithmetical Triangle,* from the 1650s and Gabriel Lamé's 1838 publication on counting triangulations of a polygon. Pascal's work is ideal for teaching the concept of mathematical induction, particularly in the context of proving that certain patterns occur in Pascal's triangle. Lamé's work is well suited for a course in combinatorics or algorithm design, where a comparison between methods of enumeration of triangulations can be drawn. Classroom projects based on these sources are outlined, showing how excerpts from the original source along with appropriate student exercises can be used for engaging instruction.

Teaching of discrete mathematics at advanced level in singapore: Teachers' perspectives

Quek Khiok Seng, Toh Tin Lam, Boey Kok Leong,Tay Eng Guan, Dong Fengming

We describe the implementation and motivation for including two discrete mathematics topics into the H3 syllabus of the new 2006 Singapore 'A' Level Curriculum for pre-university, and the training of teachers to teach these topics. We also present the results of a survey of teachers' perspectives on this implementation. Findings indicate that there is interest in discrete mathematics and a place for it in pre-university mathematics. Teachers were looking for a blend of both content and pedagogy in training workshops to help them teach discrete mathematics better. The teachers also saw it necessary to teach problem solving strategies to students, as well as to include real-life applications in their lessons. A call is made to address the related challenges of equipping teachers with problem solving strategies, and with the pedagogy of teaching problem solving to their students. Some suggestions are offered towards this end.

Teaching and learning of discrete mathematics—The Indian scenario

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India has a rich tradition of "combinatorial thoughts" (Struick, 1967). In Pingala's work Chandas-Sutra (3rd century BC), the number of letter combinations is considered under the name “Meru Prastara”, predating Pascal’s triangle. Mahavira (650AD) also contributed to the subject in his magnum opus Ganithasara Samgraha (Parameswaran, 1998). Biggs et al. (1995) point out the lack of material related to combinatorics in classical Western literature, while many original sources came from the Hindus. The disproof of the famous conjecture (1762) of Leonhard Euler on the “mutually orthogonal Latin squares” by Indian mathematicians Bose, Shrikhande and Parker in 1959 was a major breakthrough in combinatorial mathematics, ushering in the study of combinatorial designs in India. The Department of Science and Technology, Government of India, has recently established n-CARDMATH, a National Centre for Advanced Research in Discrete Mathematics. The specificity of topics and techniques of discrete mathematics help us in moulding a totally different mind set in teachers and students, bringing in sometimes elements of recreation. The “diagram tracing puzzles” and “puzzles on chess boards” are typical examples (Biggs & al, 1976). With such puzzles, we can move on to more challenging mathematical ideas so that the transition of thought from puzzles to problems is smooth. Since most of the proofs in graph theory are of a constructive type, this will be accepted with a different taste.

Biggs, N., Lloyd, E., & Wilson, R. (1976). *Graph theory* 1736-1936, OUP.

Biggs, N., Lloyd, E., & Wilson, R. (1995). The History of Combinatorics, In *Hand book of Combinatorics*, Eds. R.Graham.et.al, Elsevier.

Parameswaran, S. (1998) *The Golden Age of Indian Mathematics*, Swadeshi Science Movement.

Struik, D. (1967). *A Concise History of Mathematics*, Dover.

The role of the bijection principle in the teaching of combinatorics

Michel Spira, Universidade Federal de Minas Gerais, Brazil

The role of the bijection principle in the teaching of combinatorics is described, with the aid of examples and actual classroom instances. The bijection principle says that if there exists a bijection between two (finite) sets then these sets have the same cardinality. Underneath the apparent simplicity of this statement is hidden a powerful counting tool which, even when not explicitly stated, is essential in combinatorial thinking. Basic combinatorial problems involve, in general, counting (that is, finding the number of elements of) a finite set; the principle applies when an easily counted set is found, together with a bijection from the first set to the second. But finding both the second set and the accompanying bijection can be far from easy. The article discusses this and other related questions in a way suitable to middle and high school teachers, as well as interested students in these levels.

A technology-based approach to discrete mathematics in the classroom

Ulrich Kortenkamp, University of Education Karlsruhe, Germany

We describe a new approach to teach graph theory and graph algorithms, using interactive geometry software. Students are encouraged to use computers to explore and verify their theories, and use them as a tool for modelling real-life situations. The examples presented originated in a project of the DFG research center Matheon in Berlin (http://www.matheon.de) named Visage–Visualization of Algorithms using Geometry Software. The project is the origin of several web-based learning activities that have also been tested in class. We describe and discuss one of these exemplary units. It has been used in the class, where it proved that also low-achieving 7th-grade students are able to work mathematically, even if they have not much experience in classical mathematical topics. It gave them confidence, and they also had the experience that finding an optimal solution is not based on being lucky, but on mathematics. The immediate pedagogical opportunities of using the Visage package for teaching are: forced modelling with graphs, immediate trial and error with own models, automated checking of solutions, and additional visual hints. All these effects can be achieved in different organisational forms of teaching, depending on the available equipment and other premises.

Doing mathematics authentically and discretely. A perspective for teacher training

Stephan Hubmann, Dortmund University, Germany

An original situation was presented and discussed, with the two following questions: Which factors of the learning environment foster or hinder the learning processes of the students? Are the students capable of creating mathematical concepts, and are these concepts comprehensible with respect to conventional concepts ?

Learners'conceptions in different class situations on Königsberg's bridges problem

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The paper focuses on the Königsberg Bridges problem. This famous problem is often used as an introduction to graph theory and discrete mathematics. It is also frequently proposed as an enigma in recreational mathematics. However, its complete solution is rarely given and its mathematical depth is usually masked. Although the problem is now completely solved, it remains a subtle and interesting one, giving access to fundamental mathematical concepts like proofs, necessary and sufficient conditions, and modelling. We proposed several experiments. Three of them were made using a research problem approach with pupils in primary school, students in secondary school and mathematics teacher candidates; and another was a historical document study with undergraduate students in computer science and applied mathematics. We show that, whereas this problem and its solution are apparently simple, pupils and students at various levels encounter the same difficulties on some specific points that we describe, concerning proving and modelling. In addition, a careful look at Leonhard Euler's proof reveals that he might have encountered the same difficulties, missing an important part of the proof.

Euler L. (1736). Solutio problematis ad geometriam situs pertinentis, Commentarii academiae scientiarum Petropolitanae 8, pp 128-140.

Discrete mathematics : a mathematical field in itself but also a field on experiments. A case study : displacements on a regular grid

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The main feature of my mathematical and didactical research concerns the elaboration of definitions, a process rarely studied for itself. The guiding idea is to provisionally map the field with definitions serving as temporary markers for concept formation. My research was conducted on the following concepts: trees (a well-known concept, possibly approached in several ways), discrete straight lines (a concept still at work, for instance in the perspective of the design of discrete geometry) and a wide study of properties of displacements on a regular grid. I chose to develop this last point for two reasons. First, the study of this kind of situation brings partial answers to one of the questions of this TSG: "How can discrete mathematics contribute to make students acquire the fundamental skills involved in defining, modelling and proving, at various levels of knowledge?" A mathematical work on ("linear") positive integer combinations of discrete displacements involves skills such as defining, proving and conceiving new concepts. Second, this situation leads us to discrete mathematics but also to linear algebra, as similar concepts are involved. A new question emerges: discrete mathematics is a mathematical field in itself, but can it also be a field of experimentation in order to simultaneously investigate skills, knowledge and concepts involved in other mathematical fields?

Graph isomorphism, matrices and a Computer Algebra System : switching between representations

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We study classroom activities where prospective teachers meet a problem in Graph Theory, with an application of an advanced theorem in Linear Algebra. The support provided by a Computer Algebra System is analyzed, in particular with regard to the building of new mathematical knowledge through a transition from graphical to algebraic representation. Students use to pass from either an algebraic or analytic presentation of mathematical objects to a graphical one. In the present case, the transition from a graphical presentation (a picture of the graph) to an algebraic presentation (adjacency matrices) provides a broader frame within which a more profound understanding can be achieved. The general need for more than one representation for a mathematical object is also discussed.

Learning experimental approach by a discrete mathematic problem

Nicolas GIROUD, Joseph Fourier University, Grenoble, France

We discussed the status and teaching of the experimental approach in the classroom, defined as a set of actions and feed-back between: questions, experiments, hypotheses, conjectures and proofs. We use this definition to analyse the status of the experimental approach in the French curriculum and in two French textbooks. It appears that the curriculum gives a considerable emphasis to this approach. However, it is only addressed very weakly in the textbooks. We propose a Research Situation for the Class whose primary goal is to put students in a genuine experimental approach and presented some results about such an experimentation. The purpose of the situation was to find the largest set of cards which does not involve a certain property. The concepts involved in this situation are inductive constructions, investigating local/global maxima, lower and upper bounds, algorithmic approaches, variables, and enumeration. In order to facilitate access to the activity, we assigned the problem along with specific pieces of material to build the card games. We carried out an experiment in the tenth and eleventh grades (2 hours each). We observed that the situation placed the students in the role of experimenters: they asked their own questions and tried to answer them, they made conjectures and proofs. However, they had difficulties to check the assigned properties to be studied, and one session appeared to be insufficient.

Some specific concepts and tools of discrete mathematics

Denise Grenier, Institut Fourier, Grenoble University I, France

Discrete mathematics, as it deals with finite or countable sets, brings into play several overlapping domains, for example, number theory, graph theory, and combinatorial geometry. As a consequence of the peculiarities of discreteness versus continuum, interesting specific reasoning and new tools can be constructed, such as colouring, proof by exhaustion, proof by induction, and use of the Pigeonhole Principle (Grenier 2001, 2003). Further concepts involved in other mathematical domains are also used in a specific manner, for example, optimisation techniques and the notions of generating set or minimal set. In this TSG, I developed two of these tools, the Pigeonhole Principle and the Finite Induction Principle. The Pigeonhole Principle plays an important role in reasoning involving integer numbers. Its generalisation allows existence problems to be solved. It is unusually effective, in that it simplifies the exposure of a proof or a solution, and may appear as the only possible way of solving a problem. Further, my research has shown that French students have limited and often inaccurate knowledge about induction, which is neither taught as a concept and almost always restricted to the case of an algebraic property P(n). A consequence of these didactic challenges is that misconceptions persist in the knowledge of many students, which I tried to address through assigning original problems to students.

Grenier D. (2003). The concept of induction, Mediterranean Journal For Research in Mathematics Education, vol.2.1, 55-64, ed. Gagatsis, Cyprus Mathematical Society.

Grenier D. (2001). Learning proof and modeling. Inventory of fixtures and new problems. Actes du 9ème International Congress for Mathematics Education (ICME9).