Central *Concept* Design

Central Concept:
Right Triangle & Ratios

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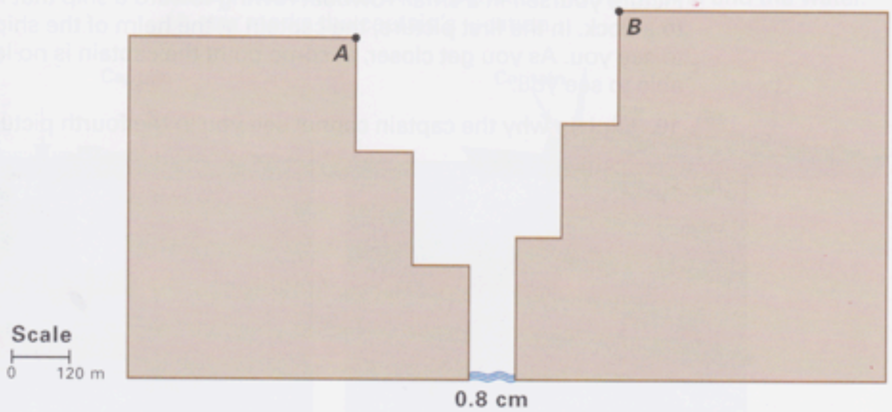
From: Mathematics in Context
Holt Rhinehart Winston 2006

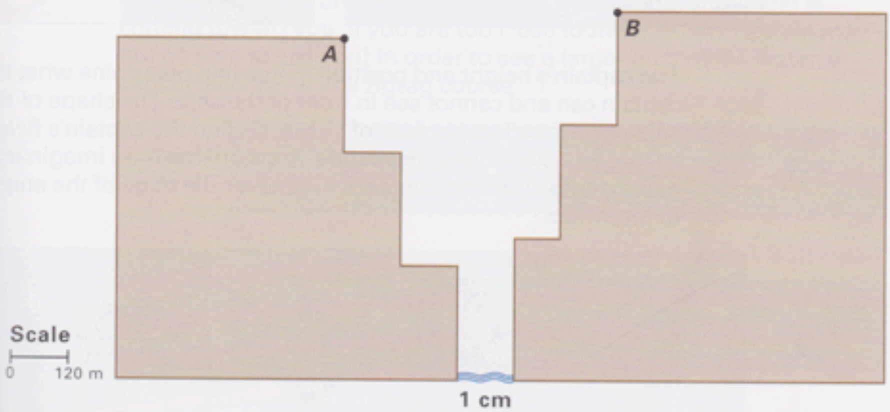
We will look more closely at that drawing on the right. Now we see it in a scale drawing of the *cross-section* of the canyon.



6. Is it possible to see the river from point A on the left rim? Why or why not?

7. What is the actual height of the left canyon wall represented in the scale drawing?

8. If the river were 1.2 centimeters (cm) wide in the scale drawing, could it be seen from point A?



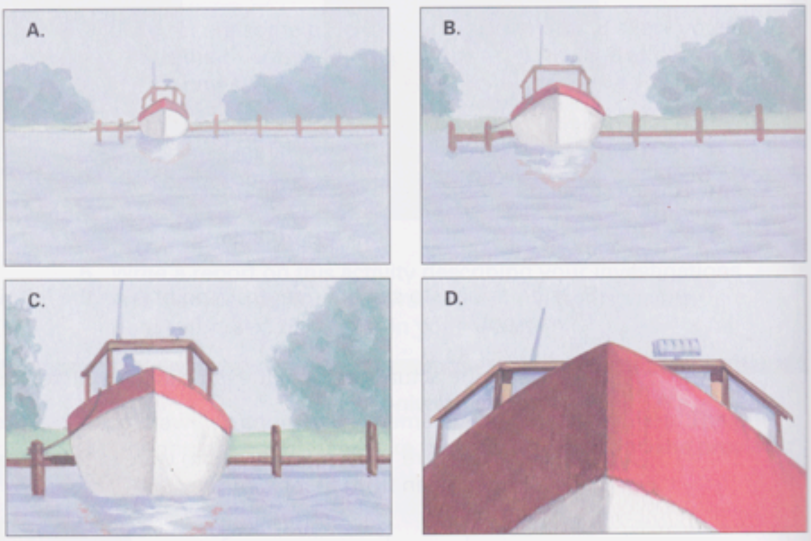
9. In the scale drawing above, the river is now 1 cm wide. Is it possible to see the river from point B? If not, which ledge is blocking your view? Explain.

Exploratory Question: Can you see the Coloroda River from the rim of the Grand Canyon?

Ships Ahoy

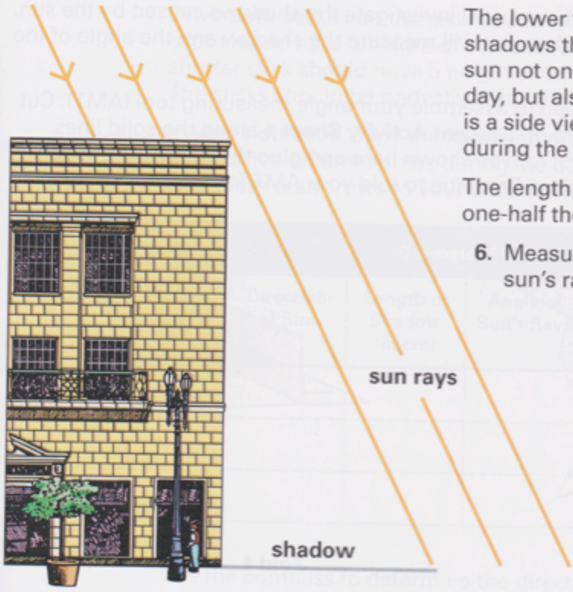
Picture yourself in a small rowboat rowing toward a ship that is tied to a dock. In the first picture, the captain at the helm of the ship is able to see you. As you get closer, at some point the captain is no longer able to see you.

10. Explain why the captain cannot see you in the fourth picture.



The captain's height and position in the ship determine what the captain can and cannot see in front of the ship. The shape of the ship will also affect his field of vision. To find the captain's field of vision, you can draw a **vision line**. A vision line is an imaginary line that extends from the captain's eyes, over the edge of the ship, and to the water.

Same question, almost: What can the captain see from his position; introducing blind spots



The lower the sun is, the longer the shadows that are cast. The height of the sun not only depends on the time of the day, but also on the season. Shown here is a side view of a building around noon during the summer.

The length of the building's shadow is one-half the height of the building.

6. Measure the angle between the sun's rays and the ground.

Around noon during the winter, the length of this building's shadow is two times the height of the building.

7. a. Draw a side view of the building and its shadow around noon during the winter.
   b. Measure the angle between the sun's rays and the ground.

Around noon during the spring, the angle between the sun's rays and the ground is 45°.

8. a. Draw a side view of the building and its shadow around noon during the spring.
   b. If the building is 40 m tall, how long is its shadow?

9. Describe the changes in the length of the shadow and the angle of the sun's rays from season to season.

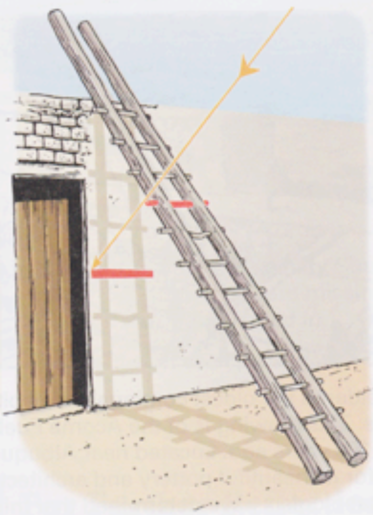
Next: identifying shadow as being similar to blind spots

C Shadows and Angles

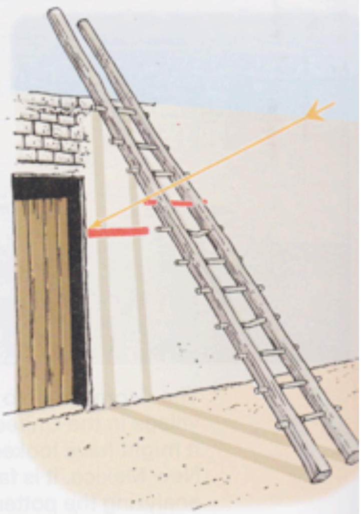
Originally, the houses in the Acoma Pueblo had no front doors; ladders were used to enter the houses on the second floor. Ladders propped against the houses formed different angles. The steepness of the ladders can be measured several ways.

Recall from Section B that the sun's rays are parallel. The drawing marked Picture A shows a ladder and its shadow. The drawing also shows how the shadow of one rung in the ladder is cast by a ray of sunlight.

Picture A



Picture B

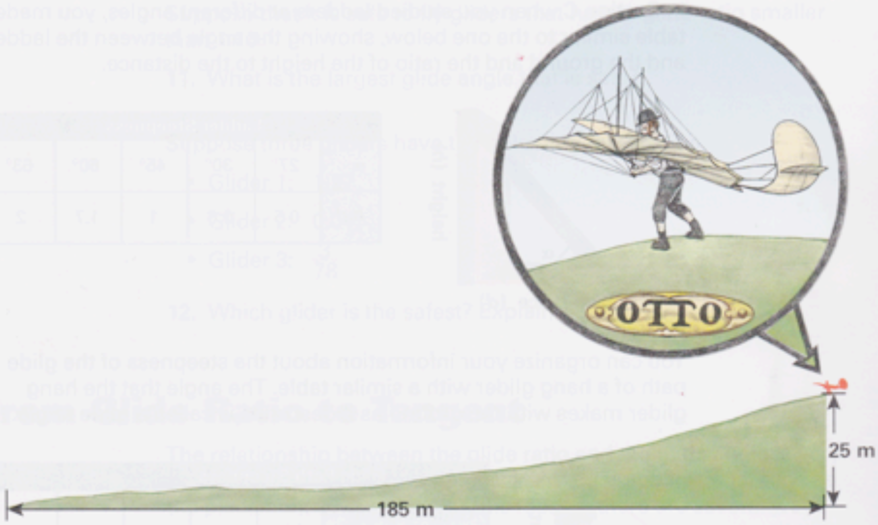


2. Use **Student Activity Sheet 10** to draw rays of sunlight that cast a shadow for each of the other ten rungs of Picture A.

The drawing marked Picture B shows the same ladder in the same position, but at a different time of day.

3. Use **Student Activity Sheet 10** to draw rays of sunlight and the corresponding shadow for each of the other ten rungs of Picture B.

Next we make the shift to steepness of ladders



Otto Lilienthal made more than 2,000 flights with hang gliders at the end of the 19th century. Suppose that on one of his flights from the Rhinower Hills near Berlin, Germany, he started from a height of 25 m and covered 185 m of ground distance as shown here. On his next flight, suppose he redesigned his glider a little, started from a height of 20 m, and traveled a ground distance of 155 m.

6. What were the glide ratios of Otto's two gliders? Which glider could travel farther?

7. Suppose that a glider has a glide ratio of 1:8. It takes off from a cliff and covers 120 m of ground distance. How high is the cliff?

8. Make scale drawings to represent the following glide ratios.

   a. 1:1 **b.** 1:2 **c.** 1:4

   d. 1:10 **e.** 1:20

The last context before we go to the abstract: glide angle

Suppose that it is safe to fly gliders that have a glide ratio smaller than 1:10.

11. What is the largest glide angle that is safe?

Suppose three gliders have the following glide ratios.

- Glider 1: 1:27
- Glider 2: 0.04
- Glider 3: $\frac{3}{78}$

12. Which glider is the safest? Explain.

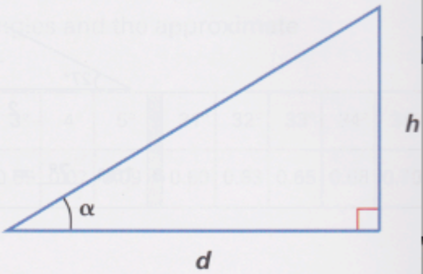
From Glide Ratio to Tangent

The relationship between the glide ratio and the glide angle is very important in hang gliding, as well as in other applications, such as the placement of a ladder. For this reason, there are several ways to express this ratio and angle.

glide ratio = $h{:}d$

glide angle = α

The ratio $h{:}d$ is also called the **tangent** of angle α, or $\tan \alpha = \frac{h}{d}$.

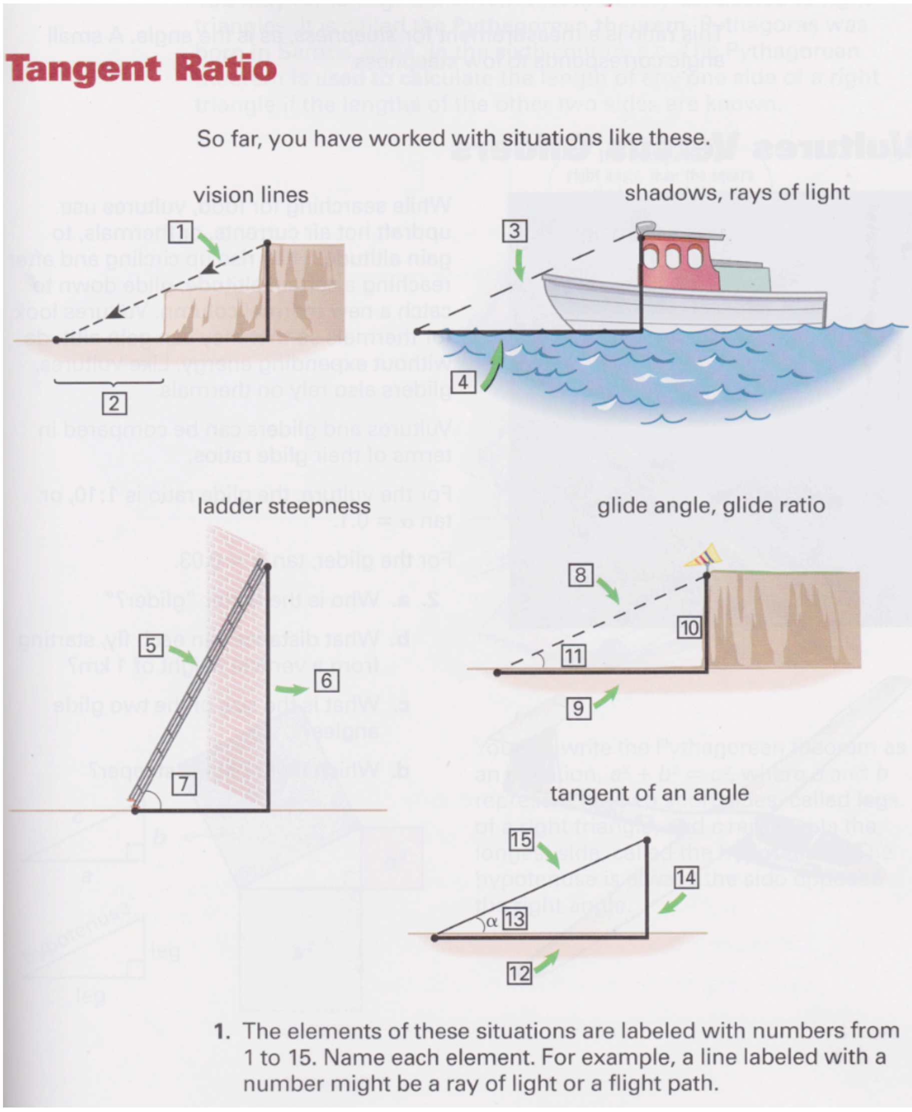


For a glide ratio of 1:1, the glide angle is 45°, so $\tan 45° = \frac{1}{1} = 1$.

Suppose that another one of Otto's hang gliders has a glide ratio of 1:7. This means that the tangent of the glide angle is 1 to 7 (or $\frac{1}{7}$).

13. Describe in your own words the relationship between the glide ratio, glide angle, and tangent.

Finally: the tangent



Tangent Ratio

So far, you have worked with situations like these.

1. The elements of these situations are labeled with numbers from 1 to 15. Name each element. For example, a line labeled with a number might be a ray of light or a flight path.

Reflecting on the same concept

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