Didactical engineering as a research methodology: from Fundamental Situations to Study and Research Paths

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Outline

1. **Didactic engineering within ATD: the problem of teaching mathematics as a modelling tool at university level**
2. **The 'a priori' analysis: mathematic and didactic design of a Study and Research Path on population dynamics**
3. **Implementation and 'in vivo' analysis**
4. **Didactic engineering from TDS to ATD: the Herbartian schema**
5. **The ecology of SRP: conditions and constraints**
6. **Three questions to the audience**

Some preliminary considerations

- In **TDS** and **ATD**, what is usually considered as ‘task’ is integrated in the basic notions of:

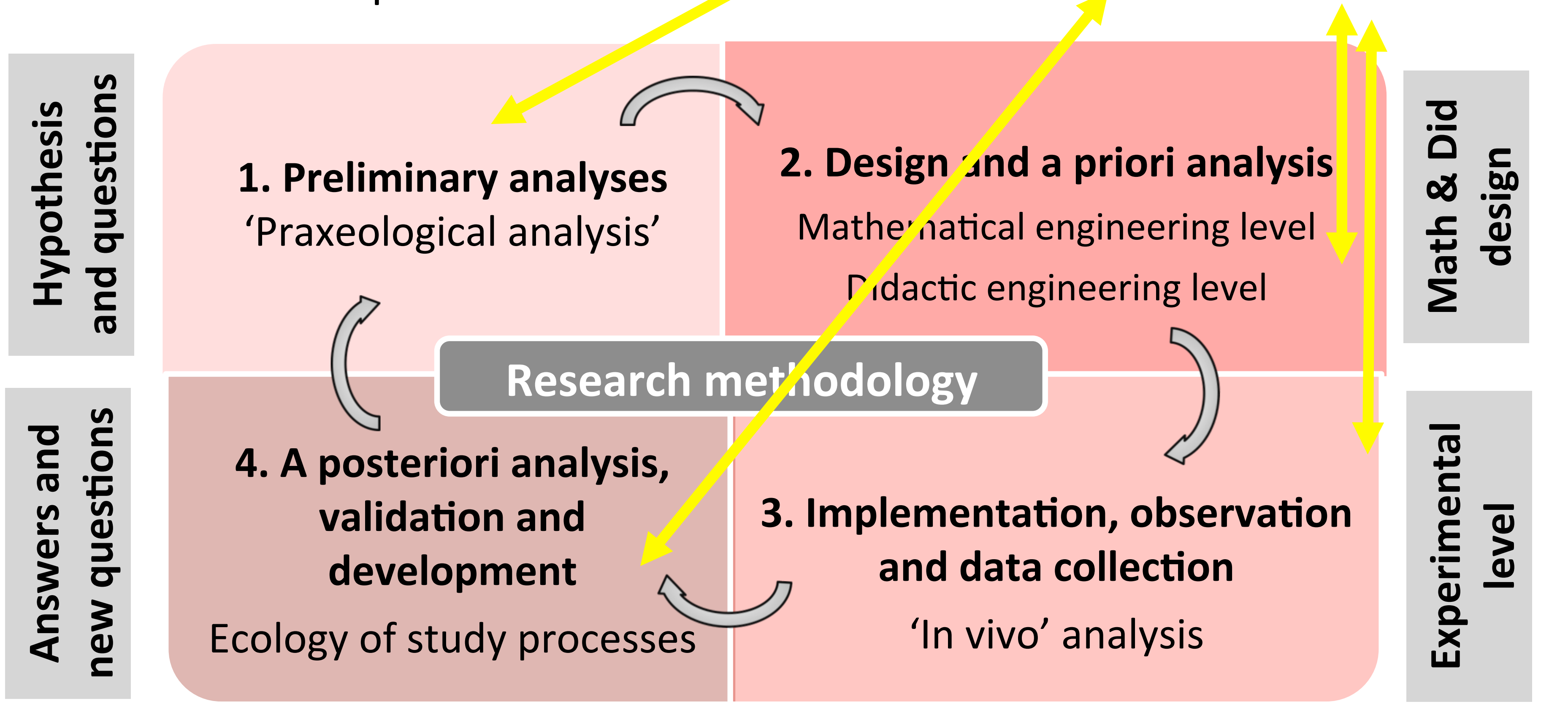
Fundamental situation	Praxeology
(game against a milieu)	(type of tasks, techniques, *technology*, theory)

- **Task design** is integrated in the whole process of **didactic engineering** (DE) developed as a research methodology by TDS and its developments since 1980 (Artigue 1990, 2002, 2009).

1. Teaching mathematics as a modelling tool

- Artigue (2009 & 2013) distinguishes four different phases:

Science of Didactics
(problems, methodologies, results, theoretical development)



1. Teaching mathematics as a modelling tool

FIRST PHASE: PRAXEOLOGICAL ANALYSIS	1.A. What mathematics is taught at Natural Sciences university degrees?

- One could think that the faculties of Natural Sciences would offer favourable institutional conditions to teach **mathematics as a modelling tool,** essential to the understanding, use and development of the Natural Sciences.
- However, reality seems far away from this purpose: despite mathematical models appear in syllabi, their teaching always arrives at the end of the process, if time is left for it!
- The ruling 'ideology' in mathematics teaching at university can be called the **'application of pre-established knowledge'**, leaving little place for the process of constructing mathematical models.

1. Teaching mathematics as a modelling tool

FIRST PHASE: PRAXEOLOGICAL ANALYSIS	1.B. What results, knowledge and know-how can educational research offer to the problem?

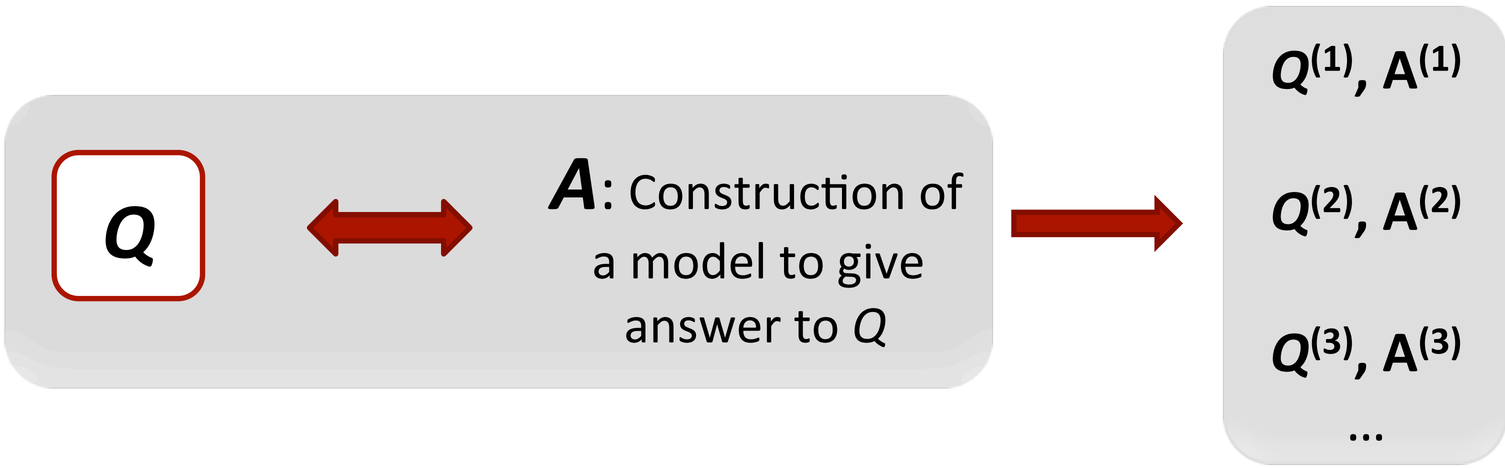
- In the last decades, there has been a large **development** of the field of **mathematical modelling and applications**, shared by many researchers and supported by the new curricular orientations introduced into our educational systems.
- Besides all the progress, the **problem of the large-scale dissemination** of modelling processes has recently been identified as both an urgent and an intricate task.

We know how to teach modelling, have shown how to develop the support necessary to enable typical teachers to handle it, and it is happening in many classrooms around the world. The bad news? 'Many' is compared with one; the proportion of classrooms where modelling happens is close to zero (Burkhardt, 2008).

1. Teaching mathematics as a modelling tool

FIRST PHASE: PRAXEOLOGICAL ANALYSIS	1.C. What epistemological principles are assumed about mathematical modelling?

- In the framework of the ATD, we start from the principle that any **mathematical activity** can be reformulated as a **modelling activity**.

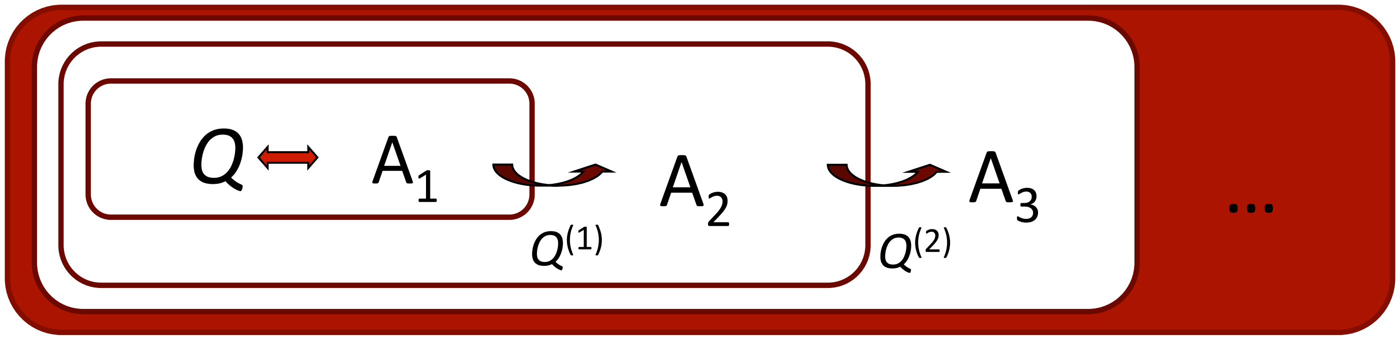


- The term ‘reality’ does not only refer to an extra-mathematical reality, it also includes ‘intra-mathematical reality’.

1. Teaching mathematics as a modelling tool

FIRST PHASE: PRAXEOLOGICAL ANALYSIS	1.C. What epistemological principles are assumed about mathematical modelling?

- The ATD proposes to reformulate mathematical modelling activity as a **process of (re)construction** and **articulation of mathematical praxeologies (A_i)** which become progressively broader and more complex with the main aim to provide an answer to some problematic questions (García, 2005 and Barquero, 2009)



1. Teaching mathematics as a modelling tool

FORMULATION OF THE RESEARCH PROBLEM

- **HYPOTHESIS: Study and research paths (SRP) appear as appropriate teaching proposals** for **mathematical modelling** to play an explicit and crucial role, emerging from questions and linking mathematical contents that appear as models to answer questions.

- **RESEARCH PROBLEM:** What kind of **conditions** could help and what kind of **constraints** hinder the integration of mathematics as a modelling tool in current educational systems at university level?

SECOND PHASE OF ANALYSIS	2.A. Mathematical engineering level

① **A generating questions Q_0 is the starting point of a SRP**

The starting point of a SRP should be a ‘lively’ question Q_0 which we will call **generating question**. The evolution of the study of the generating question leads to posing new successive questions derived from Q_0.

② **A SRP has a tree structure as a consequence of the continuous search for answers to Q_0**

The study of Q_0 and its **derived questions** leads to successive temporary answers A_i which would be tracing out the possible routes to be followed in the effective experimentation of the SRP.

3. The 'a priori' analysis: mathematic and didactic design

SECOND PHASE OF ANALYSIS	2.A. Mathematical engineering level

$$Q_0 \to \begin{cases} (Q_0^{'}, A_0^{'}) \to (Q_1^{'}, A_1^{'}) \to \cdots \to (Q_p^{'}, A_p^{'}) \\ (Q_0^{''}, A_0^{''}) \to (Q_1^{''}, A_1^{''}) \to \cdots \to (Q_q^{''}, A_q^{''}) \\ \cdots \end{cases}$$

3 The media and milieu dialectics at the core of the study of Q_0

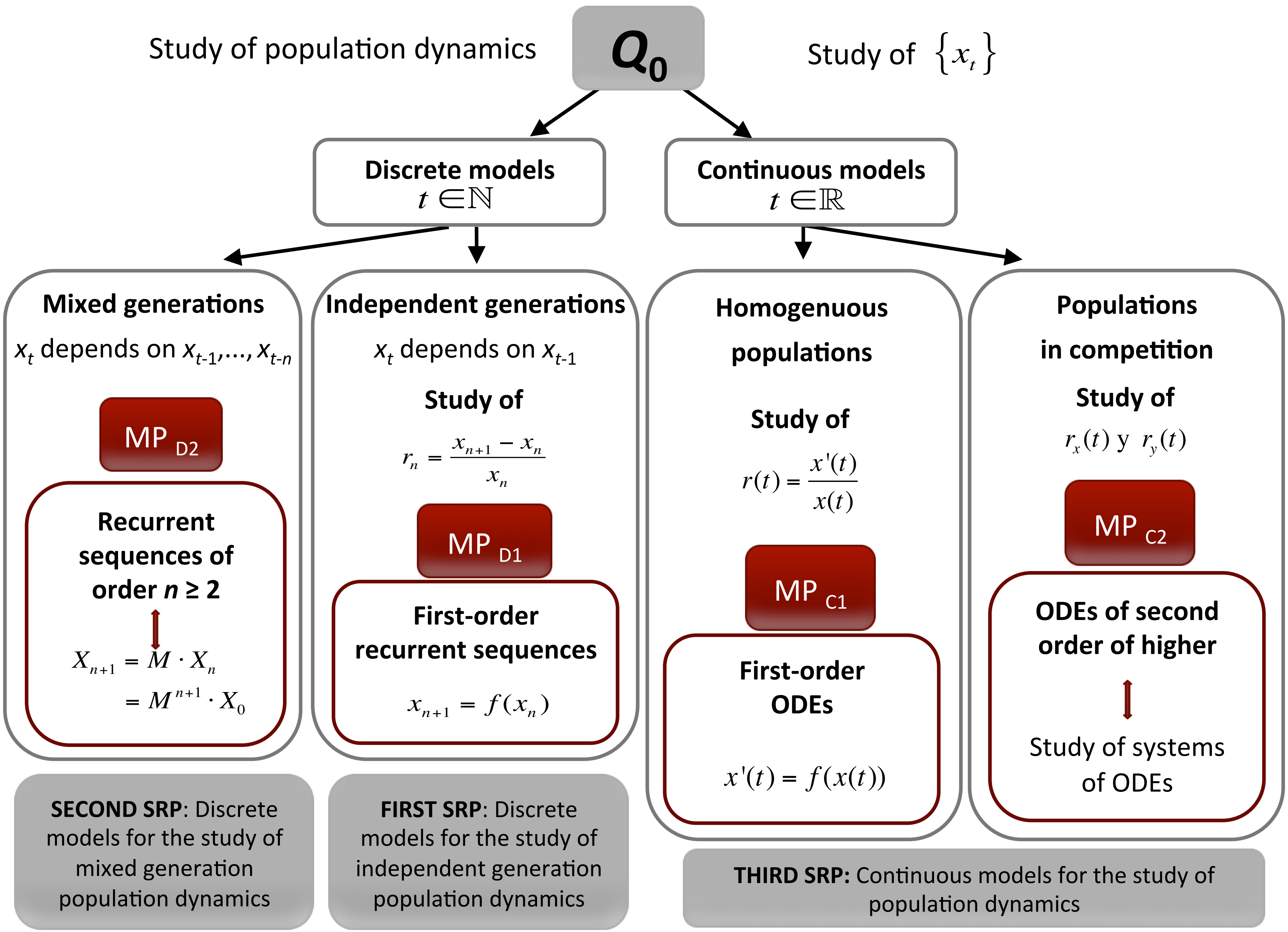
Some 'external' pre-established answers accessible through the different **means of communication and diffusion** (the **media**) will be necessary to elaborate the successive temporary answers A_i. The appropriated **milieus** will therefore be necessary to **test** and **'check' the validity** of these answers and to **adapt** them to the new study.

3. The ‘a priori’ analysis: mathematic and didactic design

A GENERATING QUESTION – Q_0

Given the size of population over some time period,

- Can we predict its size after *n* periods? Is it always possible to predict the long-term behaviour of the population size?
- What sort of assumptions on the population and its surroundings should be made?
- How can one create forecasts and test them?



Study of population dynamics
Q_0
Study of $\{x_t\}$
Discrete models $t \in \mathbb{N}$
Continuous models $t \in \mathbb{R}$
Mixed generations
x_t depends on $x_{t-1},\ldots,x_{t-n}$
MP $_{D2}$
Recurrent sequences of order $n \geq 2$
$X_{n+1} = M \cdot X_n = M^{n+1} \cdot X_0$
Independent generations
x_t depends on x_{t-1}
Study of
$r_n = \frac{x_{n+1} - x_n}{x_n}$
MP $_{D1}$
First-order recurrent sequences
$x_{n+1} = f(x_n)$
Homogenuous populations
Study of
$r(t) = \frac{x'(t)}{x(t)}$
MP $_{C1}$
First-order ODEs
$x'(t) = f(x(t))$
Populations in competition
Study of
$r_x(t)$ y $r_y(t)$
MP $_{C2}$
ODEs of second order of higher
Study of systems of ODEs
SECOND SRP: Discrete models for the study of mixed generation population dynamics
FIRST SRP: Discrete models for the study of independent generation population dynamics
THIRD SRP: Continuous models for the study of population dynamics

Q_0

Independent generations (H_0)

$$H_1 : r_n \equiv r, r \in \mathbb{R} \text{ and } Q_1$$

Discrete Malthusian model

$$x_{n+1} = \alpha \cdot x_n$$

A_1

M_1 (milieu) = { rate of growth, recurrent sequences, numerical simulations, graphical representations, … }

La primera hipòtesi vàlida ha estat trobar

$$\frac{M(n+1)}{M(n)} = c \ , \quad \forall\, c \in \mathbb{R} \ , \quad \text{on } c = \text{cte}$$

Així ens vam adonar que si trobàvem una c vàlida podríem definir la funció $M(n+1) = c\,M(n)$ útil per tots els casos.

Figure 1 - Formulation of H_1

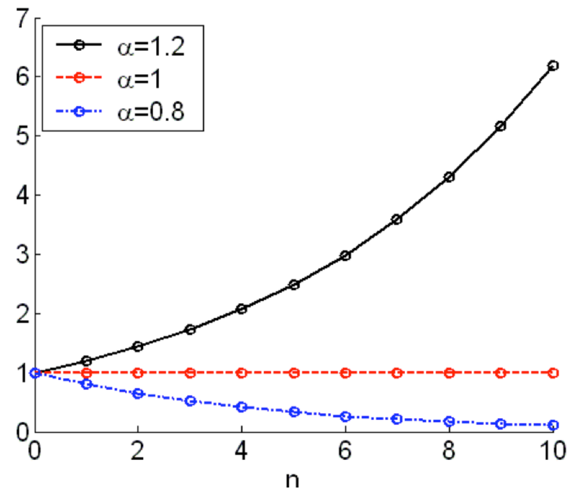


Figure 2 – Numerical simulation method in the case of the Malthusian model M_1

Q_0 ⟹ **(Q_1, A_1)** ⟹

$Q_{1.1}$ (limitation of **A_1**): How can we overcome the unrealistic fact of assuming infinite resources?

Q_0

(Q_1, A_1)

$Q_{1.1}$: Malthusian paradox

$$H_2 : r_n = a - b \cdot x_n,\ a, b \in \mathbb{R} \text{ and } Q_2$$

Discrete logistic model

$$N_{n+1} = \alpha \cdot N_n \cdot \left(1 - \frac{N_n}{K}\right)$$

A_2

Limitations of A_2

$Q_{2.1}$: What does the convergence of (x_n) depend on, in the logistic model case?

$Q_{2.2}$: What does the speed of convergence of (x_n) depend on?

M_2 (milieu) = { M_1, Q_1, A_1, non-linear rate of growth, convergence, …}

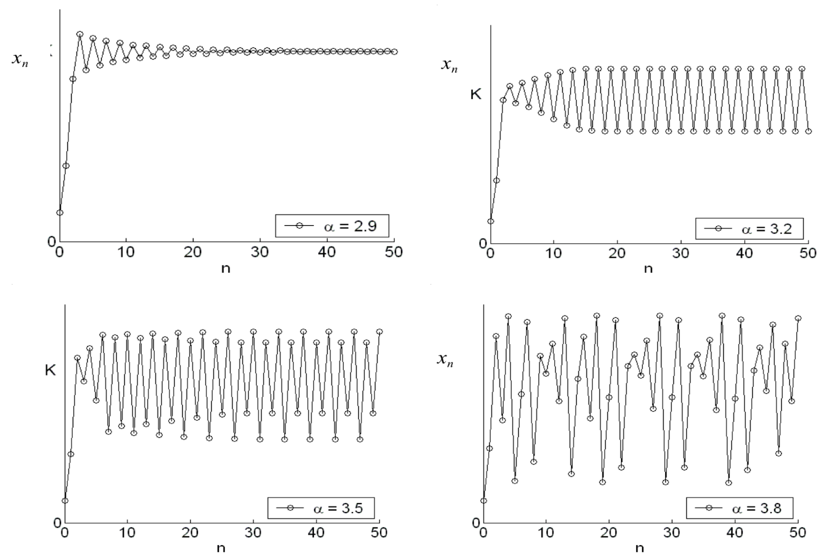
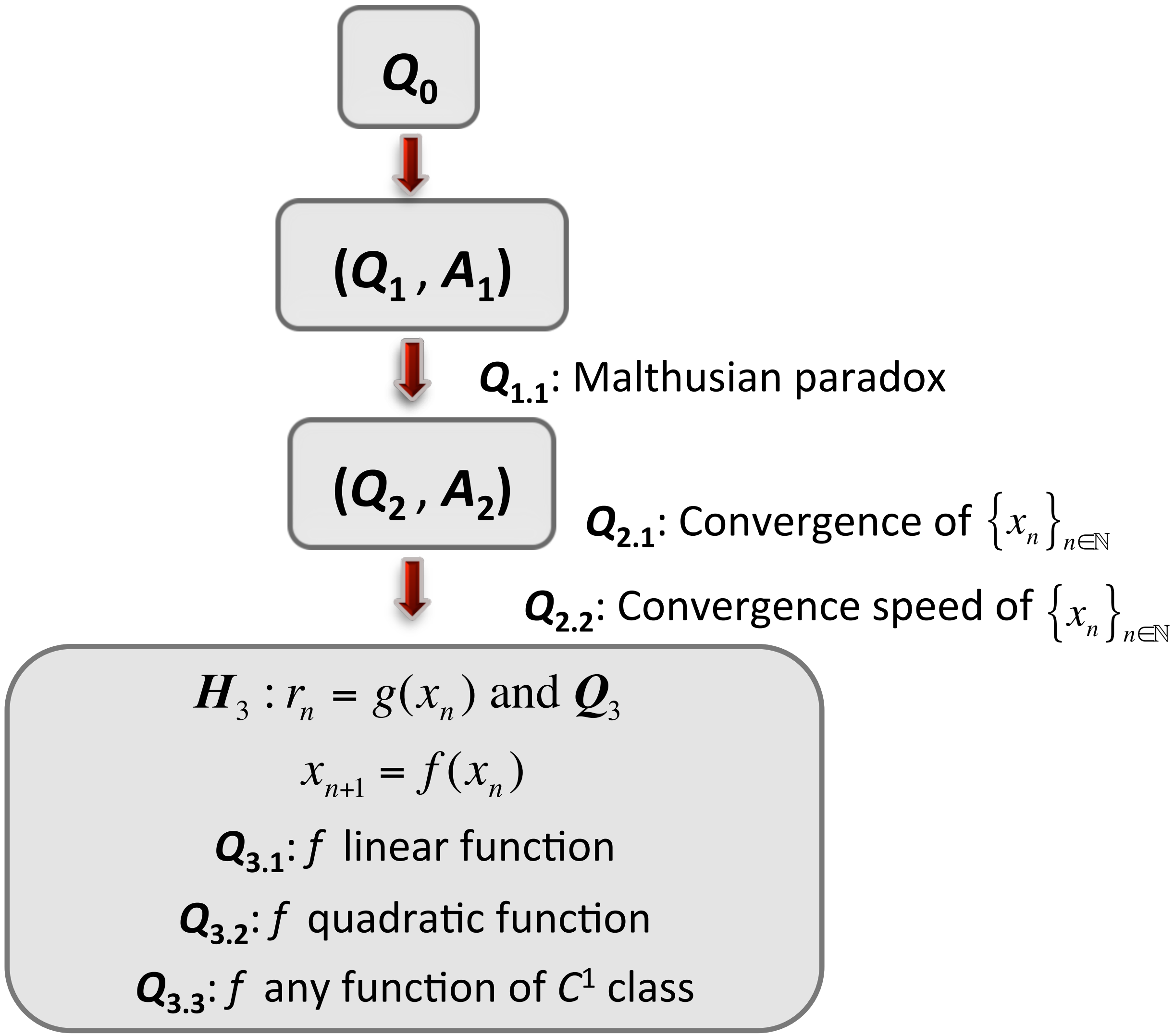
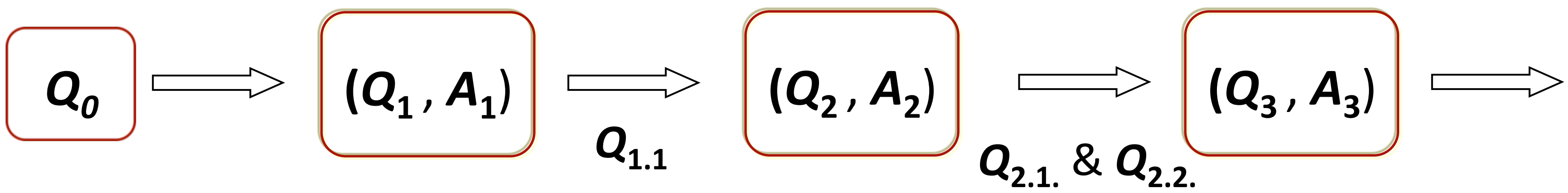
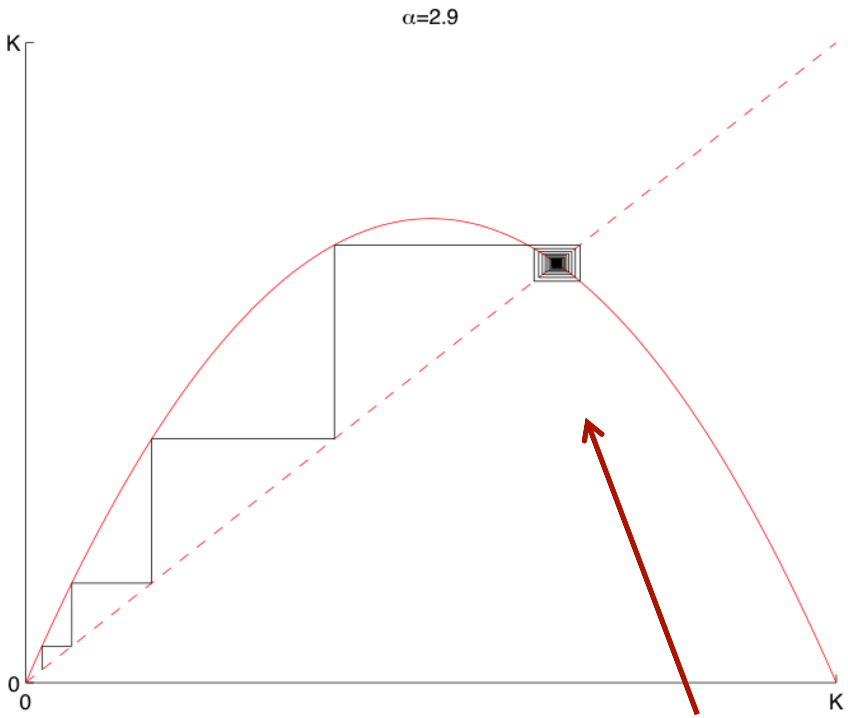
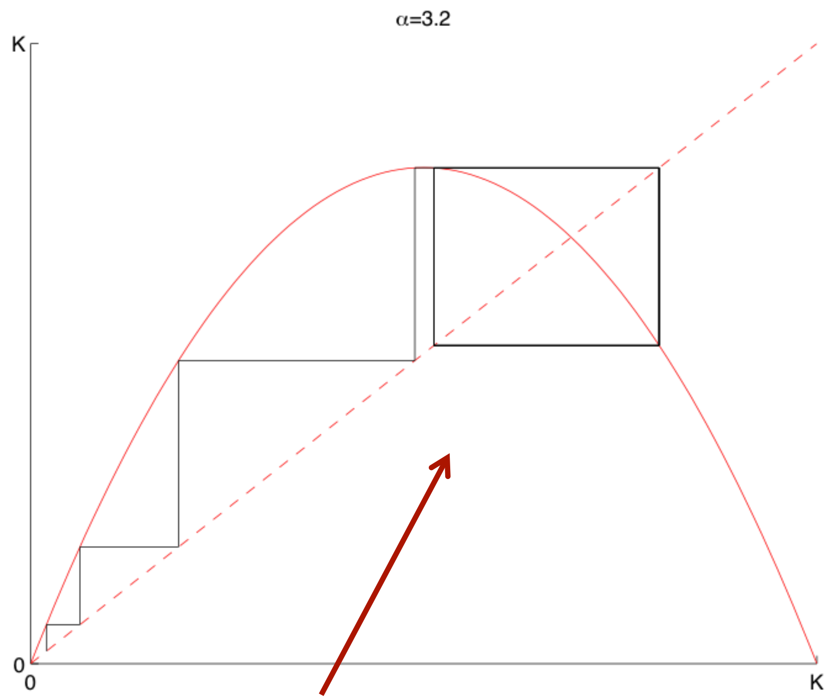


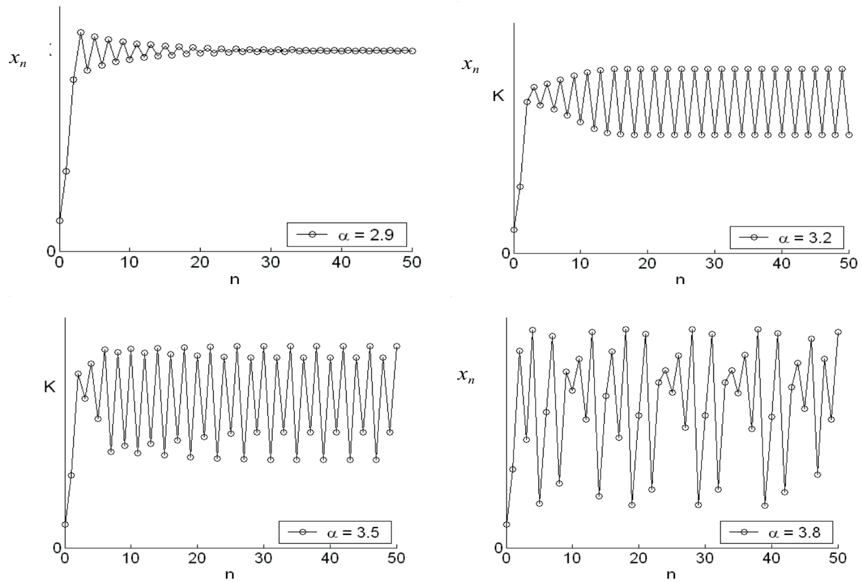
Figure 1 – Numerical simulation in the case of logistic model


Q_0
(Q_1, A_1)
$Q_{1.1}$: Malthusian paradox
(Q_2, A_2)
$Q_{2.1}$: Convergence of $\{x_n\}_{n\in\mathbb{N}}$
$Q_{2.2}$: Convergence speed of $\{x_n\}_{n\in\mathbb{N}}$
$H_3 : r_n = g(x_n)$ and Q_3
$x_{n+1} = f(x_n)$
$Q_{3.1}$: f linear function
$Q_{3.2}$: f quadratic function
$Q_{3.3}$: f any function of C^1 class


Q_0
(Q_1, A_1)
$Q_{1.1}$
(Q_2, A_2)
$Q_{2.1.}$ & $Q_{2.2.}$
(Q_3, A_3)







M_3 (milieu) = { M_2 , Q_2 , A_2 , functions, derivatives, speed of convergence, graphical simulation (spider web), …}

3. The 'a priori' analysis: mathematic and didactic design

SECOND PHASE OF ANALYSIS	2.B. Didactic engineering level

- If the study of $\boldsymbol{Q_0}$ leads to the construction of a whole **mathematical praxeological tree structure.** Its implementation requires the construction of **didactic praxeologies**, which has to create new conditions for the SRP to emerge, taking into account the restrictions of the considered teaching institution:
  - How will the **generating questions** Q_0 be taken into consideration? How to make **questions** and **answers** evolve over the time?
  - What **milieu** and **media** can be available? How to access to them?
  - How are **different responsibilities** shared in the community of study?
  - …

3. The 'a priori' analysis: mathematic and didactic design

SECOND PHASE OF ANALYSIS	2.B. Didactic engineering level

- [MESOGENESIS] How to manage the **media** and **milieu** dialectic? What answers can be obtained, through what media, against what milieu?
- [TOPOGENESIS] What is the 'role' or 'place' (*topos*) of the teacher and the students? How are they sharing the study and research responsibilities (**didactic contract**)?
- [CRONOGENESIS] How to make **questions**, **responses** and **study moments** evolve over time?

3. The ‘a priori’ analysis: mathematic and didactic design

SECOND PHASE OF ANALYSIS	2.B. Didactic engineering level The **mesogenesis** condition

- Search for ‘external’ available answers about models of populations (Malthusian growth, Verlhust model, spire-web graph, chaotic behavior etc.) in on-line encyclopaedia and textbooks
- Experimental milieu through ICT devices: Excel, CAS, etc.
- Ways to institutionalize the temporary answers provided by the students
- Internal validation of answers by working in small groups
- The teacher has to avoid validate the students answers
- …

3. The 'a priori' analysis: mathematic and didactic design

SECOND PHASE OF ANALYSIS	2.B. Didactic engineering level The **topogenesis** condition

- The *topos* of students have an important extension to their traditional responsibilities:
  - They should be able to provide their own temporary answers
  - They are required to formulate new questions and approach them
  - They should be able to introduce in the *milieu* any external work or piece of knowledge they find appropriate
- The teacher has a new role to play as 'director of the study' or 'supervisor of the inquiry'
- The class as a 'mathematical consulting company'

3. The 'a priori' analysis: mathematic and didactic design

SECOND PHASE OF ANALYSIS	2.B. Didactic engineering level The **chronogenesis** condition

- The planning of the work is not under the responsibility of the teacher only, students should take part in it: "planning proposal"
- To avoid the temptation of imposing some possible answers, the groups of students need to be invited to **defend the successive answers** they provide, although they may still be of a **temporary nature**.
- This collective study has to include new 'devices' where it would be possible to: **plan** the work, **elaborate** their answers, **compare** data and models, **write** reports with temporary answers, **defend** their final responses, …

… and the consequent **'dilatation' of temporality**

4. Implementation and 'in vivo' analysis

THIRD PHASE OF ANALYSIS	Implementation, observation and data collection

- We tested the use of these SRP during **five academic years** (from 2005/06 to 2009/10) with first year students of technical engineering degree at the Universitat Autònoma de Barcelona (UAB).
- The testing took place within the one-year course 'Mathematical Foundations of Engineering'.
- It was performed in what we called a **'Mathematical Modelling Workshop'** who was always optional for students but provided an extra point in the final grade (of each semester).

A GENERATING QUESTION – Q_0

Given the size of population over some time period,

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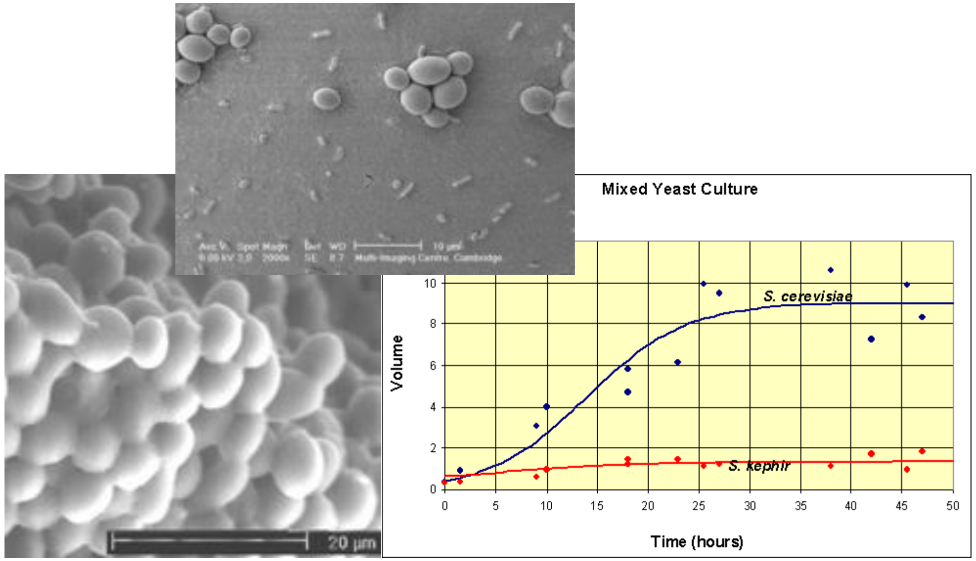
- What sort of assumptions on the population and its surroundings should be made?
- How can one create forecasts and test them?

Year	Population size
1937	8
1938	26
1939	85
1940	274
1941	800
1942	1800



4. Implementation and ‘in vivo’ analysis

THIRD PHASE OF ANALYSIS	Implementation, observation and data collection

- In the workshop, students worked in teams of **2** or **3 members**.
- Assessment of the students during the SRP:
  - In the beginning of each session, the teams had to deliver a report of all the work done during previous sessions (assumptions considered, main problematic questions treated, mathematical models used, ‘temporary’ answers).
  - At the end of each SRC, each student individually had to write a final report of the entire study (evolution of problematic questions, work in and with different models, relationship between them, etc.)
- The **weekly team’ report**, the ‘**secretary**’ and the **team of the week** had a crucial role along the workshop.

FOUTH PHASE OF ANALYSIS	A posteriori analysis and ecology

OUR GENERATIVE QUESTION – Q_0

Given the size of population over some time period,

- Can we predict its size after *n* periods? Is it always possible to predict the long-term behaviour of the population size?
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- How can one create forecasts and test them?

Year	Population size
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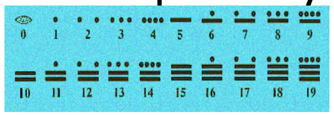
SOME CONDITIONS TO SRP

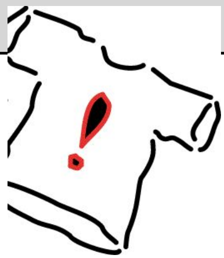
- The study of the Q_0 covers most of the curricula contents of 1st year math course.
- Tree structure of the SRC: many possible paths depending on the models considered.
- Transfer of some responsibilities to the students: work in groups, planning, formulating questions, checking hypothesis, weekly reports with temporary answers, 'secretary of the week', etc.

SOME OF THE CONSTRAINTS TO THE INTEGRATION OF SRP

- Necessity to break the rigidity of the classical structure 'lectures-problem sessions-exams' and integrate SRC in it.
- Necessity of an ad-hoc mathematical discourse available to describe the process.
- Difficulties for keeping in mind the generative question of the mathematical process.
- New devices to help the running of the new didactic contract. Some responsibilities are very difficult to transfer to the students.

Other SRP following the same research methodology

Level	SRP
Pre- and Primary school	▪ Feeding silk warms (Garcia & Ruiz 2013, ICMI 22) ▪ Numeration systems in primary teachers' training (Sierra 2006, TT) 
 **Secondary school**	▪ Saving plans (García 2005) ▪ Comparison of cellphones tariffs (Rodriguez 2006) ▪ How to earn money by selling T-shirts? (Ruiz Munzón 2010) ▪ Graphical transformations of functions (Otero et al, 2013) ▪ Supply and offer equilibrium (Otero et al, 2013)



Other SRP following the same research methodology

Level	SRP
Secondary school	▪ Measure of quantities and real numbers in teachers' training (Licera, in progress) ▪ Progressive discounts in teachers' training (Ruiz, in progress) ▪ Negative numbers and algebra (Cid, in progress) ▪ Epidemic propagation (Lucas, in progress) ▪ AMPERE Project (France)
Universities	*Business and administration degree* ▪ Sales forecast (Serrano 2011) ▪ Human Resources management (Serrano 2011) ▪ Network users dynamics (Barquero & Serrano 2013) *Mathematics degree* ▪ A urban bike sharing system (Barquero 2013)

The teaching of mathematics is an old teaching, which has trouble getting renewed. What is it suffering from? **Basically from the escape, the exhaustion of meaning**.

Taught objects are condensed in **answers to questions that we have lost**. We need to recover these questions: Why are we interested in triangles? Why do we need to simplify fractions, or to rewrite a numerical expression in a canonical form? Why are we interested in the properties of figures? **There are so many questions that have lost their answers in a school culture turned into a lifeless 'museography'**.

Chevallard, Y. (2006).

Étudier et apprendre les mathématiques: vers un renouveau

It is this **school culture** that has to be restored and then brought to life in classrooms. How to do it? By **putting the study of questions at the beginning of the study of mathematics. Questions that have to be taken seriously and truly answered** [...].

From this work, mathematical objects will emerge, which come into life not in a formal and unmotivated way, but highlighted by the part they play in a given intellectual adventure.

Chevallard, Y. (2006).

Étudier et apprendre les mathématiques: vers un renouveau

4. Didactic engineering from TDS to ATD

How to (re)introduce the rationale of mathematics at school and to promote the development of a 'real' mathematical activity?

Adidactic situations within a didactic context	Study and Research Paths (SRP)

How can we design and implement teaching and learning processes that:

- place **adidactic situations** / **problematic questions (raison d'être)** at the core of the mathematical activity developed at school,
- integrate **conditions** (at **micro** and **macro** level) favouring a **change of school paradigm:** from **"visiting works"** (*monumentalism*) to **"questioning the world"** (Chevallard 2012)?

The 'herbartian' schema (Chevallard, 2004)

- Group of **students** X and study **supervisor(s)** Y
- They start from a **question** Q [more or less "big"]
- They should elaborate their own (collective) **answer** $A^{\heartsuit}$ to Q

$$S(X\,;\,Y\,;\,Q) \rightarrow A^{\heartsuit}$$

- To elaborate $A^{\heartsuit}$, an "experimental **milieu**" is needed

$$[S(X\,;\,Y\,;\,Q) \rightarrow M\,] \rightarrow A^{\heartsuit}$$

- The milieu is composed by other bodies of knowledge $A_i^{\lozenge}$ (**labeled answers**) and other **objects** O_k

$$[S(X\,;\,Y\,;\,Q) \rightarrow \{\,A^{\lozenge}_1, A^{\lozenge}_2, \ldots, A^{\lozenge}_n, O_{n+1}, \ldots, O_m\}] \rightarrow A^{\heartsuit}$$

Taking into account a generating question Q

Meeting other questions Q’, Q”, etc. that seem to help approach Q
→ *“squeleton” of the process*

Search answers Ai◊ to Q’, Q”, etc. in the available *media*

DIVERSITY OF POSSIBLE (study and research) PATHS

Evaluation, diffusion and development of the final answer A♥

Evaluation (validation) of answers Ai◊ through an appropriate *milieu*

Elaboration of an own answer A♥

Deconstruction and reconstruction (development) of answers Ai◊ to adapt them to Q

The Herbartian schema as a general model

- **The "transmissive" pedagogy (*monumentalism*):**
  - The initial question remains in the shadow Q
  - The teacher brings an answer A^T ('concept', praxeology) legitimated by culture and validated by him/herself
  - Students assume it as their own answer $A^{\heartsuit} \approx A^T$
  - The relationship to knowledge is a "cultural copying" (*lector - reader*)

$$[S(X\,;\,Y\,;\,Q) \rightarrow \{\,A^T\}] \rightarrow A^{\heartsuit} \approx A^T$$

The Herbartian schema as a general model

- **The "naïf constructivism":**
  - The starting point is a question Q
  - Students have to build an answer $A^{♥}$ "almost from zero", only through some milieus M supplied by Y
  - The classroom is a closed universe, without any contact with the outside, the "media" and other labeled answers $A^{◊}_i$.

$$[S(X\,;\,Y\,;\,Q) \rightarrow \{\, \cancel{A^{◊}_1, A^{◊}_2, \ldots, A^{◊}_n}, O_{n+1}, \ldots, O_m \}] \rightarrow A^{♥}$$

The Herbartian schema as a general model

- **The "modernist" pedagogy:**
  - The teacher raises a question $\boldsymbol{Q}$ to which he/she disposes of a previously established answer $\boldsymbol{A^T}$
  - The teacher brings materials $\boldsymbol{O_k}$ for the study (tasks, docs, …)
  - The teacher leads the students to the answers $\boldsymbol{A_j^\Diamond}$ needed to elaborate $\boldsymbol{A^T}$, raising the appropriate sub-questions $\boldsymbol{Q_i}$ and planning the process of study
  - The students follow the teacher's indications to reconstruct their collective answer, similar to the teacher's one: $A^\heartsuit \approx A^T$

$$[S(X\,;\,Y\,;\,Q) \rightarrow \{\, A^\Diamond_1, A^\Diamond_2, \ldots, A^\Diamond_n, O_{n+1}, \ldots, O_m \}] \rightarrow A^\heartsuit \approx A^T$$

Double openness of the Herbartian schema

1. **The trajectories are not previously determined**
2. **The sharing of responsibilities is not given:**

- Who raises and chooses the initial questions $\boldsymbol{Q}$?
- Who searches the available answers $\boldsymbol{A_j}^{\Diamond}$?
- Who validates $\boldsymbol{A_j}^{\Diamond}$? Who determines the needed milieus $\boldsymbol{M}$?
- How do new questions $\boldsymbol{Q_i}$ emerge? Who decides to address them or not, and how (planning)?
- How the final answer $A^{\heartsuit}$ is delimited? Validated? Disseminated? Related to other $\boldsymbol{A_j}^{\Diamond}$(institutionalized)? Used to raise new questions?

4. Didactic engineering from TDS to ATD

Two main concrete realisations of the Herbartian schema:

STUDY AND RESEARCH PATHS

- **Question *Q* is taken "seriously"** (its answer matters) and there is no previously determined answer $\boldsymbol{A}^{\lozenge}$ to ***Q***
- ***Q*** is deployed into subquestions
- There is a search for available answers $\boldsymbol{A}_j^{\lozenge}$ in the accessible **media** and a confrontation with an experimental **milieu *M*** in order to obtain the final answer $A^{\heartsuit}$

STUDY AND RESEARCH ACTIVITIES

- The starting point is a **previously determined answer** $\boldsymbol{A}^{\lozenge}$ (a praxeology) that is to be "known" (integrated in the milieu ***M***)
- The process of study consists in finding the *rationale* of $\boldsymbol{A}^{\lozenge}$ (generating questions ***Q***) and make its components available through **6 moments of study**.

4. Didactic engineering from TDS to ATD

$$[S(X\,;\,Y\,;\,Q) \rightarrow \{\,A^T\}] \rightarrow A^{\heartsuit} = A^T$$

$$[S(X\,;\,Y\,;\,Q) \rightarrow M\,] \rightarrow A^{\heartsuit}$$

From the school paradigm of **'visiting works'** to **'questioning the world'**

To deal with this huge didactic problem, it has been proposed to design and implement new **DE realisations**:

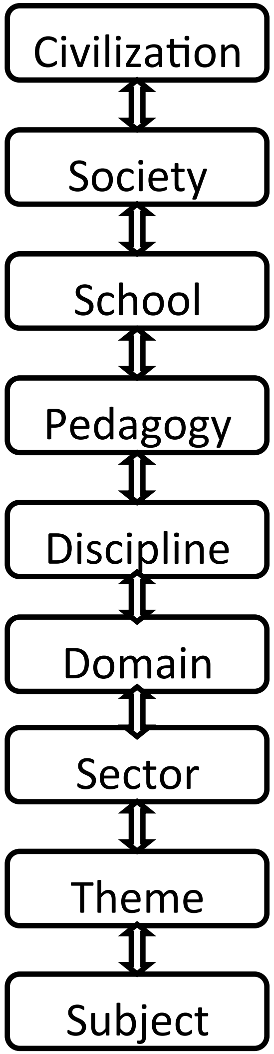
TDS	ATD
▪ **Fundamental situations** and its a-didactic and didactic dimensions ▪ **Milieu** of the situation ▪ **Dialectics:** action – formulation – validation devolution – institutionalization.	▪ **Study and Research Paths** (articulated with SR Activities) ▪ Global designs in terms of **sequences of praxeologies** ▪ **Media - milieu** dialectics ▪ The scale of levels of didactic codetermination (**ecology**)

4. Didactic engineering from TDS to ATD

- The openness assumed by **SRP,** with their generating questions *Q*, temporary answers *A* and works *O*, reflects an unlimited and permanent changing universe.
- The **variations can not be avoided**, not only because of changes in the students and the teachers, but also because of the available *media* and *milieu*. And the unavoidable "invariants" are still unknown.
- A deep '**clinical analysis'** of SRP has to be developed, under different ecological environments, to study the conditions that make their introduction and running possible, and the constraints, at all levels, that may hinder their development.

→ NEED FOR REFUNDING THE NOTION OF DIDACTIC ENGINNERING IN ATD

5. The ecology of SRP: conditions and constraints

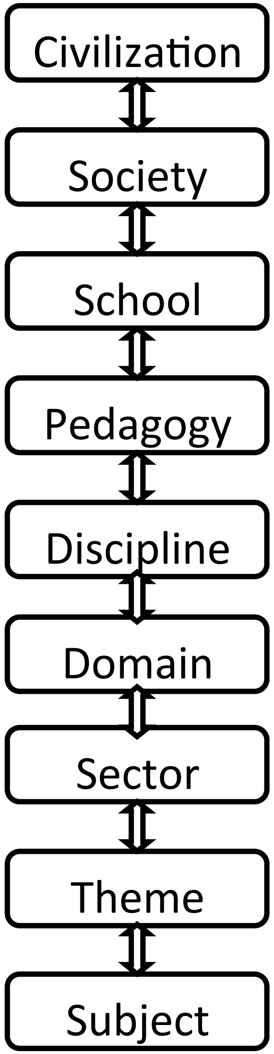


What is the main purpose of SRP experimentations?

- Possibilities for the paradigm of "Questioning the world" to exist in current educational institutions
- Concrete "forms" of implementing this paradigm, depending on the educational level and context
- Invariants or regularities observed throughout the different educational institutions
- New necessities in terms of: mathematical resources, didactic organisations (collective learning and research processes), pedagogical infrastructures, …
- Better understanding the different obstacles or constraints at different levels of the scale

  → **THE PROBLEM OF THE ECOLOGY OF SRP**

5. The ecology of SRP: conditions and constraints

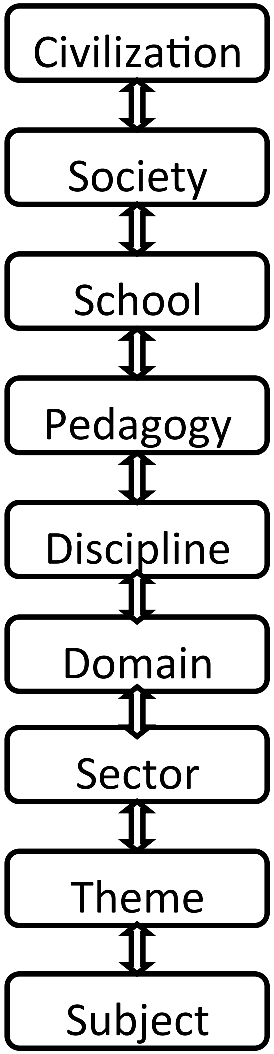


Understanding the specific 'ecology' of SRP/Situations

COMMON CONSTRAINTS:

- "Applicationism" in Natural Sciences and DTP in mathematics as dominant epistemologies
- "Monumentalism" as dominant pedagogy
- Learning as an individual cognitive process
- Syllabi organised in "themes", "sectors" and "domains", not in "Crucial alive questions"
- Frontiers between disciplines can be stronger at school than in research institutions
- The management of media at school has always been mediated by the teacher
- Reduction of the milieu to the teachers answers or to the deductive mathematical logic

5. The ecology of SRP: conditions and constraints



Understanding the specific 'ecology' of SRP/Situations:

SPECIFIC CONSTRAINTS:

- Disappearing of quantities in school (and scholar) mathematics (real numbers, measure, etc.)
- Lack of techniques to work with measure errors
- Numbers come before algebra (negative numbers)
- Models and results are not "named" (populations dynamics, saving plans)
- Excessive focus on equations instead of formulae
- Disappearance of parameters in school algebra
- Excessive focus on the derivative in detriment of the rate of growth
- All those related with the "mixture" of mathematical models and notions with other disciplines

6. Three questions to the audience

(1) In the programme of research inaugurated by TDS, the **transition to the paradigm of questioning the world** becomes crucial: mathematical contents, as well as any other subject matter, need to appear as "truly answers" to "real questions"; not as "monuments to visit".

→ Is this assumption shared by others design approaches? How?

(2) ATD proposes a big enlargement of the unit of analysis to approach the problem of **the ecology of design realisations**, moving outside the classroom to the different levels of the scale of codetermination.

→ How do other design approaches deal with the ecological problem? How have they 'experienced' it?

(3) The **ecological problem** needs to engage different partnerships: researchers and designers, the educational system, the profession of teachers and, maybe also mathematicians.

→ How to involve them, especially the profession of teachers? What role is devoted to them in task design or DE?

Didactical engineering
as a research methodology:
from Fundamental Situations to
Study and Research Paths

Thank you very much!

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