Didactical engineering as a research methodology: the TDS programme and its developments

Marianna Bosch Casabò, Universitat Ramon Llull (URL)
Berta Barquero Farràs, Universitat de Barcelona (UB)
www.atd-tad.org

ICMI Study 22 – Task Design. July 24th, 2013

Outline

Task-design based research:

(1) experimentally based in an educational context

(2) pays special attention to the description, analysis and organisation of the content to be taught.

Didactics of mathematics emerged at the beginning of the eighties with the works of the French researcher Guy Brousseau and his co-operators and has given raised to approaches as TDS, ATD, Joint-action, Double approach, Theory of Conceptual Fields, etc. **More than 30 years of research…**

In this talk, we will limit to what are for us the main approaches: TDS and ATD and to what they share related to what is known as ***didactic engineering* (DE).**

Outline

1. **Didactic engineering (DE) as a research methodology: an example**
2. **DE and the project of Didactics as an experimental science**
3. **First period (70s – mid 90s): the golden age of DE**
4. **Second period (from mid 90s on): the enlargement of the unit of analysis and the ecological approach**
5. **DE within the Anthropological Theory of the Didactic**

- **DIDACTIC ENGINEERING (Artigue 2009):**

"The notion of DE emerged in the early 80s to answer two needs:
(1) To take into account the **complexity of classroom**, at a time research mainly relied on laboratory experiments and questionnaires
(2) To thing the relationships between **research and action**."

1. DE as a research methodology: an example

5 main characteristics of DE as a theory-based intervention

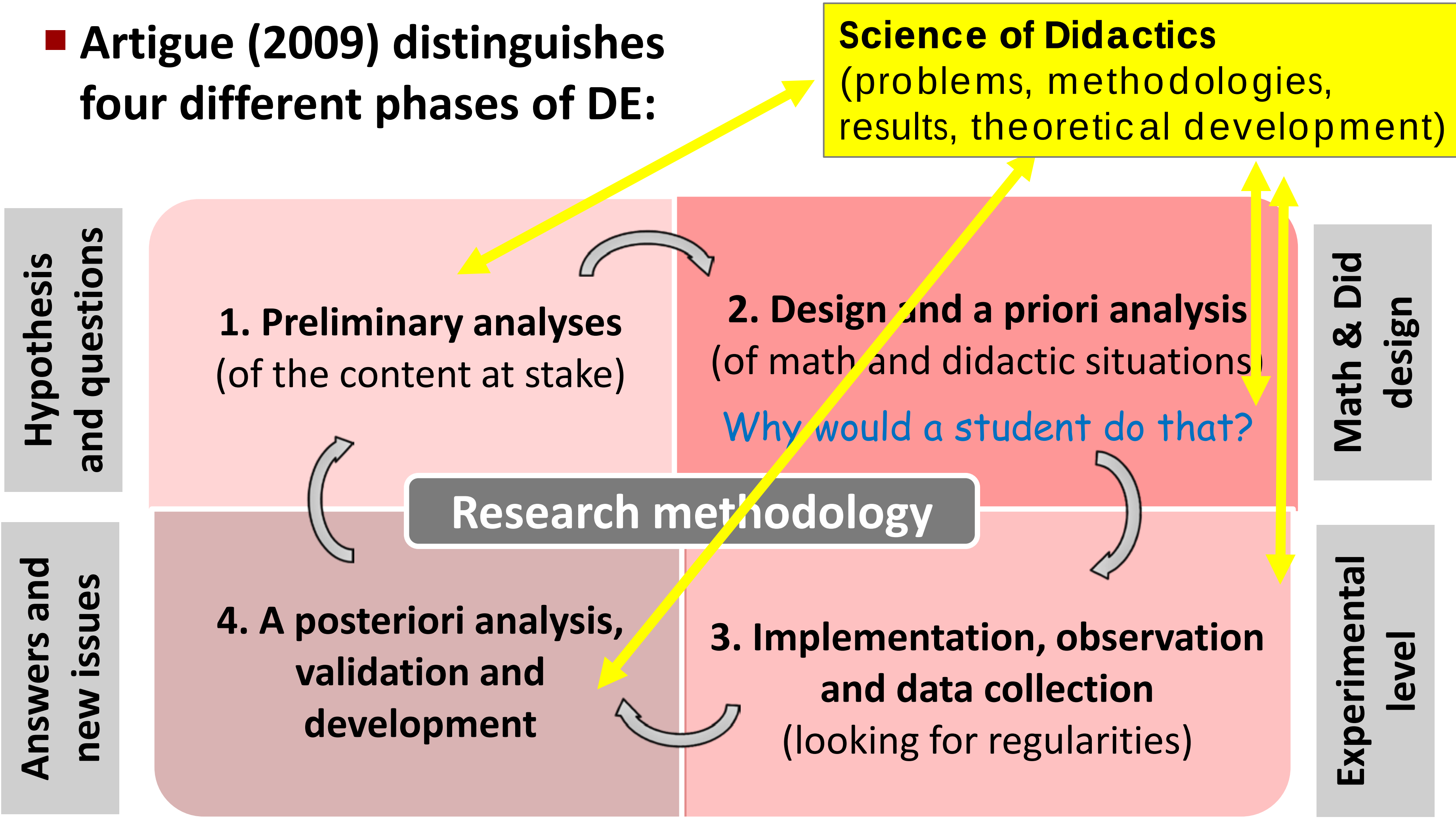
→ The central role given to the notion of ***situation***
→ The crucial attention paid to the **epistemology of knowledge**
→ The importance given to the characteristics of the ***milieu***
→ Three different functionalities of mathematical knowledge ***action – formulation – validation*** (dialectics)
→ The teachers' role: organise the relationships between the **adidactic** and the **didactic** dimensions of situations ***devolution, institutionalisation***

1. DE as a research methodology: an example

- **EXAMPLE: The measurement of quantities at Primary school**
  - Bessot A. & Heberhard M. (1983) Une approche didactique des problèmes de la mesure, *Recherches en Didactique des mathématiques*, 4.3, 293-324.
  - Brousseau N. & Brousseau G. (1987) La mesure en CM1. Compte rendu d'activités. *Bordeaux: Publications de l'IREM de Bordeaux.*
  - Brousseau G., Brousseau N. (1991-1992), Le poids d'un récipient. Etude de problèmes du mesurage en CM. Grand N, 50, 65-87.
  - Brousseau G. (2002) Les grandeurs dans la scolarité obligatoire. In J.-L. Dorier et al. (eds) *Actes de la XIe Ecole d'été de didactique des mathématiques*, *Corps, 21-30 Août 2001*, Grenoble: La Pensée Sauvage.

1. DE as a research methodology: an example

- **Artigue (2009) distinguishes four different phases of DE:**



1. DE as a research methodology: an example

FIRST PHASE: PRELIMINARY ANALYSIS	Questions, hypothesis and phenomena

- **Why teaching the measurement of quantities at primary school? What mathematical entities and practices are related to it? What social activities? → Epistemological analysis (rationale)**
- **What is taught as measurement of quantities at school? What was taught, could be taught? → Didactic transposition process**
- **What didactic *phenomena* are related to it?**

→Magnitudes or quantities disappear from school mathematical work (phenomenon of *numerisation* of measurement)

→ The choice of the unit of measure (gauge) is never raised

→A fuzzy role played by units in modelling strategies and calculations

→Lack of a mathematical theory to work with physical quantities

1. DE as a research methodology: an example

- **Hans Freudenthal (1972, pp. 197-198)**

  To count people and eggs there are natural units. To measure quantities, one needs gauges; the result of the measuring procedure is a number, which measures the quantity. There is a variety of gauges, because there is a variety of magnitudes; length, area, volume, height, mass, work, current intensity, air pressure, and monetary value are notions that become magnitudes by measuring procedures. Sometimes it is not clear why some magnitudes need different gauges. [...] A few of these gauges are learned in arithmetic instruction, and as far as he needs it, the physicist develops a rational measure system. **In between a large domain is no man's land. This is the fault of the mathematician.**

1. DE as a research methodology: an example

- **Whitney Hassler (1968) The mathematics of physical quantities (Parts I and II) *American Mathematical Monthly.***
  Develops a mathematical theory to justify calculations with quantities such as 6 cm ÷ 2 sec = 3 cm/sec , etc.

→ The "mathematical infrastructure" exists, but it does not permeate the prevailing dominant mathematical *culture* (universities, teachers, etc.)

1. DE as a research methodology: an example

SECOND PHASE: A PRIORI ANALYSIS	Design of mathematical and didactic situations

- **Fundamental (math) situation**
  - Game against a **milieu**
  - Basic **initial strategy** available
  - Cognitive and didactic **variables** generating a set of situations
  - **Action / Formulation / Validation**

What measuring consists in? What activity (game) does it enable to carry out (win)?

- **Didactic situations (conditions of reconstruction)**
  - Sequence of **didactic** and **adidactic** sit

How to get students play the game? How to relate it to the external world?

Devolution-Action-Formulation-Validation-Institutionalisation- …

1. DE as a research methodology: an example

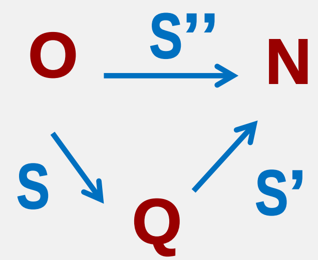
SECOND PHASE: A PRIORI ANALYSIS	**Design of mathematical and didactic situations**

- **Fundamental (math) situation**
  - Game against a **milieu**
  - Basic **initial strategy** available
  - Cognitive and didactic **variables**
  - **Action / Formulation / Validation**

Three intertwined "universes" to characterize a measure process:

- **Concrete objects and "comparison"**
- **Quantities**
- **Numbers**

3 types of relationships (situations):



1. DE as a research methodology: an example

SECOND PHASE: A PRIORI ANALYSIS	Design of mathematical and didactic situations

Comparison of concrete objects:
Paper ribbons of different lengths compared "side by side"

ACTION

The more gauges we have, the less numbers (scalars) we need

MODEL

Choice of a small set of gauges and "characte-rization" of the objects Written expressions in $\langle u_1, u_2, \ldots, u_n \rangle_K$

FORMULATION AND VALIDATION

Colored ribbons of different sizes
Brown short ribbons (gauges)
"Write a message to get a ribbon of the same size as yours"

Green = 2 Yellow + 1 Brown
Blue = 3 Yellow + 2 Brown

1. DE as a research methodology: an example

SECOND PHASE: A PRIORI ANALYSIS	Design of mathematical and didactic situations

Comparison of concrete objects:
Paper ribbons of different lengths compared "side by side"

ACTION

The less gauges we have, the more numbers (scalars) we need

MODEL

Choice of a small set of gauges and "characte-rization" of the objects Written expressions in $<u_1, u_2, \ldots, u_n>_K$

FORMULATION AND VALIDATION

MODEL

Reduction to one unit $< u >_K \cong K$
Numbers and calculation

Colored ribbons of different sizes
Brown short ribbons (gauges)
"Write a message to get a ribbon of the same size as yours"

Green = 4 Brown
Blue = 5 Brown + $\frac{1}{2}$ Brown

1. DE as a research methodology: an example

THIRD PHASE: EXPERIMENTATION	Implementation, observation, data collection

il faut d2 baguette et une ~~..~~ ou
il manque un peu de la fin

3 baguettes entieres + la mortier d'une baguette.

Il faut mètre 5 baguettes et plier la petite baguette qui
s'appelle u en trois parties égale

2 u + a little bit more
$3\ u + \frac{1}{2}\ u$
$5u + 1/3\ u$
$4\ u - \frac{1}{2}\ u$

1. DE as a research methodology: an example

THIRD PHASE: EXPERI…	Implementation, observation, …

COMPLETE FAILURE!
Students started to work with cm:
"It's about 20 cm", etc.
The adidactic turns into didactic

Comparison of co…
Paper ribbons of …
compared "side b…

MODEL

Choice of a small set of gauges a…
"characterization" of the objects
Written expressions in $<u_1, u_2, \ldots, u_n>_K$

MODEL

Reduction to one unit $< u >_K \cong K$
Numbers and calculation

Colored ribbons of different sizes
Brown short ribbons (gauges)
"Write a message to get a ribbon of the same size as yours"

Green = 4 Brown
Bleu = 5 Brown + ½ Brown

1. DE as a research methodology: an example

SECOND PHASE: A PRIORI ANALYSIS	THIRD PHASE: EXPERIMENTATION

Comparison of concrete objects:
Objects compared with a scale

Lengths are changed into weights (less familiar to the students)

MODEL

Choice of a small set of units (gauges) and "characterization" of the objects
Written expressions in $<u_1, u_2, \ldots, u_n>_K$

MODEL

Reduction to one unit $< u >_K \cong K$
Numbers and calculation

Nails and small plates as gauges:
1 plate = 14 big nails
1 big nail = 10 medium
1 medium = 15 small

Pencil case = 2 c + 8 b + 17 s
Pencil case = 34 b + 32 m + 7 s

1. DE as a research methodology: an example

- **The final result is a long sequence of 25 "activities"**
  Brousseau & Brousseau 1987 "La mesure en CM1" (grade 4)

Measurement of lengths: communication game; studying the messages
Measurement of weights: communication game; messages; work on the writings; comparing expressions; conversions; adding weights; comparing; ...
Measurement of time: time and duration; numbers in basis 60
Legal units of weight: conversions
Finding the weight of an empty recipient: first part; second part (challenge)
Measurement of lengths: adding lengths; decimal measures (coma)
Writing decimal measures […]
Operations with decimal measures […]

1. DE as a research methodology: an example

FOURTH PHASE: A POSTERIORI ANALYSIS	Results, new phenomena, new questions

- **Familiar milieus** are not always didactically productive (similar didactic phenomenon with negative numbers)
- **“The weight of the recipient”:** complex relationship between the empirical milieu and the properties deduced (students consider a linear relationship in spite of the errors of measure)
- **MORE GENERALLY: New epistemological analyses and formulations of the fundamental situation on the measurement of quantities** (Chevallard & Bosch 2000, Bahujama 2001, Brousseau 2002, Sierra 2006)
- **Extensions to Secondary level: measure & real numbers** (Licera 2011)
- **RESEARCH PROBLEM: How to change the “dominant epistemology” among teachers & mathematicians to reintroduce quantities at school?**

2. Didactic engineering and the science of Didactics

- **THE CONTEXT:** In the 80s, Guy Brousseau called for the constitution of a science called **Didactics** centred on the study of the "**conditions presiding the diffusion of the mathematical works of knowledge useful to human beings and their societies**".
  These conditions are not considered in an isolate way or through its performers (students, teachers), but **through systems**: situations, institutions, praxeologies, …

- **Didactics** appeared as a reaction to the **New Math reform** of the 60s and 70s, considered as "**a utopia totally ignoring all the difficulties and laws of the diffusion of knowledge and practices in a society**, especially the issues of time of reaction of the systems. It believed and died in the illusion of transparency of didactic facts."

L'émergence d'une science de la didactique des mathématiques: motifs et enjeux Brousseau (2010)

2. Didactic engineering and the science of Didactics

- **Didactic Engineering (DE)** is the way for Didactics to organise its relationship to the empirical world

... even if not all empirical work is DE

- **The object of DE is (Brousseau 2005):**
  - To produce, organise, and test **situations** as instruments for the didactic action of the teacher
  - To make explicit and communicate possible **options**
  - To **justify the choices** among the options by all the theoretical and experimental means of Didactics
- **Situations? Options? Choices? (Brousseau 2007)**
  - The notion of situation plays an important role as a model to understand and explain [teaching and learning] episodes and as a pattern to design sequences of lessons. However, **a "situation" is first of all a model to describe, explain and forecast collective behaviours**.

2. Didactic engineering and the science of Didactics

- **Double purpose of DE: R + D**
  - **Phenomenotechnique** (Bachelard 1967): to study didactic phenomena through their controlled production in educational settings (under ethical circumstances)
  - **Development**: to study ordinary education and make it evolve

"Experimental epistemology"

- **DE needs a double rupture:**
  - Allow oneself to **question the discipline from the inside**, without accepting the proposals of mathematicians as necessities and proposing alternative reconstructions of mathematical knowledge and activities.

Autonomous science

  - Have the same attitude towards the **other disciplines** (psychology, pedagogy, sociology, etc.) concerning the effects of their proposals on mathematical practices and knowledge.

L'ingénierie didactique au coeur de la théorie des situations Bessot in Margolinas et al (2011)

2. Didactic engineering and the science of Didactics

- **DE as a research methodology: *the way for Didactics to organise its relationship to the empirical world***

Constant **dialectic** between the **design** of new teaching proposals, the **observation** of the students and teachers activities and the development of the **theoretical approach** (new notions, new results, new phenomena or regularities to take into account):

→ **Didactic contract and its paradoxes**

→ **Institutionalisation and devolution**

→ **Didactic transposition**

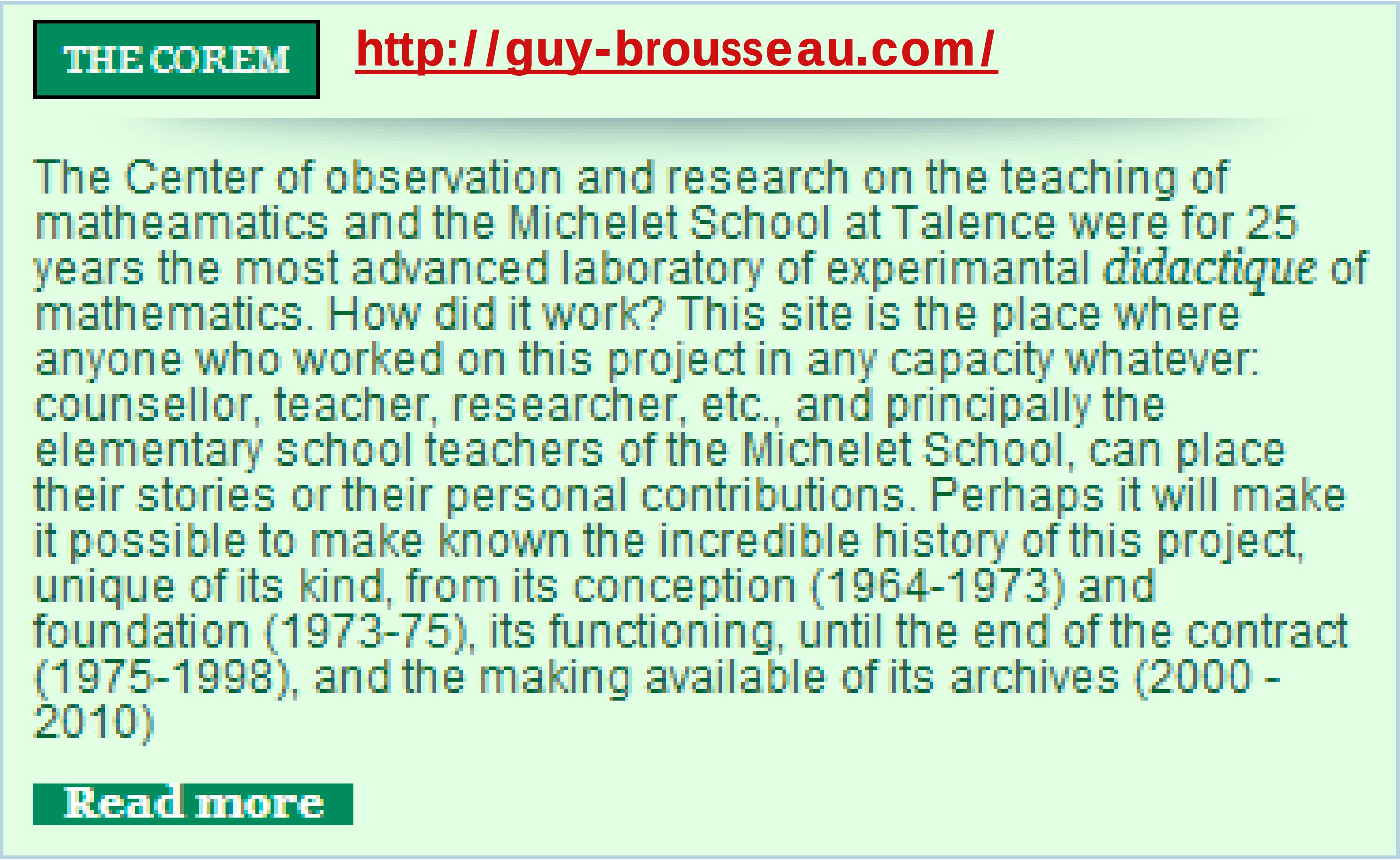
A specific "**vigilance**" to base all the results obtained in an **empirical basis**, to protect researchers from "**ideologies**" and from the **implicit institutional viewpoints** on both educational facts and mathematical knowledge.

3. First period (80s – mid 90s): the golden age of DE

DE in France:

- Design and experimentation in the classroom of (quite long) sequences of lessons at all levels, mainly primary, but also secondary and university, in collaboration with teachers and different institutions
- Institutions involved:
  - **IREM** (Instituts de Recherche sur l'Enseignement des Math.)
  - **COREM (Centre pour l'Observation et la Recherche sur l'Ens. des Maths.) and the Michelet School at Talence (Bordeaux, France, 1970-2000)**

3. First period (80s – mid 90s): the golden age of DE



3. First period (80s – mid 90s): the golden age of DE

Curricular reconstructions based on TDS (nursery & primary school)

- Logic (enumeration, classification, designation)
- Numbers and counting
- Numbers and operations
- **Measurement of quantities**
- **Fractions, proportionality, rationals & decimals**
- Geometry and space
- Statistics

There are many others, not all based on TDS and at different school levels

→*Most of these resources are available at* guy-brousseau.com/ , visa.inrp.fr/visa *(videos) and* imac.uji.es/ *(written reports)*

→*DE based on TDS is used as a basis for a national curriculum project in Chile called LEM (reading, writing, mathematics)* www.centrofelixklein.cl/

4. Second period (mid 90s -): enlargement and ecology

- ***“The difficulties met with the transmissions of DE realisations [reproducibility] have shown the necessity of considering the teacher as a full actor of the didactic situation”*** **Artigue (2009)**

→Increasing importance given to **naturalistic observations** in ordinary classrooms

→Development of **theoretical constructions** inside and outside TDS

→A substantial body of research on **teachers’ practices**: Margolinas 1999, Chevallard 1999, Robert & Rogalski 2002, Laborde & Perrin-Glorian 2005, Sensevy et al 2005, Barbé et al 2005, Cirade 2006, …

“A more flexible and realistic vision of the sharing of the mathe-matical responsibilities in the classroom between teacher and students, and a more sophisticated vision of the teacher role.”

4. Second period (mid 90s -): enlargement and ecology

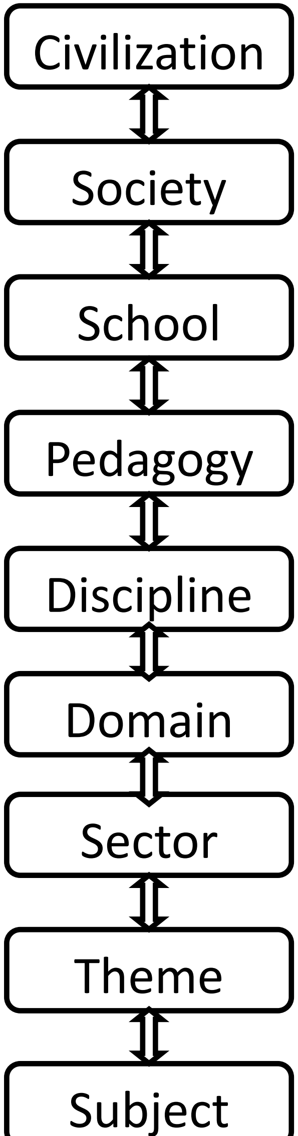
Didactics = study of the conditions presiding the diffusion of the mathematical works of knowledge useful to human beings and their societies **+ study of the institutional constraints (ecology)**

→ **Conditions specific to each domain, sector or "piece" of mathematical knowledge** and that enable to "functionalise" it: adidactic situations (reconstruction of school mathematics)

→ **Conditions related to the didactic contract** (sharing of responsibilities teacher-students): didactic situations, paradoxese

→ **Conditions related to the *pedagogical* contract** (teaching of different disciplines), **the *school* contract, the *social* contract, ... *that are implicit in most of the engineering proposals***

4. Second period (mid 90s -): enlargement and ecology



SCALE OF LEVELS OF DIDACTIC CODETERMINATION

- What is assumed "to be there" for a DE to exist? **Institutional conditions**
- **These conditions imply constraints. Which ones?**
  - Teaching disciplines? No man's lands (measure)
  - Organised in domains? Disconnections
  - Curriculum in terms of "works to visit"? Need to "functionalise" any piece of knowledge (fundamental situations, milieu) and give priority to the study of questions
  - …

→ THE PROBLEM OF THE ECOLOGY OF MATHEMATICAL AND DIDACTIC ACTIVITIES

5. DE within the Anthropological Theory of the Didactic

- **DE according to the Anthropological theory of the didactic**
  "The 'methodological' problem in didactics appears in relation to the complete scale of conditions and constraints [...] and, of course, in relation to the conditions and constraints 'brought' or 'created' by the students and the teacher within a didactic system." (Chevallard 2009)

- **We need an enlarged conception of teaching and learning processes** to avoid taking some of the conditions/constraints for granted: **Herbartian schema & Study and research paths**

5. DE within the Anthropological Theory of the Didactic

- **PEDAGOGICAL LEVEL:**
  Move from the paradigm of **"Visiting works *(oeuvres)*"** to the paradigm of **"Questioning the world" (*inquiry*)** (Chevallard 2012)
- **DISCIPLINE LEVEL:**
  Put forward the ***problematic questions*** instead of the works built to provide answers to these questions
- **The frontier between "mathematics" and "didactics" is blurred: doing mathematics includes 'study', 'research' and 'supervision'**
- **The numerous constraints at the different levels of the scale need to be taken into account. The *ecological problem* (conditions and constraints) remains crucial.**

Hans van Ginkel (2004)

Another effort that has to be made regarding curricula concerns the ***problematisation of issues****. All too often university courses limit themselves to themes […] without making the effort to inquire* ***what issues form the core of these themes****. I believe this lack of problematisation largely accounts for the slow progress of true understanding. Naming the theme and going for the easy answer tend to become in this context an institutional alibi – an alibi which does not necessarily serve the advancement of knowledge and research. The problematisation of issues also helps to* ***bring the links between the different issues into focus****. Without a clear understanding of these interlinkages no effective understanding and management of issues will be possible.*

Didactical engineering as a research methodology: the TDS programme and its developments

Marianna Bosch Casabò, Universitat Ramon Llull (URL)
Berta Barquero Farràs, Universitat de Barcelona (UB)
www.atd-tad.org

ICMI Study 22 – Task Design. July 24th, 2013

ON THURSDAY

Didactical engineering as a research methodology: from fundamental situations to study and research paths

Berta Barquero Farràs, Universitat de Barcelona (UB)
Marianna Bosch Casabò, Universitat Ramon Llull (URL)
www.atd-tad.org

ICMI Study 22 – Task Design. July 24th, 2013

REFERENCES (to be completed…)

- Artigue M. (1990). Ingénierie didactique. *Recherches en Didactique des Mathématiques*, vol. 9/3, 281-308. (English translation : Artigue, M. (1992). Didactical Engineering. In R. Douady & A. Mercier (eds.), *Recherches en Didactique des Mathématiques*, *Selected papers*, pp. 41-70.).
- Artigue, M. (2009). Didactical design in mathematics education. In C. Winslow (ed.), Nordic Research in Mathematics Education. *Proceedings from NORMA08* in Copenhaguen, April 21-April 25, 2008. pp. 7-16. Sense Publishers.
- Chevallard, Y. (2011). La notion d'ingénierie didactique, un concept à refonder. Questionnement et éléments de réponse à partir de la TAD. In C. Margolinas, M. Abboud-Blanchard , L. Bueno-Ravel, N. Douek, A. Flückiger, P. Gibel, F. Vandebrouck, F. Wozniak (Dirs), *En amont et en aval des ingénieries didactiques*. Grenoble: La Pensée sauvage.
- Chevallard, Y. (2012). Teaching mathematics in tomorrow's society: a case for an oncoming counterparadigm. *Plenary talk in 12th ICME*, Seoul, Korea.

REFERENCES (to be completed…)

- Brousseau, G. (1997). *Theory of didactical situations*. Dordrecht: Kluwer Academic Publishers.
- Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the IVth Congress of the European Society for Research in Mathematics Education* (pp. 22–30). Barcelona: FUNDEMI-IQS.
- Laborde, C., & Perrin-Glorian, M. J. (Eds.). (2005). Teaching situations as objects of research: Empirical studies within theoretical perspectives. *Educational Studies in Mathematics*, *59*(1–3).
- Margolinas, C. (1999). Les pratiques de l'enseignant: Une étude de didactique des mathématiques: recherche de synthèses et perspectives. In M. Bailleul (Ed.), *Actes de la10ème Ecole d'Eté de Didactique des Mathématiques* (pp. 10-33). Caen: IUFM de Caen et A.R.D.M.
- Margolinas, C. (2002). Situations, milieux, connaissances – analyse de l'activité du professeur. In J.-L. Dorier, M. Artaud, M. Artigue, R. Berthelot & R. Floris (Ed.), *Actes de la11ème Ecole d'Eté de Didactique des Mathématiques* (pp. 141-156). Grenoble: La Pensée Sauvage.
- Robert, A., & Rogalski, J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques: Une double approche. *Canadian Journal of Science, Mathematics and Technology*, *2*(4), 505–528.