# Proceedings of the Seventeenth ICMI Study Conference

# “Technology Revisited”

## Hanoi University of Technology
## December 3-8 2006

**Edited by Celia Hoyles, Jean-Baptiste Lagrange, Le Hung Son and Nathalie Sinclair**

# Part 1:
# General Information

Proceedings of the Seventeenth Study Conference of the International Commission on Mathematical Instruction

***Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain***

Editors: Celia Hoyles, Jean-Baptiste Lagrange, Le Hung Son and Nathalie Sinclair

Hanoi Institute of Technology
No. 1 - Dai Co Viet road - Hanoi – Vietnam

**Co-Chairs of ICMI Study 17**

Prof. Celia Hoyles Institute of Education, University of London, 20 Bedford Way, London WC1H 0AL, UK

T: +44 20 7612 6659 F: +44 20 7612 6792

c.hoyles@ioe.ac.uk

Prof. Jean-Baptiste Lagrange IUFM de Reims, 11 rue Gabriel Voisin, 51688 Reims Cedex 2, B.P. 337, France

T: +33 3 26 50 36 25 F: +33 3 26 50 30 39

jb.lagrange@reims.iufm.fr

We are grateful to our official sponsors

- Autograph
- Texas Instruments
- United Nations Educational, Scientific and Cultural Organisation (Unesco)

and to other companies supporting the conference

- Cabrilog
- Key curriculum press.

## About ICMI

The International Commission on Mathematical Instruction, ICMI, was first established at the International Congress of Mathematicians held in Rome in 1908, on the suggestion of the American mathematician and historian of mathematics David Eugene Smith. The first President of ICMI was Felix Klein. From the very beginning, the international journal L'Enseignement Mathématique was adopted as the official organ of ICMI - which it is still today. ICMI also publishes, a Bulletin twice a year. Starting with Bulletin No. 39, December 1995, the ICMI Bulletin is accessible on the internet http://www.mathunion.org/ICMI/bulletin.

### Members of the International Programme Committee

Prof. Celia Hoyles, Institute of Education, University of London, UK

Prof. Jean-Baptiste Lagrange, IUFM de Reims, France

Prof. Douglas Clements, Department of Learning and Instruction, University at Buffalo, US (up to August 2005)

Dr. Paul Drijvers, Freudenthal Institute, The Netherlands

Dr. Lulu Healy, Centre of Science & Technology, Pontificia Universidade Católica (PUC), Brazil

Prof. Cyril Julie, School of Science and Mathematics Education, University of the Western Cape, South Africa

Prof. Maria Alessandra Mariotti, Department of Science, Mathematics and Informatics, Università di Siena, Italy

Prof. John Olive, Department of Mathematics and Science Education, The University of Georgia, US (after August 2005)

Dr. Ana Isabel Sacristan, Department of Mathematics Education, Cinvestav, Mexico

Dr. Evgenia Sendova, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria

Dr. Nathalie Sinclair, Department of Mathematics, Michigan State University, US

Prof. Le Hung Son, Faculty of Applied Mathematics, Hanoi University of Technology, Vietnam

Dr. Colleen Vale, School of Education, Victoria University of Technology, Australia

Prof. Bernard R. Hodgson, Département de mathématiques et de statistique, Université Laval, Canada (*ex officio*, Secretary-General of ICMI)

## Members of the Local Committee

Prof. Dr. Nguyen Minh Hien, Minister for Training and Education Vietnam (Honor Chairman)

Prof. Dr. Hoang Ba Chu, Rector of Hanoi University of Technology (HUT) (Chairman)

Prof. Dr. Tran van Nhung, Deputy Minister For Training and Education Vietnam

Prof. Dr. Banh Tien Long, Deputy Minister for Training and Education Vietnam.

Ass. Prof. Nguyen Canh Luong, Vice Rector of HUT

Ass. Prof. Ha Duyen Tu, Vice Rector of HUT

Dr. Tong Dinh Quy, Dean of Faculty of Applied Mathematics and Informatics, HUT

Dr. Hoang Xuan Lan, Head of Department for International Relations, HUT

Magist. Nguyen Huu Do, Deputy Director of Department for Education Hanoi

Dr. Dinh Sy Dai, Director of High School Chu Van An, Hanoi

Prof. Dr. Sc. Nguyen Van Mau, Rector of Hanoi University of Sciences.

Prof. Dr. Sc. Le Hung Son, HUT, Vice President of Hanoi Mathematical Society.

Prof. Dr. Nguyen Dinh Tri, Former President of Vietnam Mathematical Society

## Foreword

The "ICMI Studies" series was launched in the mid-80s in order to investigate themes (key issues or topic areas) of particular significance to the theory or practice of contemporary mathematics education. The general aim of a Study is to provide an up-to-date presentation and analysis of the state-of-the-art concerning a theme, whether by identifying and describing current research contributions and their findings, or by identifying and discussing issues involving genuine controversies or dilemmas and the different positions towards them held by various mathematics educators.

*The typical scheme of an ICMI Study*

Once a theme has been decided upon by the Executive Committee of ICMI, the initial step is the appointment by the EC of an International Program Committee that, on behalf of ICMI, is responsible for conducting the Study. The first task of the IPC is the production of a ***Discussion Document*** to set the stage for the Study by identifying, presenting and discussing in a preliminary manner a general *problématique*, a number of key issues and sub-themes related to the theme of the Study. Readers are invited to react in writing to the Discussion Document by submitting ideas, proposals, abstracts or drafts of papers for consideration by the IPC.

On the basis of the reactions received and of the deliberations within the IPC itself, the IPC invites a limited number (50-100) of individuals to participate in an invited ***Study Conference,*** which will form a working forum for investigating the theme of the Study.

Finally, a ***Study Volume***, resulting from the contributions presented to the Study Conference as well as from the discussions that took place there, is published under the general editorship of the ICMI Study Series editors, the President and Secretary of ICMI. The nature of such a volume is *not* that of conference proceedings.

*Technology revisited*

At its annual meeting in July 2002, the ICMI Executive Committee decided on the launching of a new ICMI Study whose theme was "Technology revisited" (the word "revisited" was used because the very first ICMI Study, held in Strasbourg in 1985, was on the influence of computers and informatics on mathematics and its teaching).

The International Program Committee (IPC) first met in April 2004 to draft the Discussion Document. It was decided that a major focus of the ICMI Study 17 would be cultural diversity and how this diversity impinges on the use of digital technologies in mathematics teaching and learning, particularly in developing countries. The Discussion Document was then disseminated through the mathematics education community. About 90 papers were submitted from numerous countries around the world indicating that this major focus was well received by the community.

The submitted papers were read and reviewed by three or four members of the IPC. Invitations were then issued to a selection of individuals based on the reviews, the areas of interest and expertise, the goals and objectives of the conference, and the geographic representation.

These proceedings bring together the papers submitted by invited participants. They have generally undergone very light revision. Readers may expect that more developed versions of these papers will later be submitted to journals.

During the conference, the invited participants will contribute to working groups. Each working group will address the issues raised in the Discussion Document by considering more specifically one of the following themes:

Theme A: Implementation of curricula: Issues of access and equity

Theme B: Teachers and teaching

Theme C: Learning and assessing mathematics with and through digital technologies

Theme D: Design of learning environments and curricula.

Each theme is introduced below.

The deliberation of the working groups will form the basis of an edited book, which will continue the ICMI study series.

Finally, we wish to offer our gratitude to the groups and individuals who are helping us make this study a success:

- the Minister of Education of Vietnam and the Rector of Hanoi University of Technology for offering to organize the conference in Hanoi,
- the members of the local committee,
- the official sponsors of the conference

  Autograph

  Texas Instruments

  Unesco
- and other companies supporting the conference
  Cabrilog
  Key curriculum press.

The editors

August 2006

## Introduction to the themes

*Theme A: Implementation of curricula: Issues of access and equity*

Since the first ICMI Study, developments in digital technology have resulted in a range of applications for mathematics and mathematics teaching and learning. Furthermore, governments have developed policies to promote the learning and use of digital technologies throughout education systems in general as well as for mathematics learning in particular. Thus there has been some systemic implementation of digital technologies in mathematics education as a result of policy initiatives, alongside more scattered implementation as a result of specific innovations and initiatives. Neither centralised nor local initiatives have tended to result in widespread and sustained use of digital technologies in mathematics curricula and in teaching. Access to, and use of, digital technologies differs between countries, and within countries, according to socio-economic, gender and cultural factors.

The influence and place of digital technology at all levels of mathematics education provides a unique opportunity to examine reform and change in mathematics curricula and teaching by, for example, examining the political, economic, social and cultural factors that promote or impede access to and integration of digital technologies for quality learning in mathematics. Issues related to scaling up initiatives and the challenges of systemic change will be explored in this theme. We will also seek to understand how cultural practices in technology-integrated mathematics enhance, or erode, equity and agency in mathematics education.

*Possible questions:*

What theoretical frameworks and methodologies are helpful in understanding issues related to the widespread implementation of digital technologies for access and equity in mathematics education?

- How have mathematics curricula and values changed to reflect developments in mathematical knowledge and practices afforded by digital technologies? How should aspects of mathematics curriculum be changed in response to technology-mediated knowledge?
- How is access to digital technology impacting on the mathematical practices that occur outside of formal education settings?
- How have countries with different economic capacity or with different cultural heritage and practices implemented digital technologies in mathematics education?

- How and to what extent has the use of digital technologies in mathematics education enabled, or eroded, equity and agency in mathematics education?
- How and to what extent has technology-integrated mathematics contributed to, or reduced, differences between countries in participation and achievement in mathematics?
- What approaches, strategies or factors foster or impede the implementation of technology-rich mathematics education? What issues are involved for policy-makers, administrators and teachers for systemic change in curriculum, teaching and the organisation of technology resources in educational settings?
- What have we learned about the process of change and reform in mathematics education through our successful and unsuccessful experiences of implementing digital technologies in mathematics education?
- What can students and teachers with limited access to digital technologies for mathematics learning, or access to modest technologies, do with technology that is empowering for students?
- What is the potential for creating virtual communities for mathematics learning and permitting communication between individuals from different educational settings?
- How can the use of digital technologies in mathematics education support the learning of students with special needs?
- How can digital technologies be used in mathematics learning to respond to the diverse needs of all learners, regardless of mathematics achievement, sex, class, ethnicity or cultural background?

*Theme B: Teachers and teaching*

The integration of any new artifact into a teaching situation can be expected to alter its existing equilibrium and requires teachers to undergo a complex process of adaptation. In the case of digital technologies, the modifications required of routine practices are likely to be particularly pronounced. Not only might different pedagogical approaches be appropriate, but also the teacher needs to reconsider how the new representations and alternative learning strategies made available through technology use might change along with what could be taught, and how and when. Various frameworks, drawing from both theory and practice, are currently employed to analyse the role of the teacher in orchestrating technology-integrated mathematics learning. This theme will consider the complementarities and contrasts between these frameworks and how they are operationalised in the face of ever-evolving resources. It will also address implications of these complex issues for pre-service and for ongoing teacher professional development.

***Possible questions:***

- What theoretical frameworks and methodologies illuminate the teacher's role in technology-integrated environments for mathematics learning?
- What kinds of pedagogical approaches and classroom organisations can be employed in technology-integrated environments including distance teaching and how can they be evaluated?
- How can a focus on technological tools help us understand the ways in which mathematical practices and the roles of the teacher vary across settings?
- How can teachers be supported in deciding why, when and how to implement technological resources into their teaching practices?
- What kinds of pre-service education and professional development programs are appropriate to prepare teachers to use technology in their mathematics classrooms and to help them to sustain ongoing use?
- What can we learn from teachers who use, or who have tried to use, digital technologies for mathematics teaching?
- How are teachers' beliefs, attitudes, mathematical and pedagogical knowledge shaped and shaped by their use of digital technologies in mathematics teaching and how are these issues influenced by access to resources and by differences in culture?

*Theme C: Learning and assessing mathematics with and through digital technologies*

This theme will concentrate on developing understandings of how students learn mathematics with digital technologies and the implications of the integration of technological tools into mathematics teaching for assessment practices. Its foci will include consideration of how digital technologies might be employed to open windows on learners' developing knowledge, and on how interactions with digital tools mediate learning trajectories. Additionally, the theme will address the challenges involved in balancing use of mental, paper-and-pencil, and digital tools in both assessment and teaching activities.

*Possible questions:*

- What theoretical approaches and methodologies help to illuminate students' learning of mathematics in technology-integrated environments? What are the relationships between these approaches and how do they compare or contrast with other theories of mathematics learning?

- How does the use of different digital technologies influence the learning of different mathematical concepts and the shape of the trajectories through which the learning develops?
- How can technology-integrated environments be designed so as to capture significant moments of learning?
- How can the assessment of students' mathematical learning be designed to take into account the integration of digital technologies and the ways that digital technologies might have been used in the learning of mathematics?
- How can and should learning and assessment practices reflect differences in resource level and in cultural heritage?
- How can the benefits of existing technologies be maximised for the benefit of mathematics teaching and learning?
- What is the potential contribution to mathematics learning of different levels of interactivity and different modalities of interaction, and how might this potential be realised?
- What is special about the potential of collaborative study of mathematics whilst physically separated, and how might this potential be harnessed so as to support mathematics learning?
- What is the potential for creating virtual communities for mathematics learning and permitting communication between individuals from different educational settings?
- What new types of mathematical knowledge and practices emerge as a result of access to digital technologies, particularly computational, dynamic visualisation and communication technologies, for example in a mathematics laboratory?

*Theme D: Design of learning environments and curricula*

The purpose of this theme is to focus closely on the issues and challenges involved in designing mathematics learning environments that integrate digital technologies while recognising that the tools made available in such environments can and do shape mathematical activity in ways that to some extent are predicable and in some not. In addition to considering the specific affordances and constraints of different digital technologies for structuring mathematical learning experiences (including various software packages, hardware configurations and the Internet), this theme will consider the implications of design decisions on tools, curriculum, teaching and learning.

*Possible questions:*

- What theoretical frameworks and methodologies are helpful in understanding how design issues impact upon the teaching and learning of mathematics?
- How does the use of different technology-integrated environments both influence the learning of different mathematical concepts and shape the trajectories through which the learning develops?
- How can technology-integrated environments be designed so as to foster significant mathematical thinking and learning opportunities for students?
- What kinds of mathematical activities might different digital technologies afford and how can learning experiences (including the tools, the tasks and the settings) be designed to take advantage of these affordances?
- How can technology-integrated learning environments be designed so as to remain sensitive to persistent challenges, for example swift and inevitable obsolescence and ongoing maintenance costs?
- How can technology-integrated learning environments be designed so as to influence and change curriculum, and how can this be achieved consistently over time?
- How are new types of technology-mediated mathematical knowledge and practices related to current classroom curricula and values, and how should aspects of mathematics curriculum therefore be deleted or changed?

# Discussion Document for the Seventeenth ICMI Study "Technology Revisited"

## Introduction and rationale for ICMI Study 17

This document announces a new study to be conducted by the International Commission on Mathematical Instruction. The focus of this study, the seventeenth led by ICMI, will be the use of digital technologies in mathematics teaching and learning in countries across the world. ICMI Study 17 will seek to distinguish and address major theoretical challenges that face researchers working in this area, alongside practical challenges associated with implementing digital technologies in ways that fulfil their potential for teaching and learning mathematics.

In London in April 2004, the International Programme Committee (IPC) for ICMI Study 17 agreed upon three principles. First, it was decided that cultural diversity and how this diversity impinges on the use of digital technologies in mathematics teaching and learning particularly in developing countries would be one major focus for ICMI Study 17. Thus, the Study will specifically seek contributions from authors from developing countries and from those who offer a broad range of experiences. This orientation will illuminate our understandings of how far the expectations for digital technologies have been realized, in different phases of education, different countries, and different contexts within and outside educational institutions. We also judge that this focus will help the Study in the task of identifying and elaborating the resources needed to optimise the chances of sustained strategic change for the benefit of mathematics teaching and learning. Second, it was decided to delineate a set of themes to serve as the organising framework for the Study conference and for the subsequent ICMI publication. And third, the IPC agreed on a set of approaches through which participants in the Study will be invited to contribute to the themes.

This discussion document first presents the background to the Study, the challenges faced and the scope envisaged for the work. These sections then lead on to the descriptions of the seven organising themes of the Study and the five approaches that have been distinguished. Because the IPC anticipates that the Study conference will be organised around discussion within the themes (with some overarching sessions), each proposed contribution to the Study will be asked to state the theme into which it will fit. The IPC will also expect that any contribution to a theme would encompass a broad range of approaches. Finally the discussion document outlines the organisation, timing and location of the Study conference and the timetable of milestones leading up to the conference.

*The first ICMI Study*

This is the second time that an ICMI study has centred on the use of computers in mathematics education. In fact, the first ever ICMI study (undertaken in 1985) was titled, "The Influence of Computers and Informatics on Mathematics and its

Teaching". This Study represented one of the first attempts to develop a critical view of the role and influence on what was termed 'informatics' on mathematics education. The Study had a substantial impact, with the study volume first published by Cambridge University Pressi, but once out of print, was reissued in 1992 by UNESCOii. This latter volume included an overview written by Burkhardt and Fraser, which identified several questions on which future studies might focus: "Where are we going? Where do we want to go? Why? How do we know? How may we find out more? How do we get it to happen?" Authors also noted a major mismatch in timescale between the fast pace of change of technology, the slower changes in research in mathematics and the still slower changes in mathematics education itself and the curricula, particularly in relation to integrating technology into teaching and learning. Thus, in the seven years since the original 1985 meeting some aspects had moved swiftly while others had remained essentially static.

Some 20 years later, ICMI has felt the need to lead another critical reflection in this area. Consideration of the first Study provides an interesting starting point: for example, even a cursory glance reveals that the authors worked in a rather restricted set of countries (Europe and North America), and the focus of the papers was almost exclusively on using computers to model in an exploratory manner rather advanced mathematical ideas. Many authors pointed to the potential of using 'symbolic manipulators' in courses of calculus or linear algebra in order to allow students to focus on conceptual rather than procedural or technical issues.

It is worthwhile noting that despite the fact that many authors identified the potential of the systems they described, many also noted that there was little evidence of any significant impact on the mathematics curriculum of secondary schools and universities (primary-level mathematics was not considered). Reasons put forward included mathematicians' lack of experience in using the systems and the absence of strategic approaches to change.

*Digital technologies: Further developments and questions of impact*

Since 1992, there have been substantive developments in digital technologies, both in terms of hardware and software, encompassing for instance, calculator technology and the use of the Internet, as well as computers of all types, and including digital technology widely used in society such as mobile phones and digital cameras. These developments together with associated software have potential implications for mathematics teaching and learning at all phases of education, and indeed outside the formal contexts of education. ICMI Study 17 seeks to take stock of these developments and assess their impact in the broadest terms. Our intention is that the interpretation of the term digital technology will be broad: encompassing for instance, calculator technology and the use of the Internet as well as computers of all types.

Alongside developments in digital technologies, there have been changes in the goals, objectives and orientation of studies using them, as well as a broadening of the perspectives, theoretical frameworks and methodologies adopted. Recently, it might be argued that the pace of technological development has increased still further. Digital technologies are becoming ever more ubiquitous and their influence touch most, if not all, education systems. In many countries, it is hard to conceive of a world without high-speed interactivity and connectivity.

All these developments have spawned an increasing range of studies, some focused on the impact of specific software, others looking more broadly at the interaction of teachers, students and technologies. As Hoyles and Noss (2003) claimed: "there are major research issues for mathematics education that are shaping and being shaped by the issues confronting 'technologists'"iii How far these issues have been addressed and their potential realised for the improvement of mathematics teaching, learning and the curriculum, remains a subject of debate that will continue in ICMI Study 17.

*Key challenges for ICMI Study 17*

Through ICMI Study 17, we will seek to identify and analyse some of the challenges in mathematics teaching and learning, practically and theoretically, in the light of the use of digital technologies. Most digital technologies do not make explicit how they work or how they can be used in mathematics education. This means that taking account of their design, particularly in terms of implications for epistemology, is a central challenge. But, as we attempt to incorporate new technological tools into teaching and learning, we also intend to make progress in trying to understand how the related epistemological structures are mediated by learning communities, and reciprocally, how learning communities are shaped by the artefacts and technologies in use.

In ICMI Study 17, we will not only seek to recognise the diversity in available software and hardware for use in mathematics education, but also consider seriously the influence of diverse curricula organisations, from highly centralised to locally autonomous, and of the availability of resources in different countries – whether it is access to handheld devices, computers or to the web. ICMI Study 17 will also seek to take account of cultural diversity and how issues of culture alongside those related to teacher beliefs and practice all shape the way digital technologies are used and their impact on mathematics and its teaching and learning.

## 2. Scope of the Study

Digital technologies are evolving fast and pervade more and more aspects of social life, human thinking and knowledge. Twenty years after the first ICMI study, the 'stand-alone' computer is no longer paradigmatic tool and citizens experience everyday a variety of computerised tools (from small electronic appliances more or less sophisticated calculators, mobile communication devices to worldwide networks). All this hardware is accompanied by an ever growing diversity of software: from applets that most people can build and use, to large applications designed and implemented by large transnational teams, few of which with educational goals.

In this varied and evolving context, ICMI Study 17 will seek a balance between two, potentially contradicting, aims:

- to reflect on actual uses of technology in mathematics education, avoiding mere speculation on hypothetical prospects,
- to address the range of hardware and software with a potential to impact upon or contribute to mathematics teaching and learning.

It is anticipated that this balance will be achieved through the elaboration of themes by which the Study will be organised, and by specifically encouraging contributions from diverse cultures with differing experiences, most notably from developing countries. It is timely to evaluate ideas and new prospects offered by tools as well as provide updated analyses. In addition, to complement the current body of knowledge, we need to address new developments, involving for example connectivity, and their impact on and contribution to mathematics epistemology and mathematics teaching and learning.

Mathematics Education itself has evolved since the first study. Epistemological studies of mathematics and psychological approaches to learning mathematics are extensively supplemented by research within primary, secondary and university mathematics contexts as well as contexts outside of educational institutions. Issues around the teacher and classroom practices have been widely addressed using a number of theoretical frameworks, some specific to mathematics, some more general. The Study will aim to consider mathematics education in all these diverse and complementary aspects.

While we noted the first Study was largely focused on modelling mathematics, more recently work has focussed much more generally on the multitude of ways technology can shape teaching and learning mathematics, while reciprocally being shaped by its use. For example, studies have looked at the complex process of instrumental genesis, the role of the teacher and the connection of tool use and traditional techniquesiv. New robust paradigms for thinking about tool use in the context of mathematics education are beginning to emerge and ICMI Study 17 aims to take a further step forward in this direction.

We now turn to descriptions of the seven themes around which ICME 17 will be organised.

**3. The Themes of ICMI Study 17**

The themes around which ICME 17 will be organised have been described above.

**4. Approaches to the Themes**

Different and complementary approaches to the themes of ICMI Study 17 are expected to follow from the diversity in its scope. Our broad classification distinguishes six approaches. Contributions to any theme will be expected to take seriously more than one approach; one approach might define the choice of theme, while other complementary approaches ensure a rounded contribution.

*1. Impact on mathematics*

Technology modifies mathematical practices in research, influences the development of new mathematical domains and changes the relationship between mathematics and society. Thus, mathematics, its different facets and domains, and their evolution are central to this approach.

*2. Roles of different digital technologies*

The Study cannot ignore the diversity of technological tools now available. Contributions should discuss the influence of a given technology on research findings, and make appropriate comparisons (for example, between programmable microworlds and expressive tools (see Hoyles & Noss, 2000), between handhelds and more traditional computer use, between face-to-face or distance teaching and learning or learning through the web). Technological tools however should not to be taken as given and considerations of curriculum and classroom practices should help to examine a tool's design in view of approaches to teaching and learning.

*3. Contribution to learning mathematics*

This approach brings a focus on questions such as how does a human being learn mathematics with technology and what mathematics does (s)he learn. These questions could be addressed in terms of cognition or affect, with regard to mathematical fields, activities and contexts at different school levels, or in contexts in and out of school.

*4. The role of the teacher*

Some consideration of the teacher is essential to developing any understanding of the integration of digital technologies in classrooms. Investigation can take different forms. It might involve observation and analyses of effective uses of digital technologies by teachers in naturalistic settings; interventionist design experiments, “didactical engineering", that implement and evaluate new curricula and teacher practice; or comparative analyses in programmes of teacher education of different approaches to the use of digital technologies in terms of their effectiveness and impact.

*5. Theoretical frameworks*

There is a diversity of theoretical frameworks in technology and mathematics education, some inspired by more general conceptualizations and others concerning paradigms for thinking about tool use. Contributions to the Study based on empirical research should seek to make explicit their theoretical foundations and the implications of these frameworks. More theoretically oriented contributions might work on convergences between frameworks. Alongside these different theories are also different paradigms of research that must be mobilised to address a range of research questions and to achieve a range of different outcomes.

*6. Connectivity and virtual networks for learning*

Digital technologies have already changed the ways we think about interacting with mathematical objects, especially in terms of dynamic visualisations and the multiple connections that can be made between different kinds of symbolic representation. At the same time, we are seeing rapid developments in the ways that it is possible to interact and collaborate through technological devices. This approach seeks to consider the potential and challenges for mathematics education of the increasing levels of connectivity, both within and between classrooms, will be considered.

---

[i] Churchhouse, R. F. (ed). 1986. *The Influence of Computers and Informatics on Mathematics and its Teaching.* ICMI Study Series, Cambridge University Press

[ii] Cornu, B. & Ralston, A. (eds). 1992. *The Influence of Computers and Informatics on Mathematics and its Teaching.* 2nd edition, UNESCO (Science and Technology Education No. 44)

[iii] Hoyles, C. & Noss, R. 2003. What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick and F. K. S. Leung (Eds.) *Second International Handbook of Mathematics Education*. Dordrecht: Kluwer Academic Publishers

[iv] Guin, D., Ruthven, K. & Trouche, K (eds). 2005. *The Didatical Challenge of Symbolic Calculators: Turning a Computational Device into a Mathematical Instrument* Springer USA