

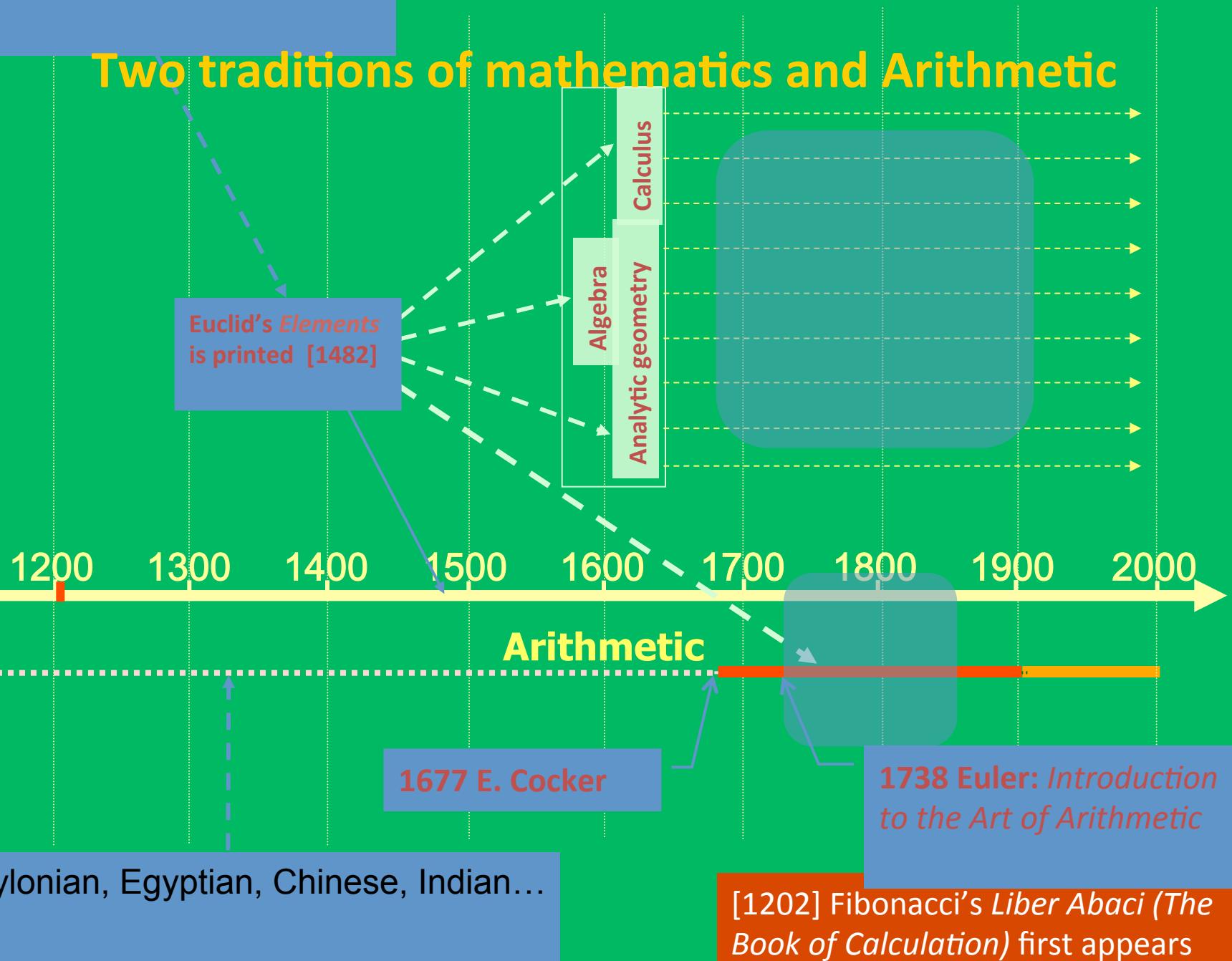
# The fundamental idea of mathematical tasks design in china: the origin and development

Xuhua Sun

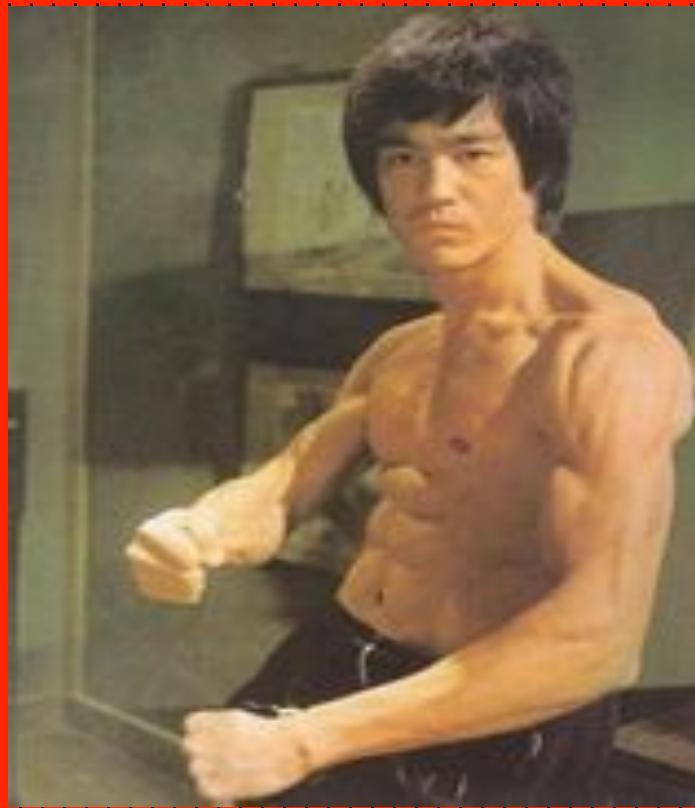
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## Two traditions of mathematics and Arithmetic



An important fact:  
gong-fu education stresses its base-  
mabu (马步) in China



# Chinese math education

- Why have Chinese experts named their curriculum goal “**two base**”(双基) ?
- Why is Chinese instruction described as “**teaching with variation**”(变式教学) by Chinese experts ?

# An hidden framework

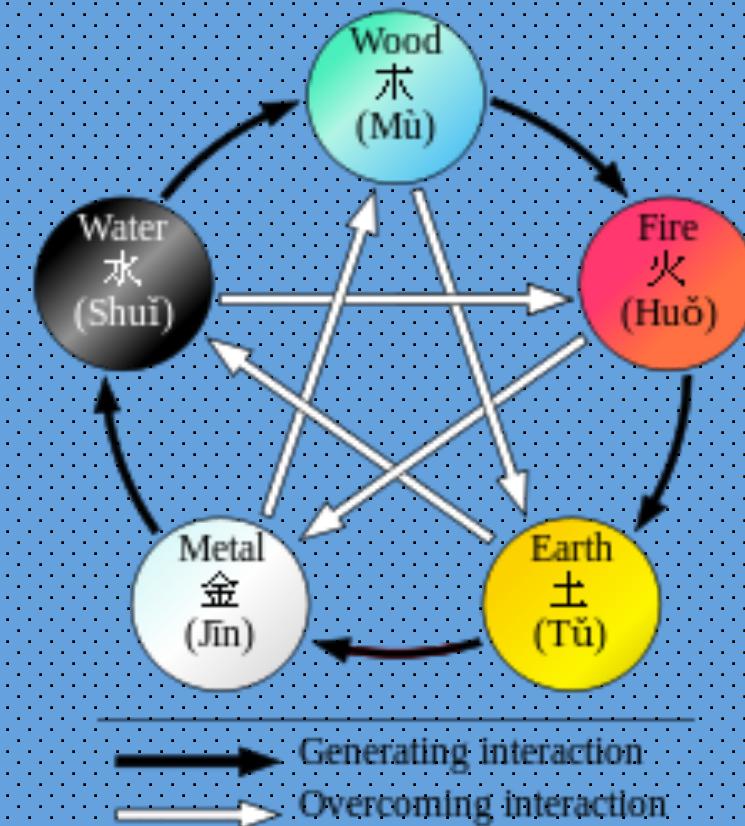
- Obviously, the invariant (double base) and variation (变式 bianshi teaching) naturally became an important mathematical task design framework in china.
- Why ‘naturally’? Why not so naturally in other countries?

# The hidden curriculum belief

- There are the invariants beyond the multiple variations (萬變不離其宗).
- Therefore, a good curriculum should play a role to abstract the invariant concepts from a varied situation and apply these invariant concepts to the varied situations (in Chinese 變中發現不變, 以不變應萬變).

# The hidden belief would trace back to the origin of Chinese culture

- Yin-yang theory .
- The five elements theory.
- I Ching
- All these stress the relationship between the invariant and variant elements



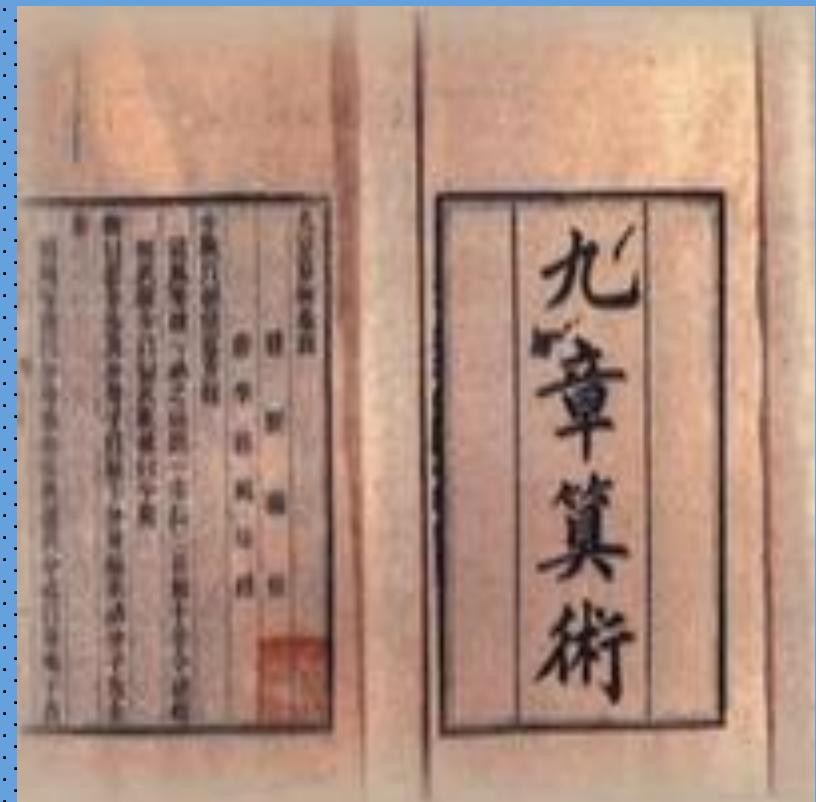
# *I Ching* as the root of all subjects

- *I Ching* has played important role in shaping Chinese culture as **holistic fundamental idea of mathematics , science, medicine, cosmology and philosophy** (**Needham, 1959**).
- Leibniz invented the computer language system by researching *I Ching*



# Jiuzhang Suanshu (JZSS)

- JZSS is a representative subject –mathematics influenced by *I Ching*
- Nine chapters is Chinese two-thousand-year textbook
- Missing Chinese history  
In M. Kline.  
*Mathematical Thought from Ancient to Modern Times*



# Elements vs. Jiuzhang Suanshu

- Pure mathematics vs. applied mathematics
- Deductive reasoning vs. inductive reasoning
- Theory-orientation vs. practice-orientation
- Does it indicate two kind models of task design?!

# The idea of “categorizing”

- The idea of “categorizing” includes both the **product** of “categorizing” and the **process** of “categorizing”
- was called “**grasping ways beyond categories**”; “**categorizing in order to unite categories**” (以法通類, 以類相從) in ancient Chinese.

# “categorizing model” in JZSS and its commentary

- Categorizing variant problems into their categories(歸類)
- The 246 problems were categorized into 9 categories .

# categorizing model : categorizing problems in JZSS

- 1. 方田 Fangtian - Rectangular fields;
- 2. 粟米 Sumi - Millet and rice. Exchange of commodities at different rates; pricing;
- 3. 衰分 Cuifen - Proportional distribution. Distribution of commodities and money at proportional rates;
- 4. 少廣 Shaoguang - The lesser breadth. Division by mixed numbers;
- 5. 商功 Shanggong - Consultations on works. Volumes of solids of various shapes;
- 6. 均輸 Junshu - Equitable taxation;
- 7. 盈不足 Yinguzu - Excess and deficit. Linear problems solved using the principle known later in the West as the rule of false position;
- 8. 方程 Fangcheng - The rectangular array. Systems of linear equations,
- 9. 勾股 Gougu - Base and altitude; Problems involving the principle known in the West as the Pythagorean Theorem.

“Categorizing model” as the most important principle for task design in china

It has played important role  
in mathematics history  
In mathematics education history

# “categorizing problem strategy” played important role in Chinese math education history

- All word problems were categorized into several categories in many classics mathematics books (算经十书) in ancient China.
- Before 1878, all word problems in all teaching material were categorized into several categories in China.

# Before the curriculum was imported from western system, Chinese traditional task design framework before 1807

1. Difference /sum category;
  2. Speed category,
  3. Tree- planting category;
  4. Age category;
  5. Availing category;
  6. Engineering category;
  7. Profit category (Wang, 1996).
- **Commence:** categorization frame in China is, not western knowledge content, but its associated context

After 1878, an associated pedagogy  
stressing the process of categorization  
was developed in China

- Expanding problem
- From a single one
- to a category of problems

The Chinese hidden task design principle from “categorizing problem”

Its explicit design skills:

*One Problem Multiple Solutions* 一题多解,

*One Problem Multiple Changes* 一题多变,

*Multiple Problem One Solution* 多题一解

(Sun, 2007; 2011).

Most Popular worksheet:

*One lesson three levels of exercises* 一课三练

# one problem multiple solutions



# one problem multiple changes

5



$$10 + 3 = 13$$
$$3 + 10 = 13$$

$$13 - 3 = 10$$
$$13 - 10 = 3$$

6



$$11 + 2 = 13$$

$$13 - 2 = 11$$



1.



$$10 + 1 = \square$$

$$1 + 10 = \square$$

$$11 - \square = \square$$

$$11 - \square = \square$$

2.  $10 + 7 =$

$11 + 3 =$



$$\square + \square = \square$$

$$\square + \square = \square$$

$$\square - \square = \square$$

$$\square - \square = \square$$

$14 - 4 =$

$13 - 2 =$

# Western example vs Chinese example

## Separate addition and subtraction in west

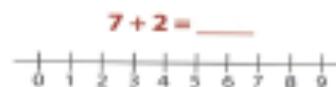
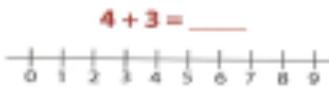
Observa o exemplo.



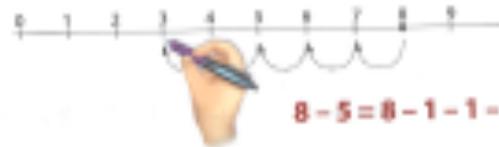
$$2 + 3 = 2 + 1 + 1 + 1 = 5$$



Usa as rectas numéricas para efectuaras as adições.



Observa o exemplo.



$$8 - 5 = 8 - 1 - 1 - 1 - 1 = 3$$



Utiliza as rectas numéricas para efectuaras as subtracciones.

7 - 3 = \_\_\_\_\_

A horizontal number line with tick marks from 0 to 9. Below it is one empty box for writing the answer.

6 - 2 = \_\_\_\_\_

A horizontal number line with tick marks from 0 to 9. Below it is one empty box for writing the answer.

## Combine addition and subtraction in China

5



$$10 + 3 = 13$$

$$3 + 10 = 13$$

$$13 - 3 = 10$$

$$13 - 10 = 3$$

6



$$11 + 2 = 13$$

$$\square + \square = \square$$

$$13 - 2 = 11$$

$$\square - \square = \square$$



1.



$$10 + 1 = \square$$

$$1 + 10 = \square$$



$$\square + \square = \square$$

$$\square + \square = \square$$

$$11 - \square = \square$$

$$\square - \square = \square$$

$$11 - \square = \square$$

$$\square - \square = \square$$

2.

$$10 + 7 =$$

$$14 - 4 =$$

$$11 + 3 =$$

$$13 - 2 =$$

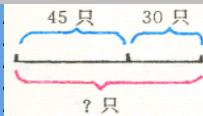
68

# An good example in the theme E (Mariolina,Sun,Alessandro,2013)

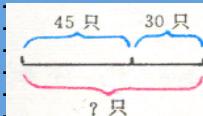
*A summary system of problems with variation in second grade*

First solve the nine problems below. Then explain why they have been arranged in rows and columns in this way, finding relationships

**(1) In the river there are 45 white ducks and 30 black ducks. All together how many ducks are there?**



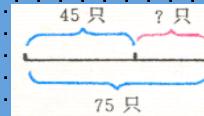
**(1) In the river there is a group of ducks. 30 ducks swim away. 45 ducks are still there. How many ducks are in the group (at the beginning)?**



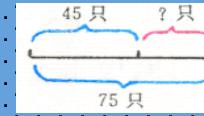
**(1) In the river there are 30 black ducks. White ducks are 15 more than black ducks (black ducks are 15 less than white ducks). How many white ducks are there?**



**(2) In the river there are white ducks and black ducks. All together there are 75 ducks. 45 are white ducks. How many black ducks are there?**



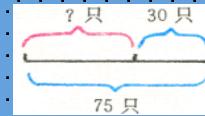
**(2) In the river there are 75 ducks. Some ducks swim away. There are still 45 ducks. How many ducks have swum away?**



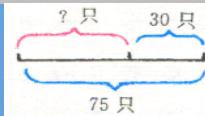
**(2) In the river there are 30 black ducks and 45 white ducks. How many white ducks more than black ducks (How many black ducks less than white ducks)?**



**(3) In the river there are white ducks and black ducks. All together there are 75 ducks. 30 are black ducks. How many white ducks are there?**



**(3) In the river there are 75 ducks. 30 ducks swim away. How many ducks are still there?**



**(3) In the river there are 45 white ducks. Black ducks are 15 less than white ducks (white ducks are 15 more than black ducks). How many black ducks are there?**



# Summary

- Both the “variation theory” in Chinese culture and the variation theory proposed by Ference Marton emphasize to discern **the invariants and variation** by task design
- The main difference is the first one is from **arithmetic /algebra history** perspective towards a certain goal of deep concept connection. The second one is from learning perspective towards a certain goal of deep learning (Marton and Booth,1997).

# More reference

- Sun, X. H. (2011). Variation problems and their roles in the topic of fraction division in Chinese mathematics textbook examples. *Educational Studies in Mathematics*, 76(1), 65-85.
- Sun, X. H. (2007). Spiral variation (Bianshi) curricula design in mathematics: Theory and practice. Unpublished doctoral dissertation. Hong Kong: The Chinese University of Hong Kong (in Chinese).

# The end

- Acknowledge:
- Thank Anne Watson for the editing work and the proposed presentation.
- Welcome to discuss with me by email
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