

Abel Visiting Scholarship Program

I have visited the Institut de recherche mathématique de Rennes (IRMAR) from 5 September to 30 October 2022. The visit was supported by IMU-Abel grant and University of Rennes 1. My host professors were Martin Costabel and Monique Dauge. Our main research topic was working on the project “Numerical Analysis of Volume Integral Equations”.

In this project, we study the numerical methods for approximating volume integral equations arising from electromagnetic scattering from penetrable objects. The perturbation of a constant medium is then confined to a bounded domain, and correspondingly, the VIE is posed in a bounded domain, while the behavior of the field in the exterior domain, in particular the behavior at infinity (radiation condition) is automatically taken into account by choosing a Green function.

A popular method for the numerical solution of these equations is the Discrete Dipole Approximation (DDA) which is often used by physicists and engineers. DDA is a simple algorithm that can be easily written and implemented using any programming language (in our project we use Julia). It replaces the integrals in each subvolume (square or circle in 2D and cube or sphere in 3D) with a one-point quadrature rule and collocates the equation at the center (corner) of the subvolumes to obtain a linear system of equations. There are many papers in the literature that discuss the application of DDA in various fields.

DDA is a numerical algorithm and there is a question about the consistency, stability and convergence of the method. In this project, we are interested in investigating the stability of the DDA method. As a first step, we have tried to find an answer to this question in the quasi-static case by taking the numerical symbol of the DDA method as the characteristic function of the infinite Toeplitz matrix associated with the DDA discretization of the principal part of the electric VIE. We also investigated the issue of stability for nonzero wavenumbers and obtained some results. For a consistent algorithm, stability means convergence and vice versa. In this research, we also revisit the study of the consistency error of DDA.

The next step in our project is to find an estimate for the convergence rate of DDA; this is one of the ways to continue this nice project. In addition, we are working on the spectral behavior of the piecewise constant collocation method for these VIE integral equations to compare with DDA and then try to answer the stability and convergence questions.

During this two-month visit, we had our regular meetings almost every day of the week. On October 20, 2022, I gave a talk entitled “Discrete Dipole Approximation of Volume Integral Equations: a perfect method for perfect equations?”. We are also preparing the following papers for submission:

- a. M. Costabel, M. Dauge & K. Nedaiasl, ”STABILITY ANALYSIS OF A SIMPLE DISCRETIZATION METHOD FOR SOME STRONGLY SINGULAR INTEGRAL EQUATIONS”, in preparation.
- b. M. Costabel, M. Dauge & K. Nedaiasl, ”ON THE STABILITY OF THE DISCRETE DIPOLE APPROXIMATION IN TIME- HARMONIC DIELECTRIC SCATTERING”, in preparation

I should mention that we held our regular online meetings, at least one every two weeks from March 2020 to January 2022. We will continue our collaboration in this direction through online platforms until our next visit. During this scientific visit, we not only talked about “mathematics” but also made two excursions to the island of Monte Saint-Michel in Normandy and to the port city of Saint-Malo in Brittany. We attended two concerts, one in Monte-Saint-Michel and the other in Betton, where the French classical pianist Vanessa Wagner performed. All of these concerts were great experiences.

Finally, I would like to thank the selection committee of the Abel Visiting Scholar Program, University of Rennes 1, and my colleagues in the mathematics department of IASBS for supporting, hosting, and sharing my courses during this research stay.

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