Activity report

The main objective of my visit to the University of Crete has been to work with Mihalis Kolountzakis. In [1] we had studied the relationship between multi tiling sets and the existence of Riesz basis of exponentials in locally compact abelian groups. This connection, in the case of $\mathbb{R}^n$ was discovered by Grepstad and Lev in [3] and also studied by Kolountzakis in [5]. Thus, firstly we have discussed different aspects of our approaches to the above aforementioned relationship. One of the advantages of our approach is that it permits simplified the problem by using the symmetries that appear. For instance, this reduction let us prove a kind of converse of the result obtained by Gresptad and Lev. So, our first goal has been to use our techniques, based on the tools developed around the so called *shift invariant spaces*, to study a recent result by Iosevich and Mayeli proved in [4]. This study leads to an improvement of that result. More precisely, we have been able to get the same conclusions but assuming weaker conditions.

Later on, we have turned our attention to multi tiling problems and Gabor orthonormal bases. In [2] necessary and sufficient conditions on a set Λ are obtained in order to assure that

$$\{e^{2\pi i\lambda(\cdot)}\chi_{[0,1)}(\cdot - t): \quad (t, \lambda) \in \Lambda\}$$

is an orthonormal basis for $L^2(\mathbb{R})$. In this line of research, our plan is to be able to replace the set $[0, 1)$ by more general sets. The main tool to study this problem is the so called short time Fourier transform. This transform plays the role played by the usual Fourier transform in the problems of tilings and orthonormal basis of exponentials. We have obtain some results, which are part of an ongoing research collaboration. I would like to point out that for me, it is very important to have this opportunity of working with Professor Kolountzakis, because he is one of the leaders in this subject and the discussions with him are very fruitful for me.

During my stay in Crete, I have also had the chance to meet Sigrid Grepstad and Nir Lev. Since they were the first ones to discover the relationship between multi tiling sets and Riesz basis of exponentials, it has been very interesting for my to discuss with them the different aspects of our work.

Finally, I would like to add that I have participated in the Analysis Seminar of the Department of Mathematics, where I have given a talk (I attach to this report a certificate).

References

[1] Agora E., Antezana J., Cabrelli C., Multi-tiling sets, Riesz bases, and sampling near the critical density in LCA groups. Preprint arXiv:1404.2317.

[2] Gabardo, J.P.; Lai, Chun-Kit; Wang Y. Gabor orthonormal bases generated by the unit cubes, Preprint arXiv:1411.7765

[3] Grepstad S., Lev N., Multi-tiling and Riesz bases. Adv. Math. 252 (2014), 1-6.

[4] A. Iosevich, A. Mayeli, Exponential bases, Paley-Wiener spaces and applications. J. Funct. Anal. 268 (2015), no. 2, 363-375.

[5] Kolountzakis M., Multiple lattice tiles and Riesz bases of exponentials. Proc. Amer. Math. Soc. 143 (2015), 741-747