Title of School: Combinatorial Commutative Algebra
Dates: March 05 – 14, 2018
Location: Lahore (Pakistan)

I. Summary
The CIMPA School on Combinatorial Commutative Algebra was held at Department of Mathematics, COMSATS Institute of Information Technology (CIIT), Lahore Campus on March 05-14, 2018. A total of 78 researchers including 6 speakers, 11 international students (India (3), Iran (1), Indonesia (2), China (1), Morocco (1), Malaysia (1), Vietnam (1) and Italy (1)) and 60 national students have participated in the event. The opening ceremony of the event was held on March 05, 2018 in which Prof. Dr. Arshad Saleem Bhatti, Dean of Sciences CIIT Islamabad, Prof. Dr. Jorge Mozo Fernández, representing CIMPA, Laura Mayoral Aguilera, Spanish Consul, representing the Spanish Embassy in Pakistan, Prof. Dr. A. D. Raza Choudhary, ex Director General Abdus Salam School of Mathematical Sciences, GCU Lahore-Pakistan were guests of honor. The school lectures were held at seminar room of science block. The seminar room has 125 fixed chairs, two big white board, a multimedia projector, sound system and fully air-condition system. Each day was divided in two sessions. In each morning session there used to be three lectures of one hour each while evening sessions were dedicated to tutorials and short presentations by the participants. Ten students have presented their research while many students had actively participated in the tutorial sessions. There were two excursions including visits to the Army Museum, Badshahi Mosjid and Wagha Border (Pakistan India Border). Out of 6 speakers, 3 had stayed within the university campus while remaining speakers had stayed at a nearby hotel along with other international participants. There were 12 Pakistani students who came from outside Lahore to participate in the school. The CIIT Lahore campus management had provided them full residence. The transport to all students from Lahore and to those staying in nearby hotels was also provided by CIIT Lahore Campus.

II. Scientific Content
The CIMPA School on Combinatorial Commutative Algebra consisted of six courses with a number of lectures for each as per following detail:

**Course 1.** Volkmar Welker (Germany): Orthogonal Representation of Graphs, Combinatorics and Algebra.

Motivated by questions in algebra and combinatorics we study two ideals associated to a simple graph $G$ on $n$ vertices:

- the Lovasz-Saks-Schrijver ideal defining the orthogonal representations in $\mathbb{R}^d$ of the graph complementary to $G$ and
- the ideal of the $(d + 1)$-minors of a generic symmetric $n \times n$ -matrices with 0s in positions prescribed by the graph $G$.

We show that these two ideals are closely related and that algebraic properties such as being radical, prime or a complete intersection/having expected height transfer from the Lovasz-Saks-Schrijver ideal of orthogonal representations in $\mathbb{R}^d$ to the ideal of $(d + 1)$-minors. For $d = 1$ the Lovasz-Saks-Schrijver ideal of $G$ is known as the edge ideal of $G$ and for $d = 2$ and $G$ bipartite it is up to coordinate transformation the binomial edge ideal of $G$.

An orthogonal representation of a graph $G$ in $\mathbb{R}^d$ is a map $\phi$ from the set of vertices of $G$ to $\mathbb{R}^d$ such that vertices not connected by an edge in $G$ are mapped to orthogonal vectors. The Lovasz-Saks-Schrijver ideal of a graph $G$ is then generated by the equations expressing the orthogonality relations of an orthogonal representation of the complement of $G$. Orthogonal representations were first studied by Lovasz in connection to graph entropy. In work with Saks and Schrijver he also studied the geometry of the variety of orthogonal graph representations. The latter is the reason for naming the ideals Lovasz-Saks-Schrijver ideals.

For Lovasz-Saks-Schrijver ideals we link radicality, complete intersection and primality to combinatorial properties of $G$ and show that they always hold for $d$ large enough. For specific classes of graph, such a forests, we can give a complete picture and classify the radical, prime and complete intersection Lovasz-Saks-Schrijver ideals.

**Course 2.** Tim Römer (Germany): Lattice Polytopes.

In these lectures we discuss lattice polytopes and associated rings. The latter are toric algebras. The interplay between combinatorial properties of the polytopes and the corresponding algebraic ones of the algebras is an active area of research in combinatorial commutative algebra. We discuss examples of such results and useful methods to study these
objects. Moreover, some lattice polytopes of interest are considered to which we apply the theory.

Course 3. Sara Saeedi Madani (Iran): Binomial Edge Ideals
In these lectures we introduce a class of binomial ideals attached to graphs, called "binomial edge ideals", and we study various algebraic properties and invariants of them. We try to understand those properties and invariants via the combinatorial properties of the underlying graph. Finally, we present some open problems in this area.

Course 4. Santiago Zarzuela Armengou (Spain): Computing Local Cohomology
Local cohomology is difficult to compute explicitly. One can reduce to a simple set-up as in the Hochster's formula for the local cohomology of a Stanley-Reisner ring. The poset defined by the primary decomposition of the defining ideal provides the needed combinatorial information. These ideas can be extended in several directions, always with the above poset as the combinatorial object where to look. In these lectures we shall review some of these constructions, which often involve the explicit computation of the derived functors of the direct and inverse limits over a finite poset.

Course 5. Nguyen Dang Hop (Vietnam): Powers and Symbolic powers of ideals
The study of powers and symbolic powers of ideals in a ring is a classical topic in commutative algebra. Let $R$ be a polynomial ring over a field with the standard grading, and $I$ a homogeneous ideal of $R$. There are two aspects in the study of powers and symbolic powers of $I$. Firstly, there is the asymptotic study, which focus on the eventual behavior of large enough powers of $I$. We usually expect uniform behavior of the large enough powers. But there is also the wholesale study, in which we ask about the common feature of all the powers. An example in the wholesale study is: Characterize the sequence $(\text{depth } R/I^n)$, where $n$ ranges over the natural numbers? The second aspect is less well-understood, as usually we don't expect that the small powers of $I$ to behave in a simple manner. We will discuss both asymptotic and wholesale aspects of powers and symbolic powers. In particular, we will focus on the depth and the Castelnuovo-Mumford regularity of these powers. We will use tools like local cohomology, simplicial homology, usually only in simple ways, to answer some algebraic questions on powers (and symbolic powers) of ideals.

Course 6. Rashid Zaare-Nahandi (Iran): Ideals with Linear Quotients and the Simon Conjecture
Let $I$ be a monomial ideal in the ring of polynomials $k[x_1,\ldots,x_n]$ generated by $u_1,\ldots,u_m$. It is called an ideal with linear quotients if the colon ideal $(u_1,\ldots,u_i):(u_{i+1})$ is generated by linear forms, for each $1 \leq i < m$. A hypergraph $G$ is called a clutter if cardinality of all hyper edges are the same. For each clutter an edge ideal $I(G)$ generated by equidimensional monomials is corresponded. In the first lecture, we introduce the notion of a simplicial element and chordality in clutters. In the second lecture, properties of ideals with linear quotients will be discussed and a theorem will be proved that any equidimensional ideal with linear quotients can be obtained by a simplicial order in a complete clutter. In the third lecture, a conjecture introduced by Simon on extendable shellability property of clutters and its relation to simplicial orders will be discussed. This work is joint with Mina Bigdeli, Jurgen Herzog and Ali Akbar Yazdan Pour.

Following 10 students have presented their research work during the school.

1) Carla Mascia (Italy)
2) Azeem Khadam (Pakistan)
3) Asma Khalid (Pakistan)
4) Tahira Majeed (Pakistan)
5) Naqeeb ur-Rehman (Pakistan)
6) Rabia Nazir (Pakistan)
7) Muhammad Imran Bhat (India)
8) Fatimah Abdul Razzak (Indonesia)
9) Rezwan-ul-Shaban (India)
10) Hafizullah (Pakistan)

III. Participants

This school turned to be very successful in terms of participation, where out of total of 61 participants, 11 foreign and 50 Pakistani participants joined the school. Initially 19 participants were approved by CIMPA but at the end only 11 managed to turn up. Out of these only 8 were partially or fully funded by CIMPA to cover either of their travel, food and lodging cost. The foreign participants; 8 male and 3 female were from Iran (1), India (3), Indonesia (2), Italy (1), Congo (1), China (1), Vietnam (1) and Malaysia (1). The names of participants are as under:

1. Volkmar Welker Philipps-Universität Marburg, Fachbereich Mathematik
2. Tim Römer  
University of Osnabrück, Osnabrück, Germany

3. Sara Saeedi Madani  
Department of Mathematics and CS, Amirkabir University of Technology, Tehran, Iran

4. Santiago Zarzuela Armengou  
Departament d’Àlgebra i Geometria, Universitat de Barcelona, Barcelona, Spain

5. Nguyen Dang Hop  
Fakultät für Mathematik, Otto von Guericke Universität Magdeburg, Magdeburg, Germany

6. Rashid Zaare-Nahandi  
Department of Mathematics, Institute for Advanced Studies in Basic Sciences, Zanjan, Iran

7. Jorge Mozo Fernández  
Facultad de Ciencias - Campus Miguel Delibes Paseo de Belén, Valladolid, Spain

8. Ali Akbar Yazdan Pour  
Department of Mathematics, Institute of Advanced Studies in Basic Sciences, Zanjan, Iran

9. Khalid Nazir  
University of Kashmir, Srinagar, India

10. Mohmad Imran Bhat  
University of Kashmir, Srinagar, India

11. Rezwan ul Shaban  
University of Kashmir, Srinagar, India

12. Fatimah Abdul Razak  
School of Mathematical Sciences, Universiti Kebangsaan Malaysia, Bangi, Malaysia

13. Edward Bankoussou Mabiala  
Université Hassan II, Maarif Casablanca, Morocco

14. Bac Nguyen Trong  
University of Economics and Business Administration, Thai Nguyen University, Vietnam

15. Yoshua Yonatan Hamonangan  
Institut Teknologi Bandung, Bandung, Indonesia

16. Mamika Ujianita Romdhini  
Mataram University Jl. Mataram Indonesia

17. Muhammad Asif  
Xiamen University, Fujian Province, China

18. Carla Mascia  
Department of Mathematics, University of Trento, Italy

19. Naqeeb ur Rehman  
Allama Iqbal Open University, Islamabad

20. Shabnam Malik  
Forman Christian College (A Chartered University), Lahore

21. Ibraheem Farheen  
Forman Christian College (A Chartered University), Lahore
22. Afshan Adil

23. Aqsa
COMSATS Institute of Information Technology, Islamabad

24. Aqsa Bashir
ASSMS, GC University, Lahore, Pakistan

25. Asia Rauf
Govt. College Women University Faisalabad, Pakistan

26. Asma Khalid
ASSMS, GC University, Lahore, Pakistan

27. Azeem Khadam
ASSMS, GC University, Lahore, Pakistan

28. Faraha Ashraf
ASSMS, GC University, Lahore, Pakistan

29. Fareeha Ambar
Lahore University of Management Sciences (LUMS), Lahore

30. Ghazanfar Abbas
ASSMS, GC University, Lahore, Pakistan

31. Hafiz Ullah
CIIT Islamabad, Pakistan

32. Hafiza Mehreen Zafar
Lahore College for Women University, Jhang Campus

33. Haseeb Wali
CIIT Islamabad, Pakistan

34. IMRAN ANWAR
ASSMS, GC University, Lahore, Pakistan

35. Maria Naseem
University of Central Punjab, Lahore, Pakistan

36. Maria Farooq
Lahore University of Management Sciences (LUMS), Lahore

37. Mariam Imtiaz
University of Engineering &Technology, Lahore, Pakistan

38. Muhammad Zahid
ASSMS, GC University, Lahore, Pakistan

39. Nadia Shoukat
ASSMS, GC University, Lahore, Pakistan

40. Nimra Javed
ASSMS, GC University, Lahore, Pakistan

41. Rabia Nazir
Govt. College University Faisalabad, Pakistan

42. Rafiah Zafar
Lahore College for Women University Jhang Campus

43. Raza Ali
ASSMS, GC University, Lahore, Pakistan

44. Rehana Ashraf
Lahore College for Women University, Jhang Campus

45. Rida Irfan
COMSATS Institute of Information Technology, Sahiwal

46. Rizwan Jahangir
ASSMS, GC University, Lahore, Pakistan

47. Sadia Akhtar
Lahore College for Women University, Jhang Campus,

48. Sajjad Khan
CIIT Islamabad, Pakistan

49. Sidra Razzaq
Lahore College for Women University, Jhang Campus,

50. Syed Fazal Abbas Shah
ASSMS, GC University, Lahore, Pakistan
51. Tahira Majeed  
COMSATS Institute of Information Technology, Lahore

52. Tusif Ahmed Malik  
ASSMS, GC University, Lahore, Pakistan

53. Zahid Iqbal  
National University of Science and Technology, Islamabad

54. Zainab Ali  
Lahore University of Management Sciences (LUMS), Lahore

55. Zohaib Nadeem Sheikh  
ASSMS, GC University, Lahore, Pakistan

56. Zunaira Kosar  
ASSMS, GC University, Lahore, Pakistan

57. Asif Allah Ditta  
COMSATS Institute of Information Technology, Lahore

58. Dilwar Ali  
COMSATS Institute of Information Technology, Lahore

59. Wakeel Ahmad  
COMSATS Institute of Information Technology, Lahore

60. Saad Ahmad  
COMSATS Institute of Information Technology, Lahore

61. Aqsa Farooq  
COMSATS Institute of Information Technology, Lahore

62. Iqra Farman  
COMSATS Institute of Information Technology, Lahore

63. Muhammad Awais  
COMSATS Institute of Information Technology, Lahore

64. Sabir Hussain  
COMSATS Institute of Information Technology, Lahore

65. M. Fouz Farooq  
COMSATS Institute of Information Technology, Lahore

66. Iram Saleem  
COMSATS Institute of Information Technology, Lahore

67. Hina Javaid  
COMSATS Institute of Information Technology, Lahore

68. Haseeb Ahmad  
COMSATS Institute of Information Technology, Lahore

69. Ali  
COMSATS Institute of Information Technology, Lahore

70. Arhum Maqbool  
COMSATS Institute of Information Technology, Lahore

71. Ammar Mujahid  
COMSATS Institute of Information Technology, Lahore

72. Iqra Siddique  
COMSATS Institute of Information Technology, Lahore

73. Ayesha  
COMSATS Institute of Information Technology, Lahore

74. Malik Ali Raza  
COMSATS Institute of Information Technology, Lahore

75. Saliha Manzoor  
COMSATS Institute of Information Technology, Lahore

76. Zahra Manzoor  
COMSATS Institute of Information Technology, Lahore

77. Asma  
COMSATS Institute of Information Technology, Lahore

78. Azmat  
COMSATS Institute of Information Technology, Lahore
IV. Financial Report

<table>
<thead>
<tr>
<th>Sr. #</th>
<th>Funds</th>
<th>Granted (in Euros)</th>
<th>Spending (in Euros)</th>
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<td>19640</td>
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Table 1: Summary of funds granted and spending

V. Pictures

Group Photo
Cimpa Presentative Prof. Dr. Jorge Mozo Fernández addressing the audience at the opening ceremony

From left to right: Prof. Santiago Zarzuela Armengou (Spain), Prof. Tim Romer (Germany), Prof. Volkmar Welker (Germany), Ms. Laura Mayoral Aguilera (Spanish Consul), Prof. Jorge Mozo Fernández (Spain), Prof. A. D. Raza Choudhary (Ex DG ASSMS), Prof. A. S. Bhati (Dean of Sciences, CIIT) at the opening ceremony
Ex DG ASSMS Prof. Dr. A. D. Raza Choudhar addressing the audience

International participants at the opening session
Registration desks

Registration team
Prof. Volkmar Welker (Germany) opening lecture