16th Discussion Meeting in Harmonic Analysis

The Department of Mathematics, IISER Bhopal organised an international conference ‘**16th Discussion Meeting in Harmonic Analysis**’ during **December 16-19, 2019**. This conference is the most important activity in the area of harmonic analysis in India and is conducted once in every two years. The main goal of the conference is to provide the young researchers a platform to interact with the leading experts in the area from around the world. There were around 95 participants in this conference from around the world.

Prof. Malabika Pramanik (University of British Columbia, Canada) delivered the plenary lecture series on the topic ‘*Directional Operators in Harmonic Analysis and Configuration of Sets*’. The conference featured about 25 research talks by invited speakers and participants along with a poster session by young researchers. The event was funded by National Board for Higher Mathematics (NBHM), Science and Engineering Research Board (SERB), International Mathematical Union (CDC-IMU), and IISER Bhopal.

The details of the talks are given in the attached annexure.

16^{th} Discussion Meeting in Harmonic Analysis

Indian Institute of Science Education and Research, Bhopal

December 16-19, 2019

Thematic lectures

Configurations in sets

Malabika Pramanik
(University of British Columbia, Canada)

When does a given set contain a copy of your favorite pattern (for example, specially arranged points on a line or spiral, or the vertices of a polyhedron)? Does the answer depend on how thin the set is in some quantifiable sense? Problems involving identification of prescribed configurations under varying interpretations of size have been vigorously pursued both in the discrete and continuous setting, often with spectacular results that run contrary to intuition. Yet many deceptively simple questions remain open. In the first talk, I will survey the literature in this area, emphasizing some of the landmark results that focus on different aspects of the problem. The remaining three talks will be devoted to the discussion of representative results in the area.

Monday, December 16, 2019 (Day 1)

Maximal Ergodic Inequalities For Some Positive Operators On Noncommutative L_p-spaces

Samya Kumar Ray

(Wuhan University, China)

In this talk, we discuss one-sided maximal ergodic inequalities for large class of positive operators on noncommutative L_p-spaces, which do not fall into the category of noncommutative Dunford-Schwartz ergodic inequalities for positive contractions considered by Junge and Xu. Our methods partly rely on various characterization theorems associated to Lamperti operators. By using characterization theorem for completely Lamperti contractions, we establish a simultaneous dilation theorem for them. This enables us to obtain a dilation theorem for a substantial class of positive contractions on noncommutative L_p-spaces. Then, we prove a maximal ergodic inequality for positive complete isometries. Utilizing the dilation theorem, we show that before mentioned class of positive contractions satisfy maximal ergodic inequalities. We also discuss an one-sided maximal ergodic inequality for powerbounded doubly completely Lamperti operators. We achieve this by proving a structural theorem for doubly completely Lamperti operators, which allows us to use the maximal ergodic inequality for positive completely Lamperti contractions. Finally, we show that the concrete examples of positive contractions which were considered by Junge and Le Merdy for which dilation fail, still satisfy maximal ergodic inequalities. We also discuss some other examples, showing sharp contrast to classical situation.

Reshetnyak/Plancherel formula for momentum ray transforms

Ramesh Manna

(TIFR Centre For Applicable Mathematics, Bangalore, India)

In this talk, we discuss Reshetnyak formula for momentum ray transforms. The momentum ray transform integrates a rank m symmetric tensor field f over lines in $\mathbb{R}^n$ with the weight t^k. The case $k = 0$ is the classical ray transform. We give an overview of the results which have been proven to date. Finally, using the Reshetnyak formula, we obtain a stability estimate in terms of the H_t^s-norm, which is a modification of the Sobolev norm weighted differently at high and low frequencies. This is a joint work with Venky Krishnan, Suman Kumar Sahoo and Vladimir Sharafutdinov.

Monotonicity of Markov semigroups and H^∞-calculus

Tao Mei

(Baylor University, USA)

I plan to explain a recent discovery of a monotonicity that is satisfied by all subordinated Markov semigroup of operators.

For the classical Poisson semigroup P_t generated by the square root of Laplacian, this property reads as $sP_t < tP_s$ for any pair of parameters $s < t$. I will explain how this monotonicity helps us to prove an endpoint result of bounded H^∞ -calculus for general "Laplacian" operators.

Analysis on Semihypergroups: An Overview

Choiti Bandyopadhyay

The theory of semihypergroups and hypergroups allows a detailed study of measure algebras that can be expressed in terms of a convolution of measures on the underlying spaces. In particular, the class of semihypergroups contains many important examples of coset and orbit spaces in locally compact groups, which do not have enough structure to be a semigroup or a hypergroup. The lack of any extensive prior research since its inception in 1972 and the significant examples it contains, opens up a number of intriguing new paths of research on semihypergroups.

In our talk, we will give a brief overview on how some well-known algebraic and analytic concepts and language of classical semigroup and group theory can be translated for semihypergroups, and investigate where the theory deviates from the classical theory of semigroups. In particular, we will discuss ideals and homomorphisms, spaces of almost periodic and weakly almost periodic functions and free-product structures in the category of semihypergroups.

Restriction estimates to complex hypersurfaces

Juyoung Lee

(Seoul National University, South Korea)

There are many results of the restriction problem on hypersurfaces in $\mathbb{R}^n$, by using various methods such as bilinear or multilinear estimates. However, there are few results for surfaces with codimension bigger than 1. In this talk, we are concerned with the restriction problem of surfaces with codimension 2 in $\mathbb{C}^n(\approx \mathbb{R}^{2n})$, which is generated by a graph of a holomorphic function. We used a Bourgain and Guth's method in [1]. Our result generalizes a result of Bak, J. Lee, and S. Lee. [2]
This is a joint work with Sanghyuk Lee.

Bibliography:

[1] J. Bourgain, L. Guth, *Bounds on oscillatory integral operators based on multilinear estimates,* Geom. Funct. Anal. **21** (2011), 1239–1295.
[2] J. Bak, J. Lee, S. Lee, *Bilinear restriction estimates for surfaces of codimension bigger than 1*, Analysis and PDE, Vol. **10** (2017), No. 8, 1961–1985.

Fourier analysis of vector measures and their associated L^p spaces

N. Shravan Kumar

(Indian Institute of Technology Delhi, India)

In this talk, we introduce the Fourier transform on the L^1 space associated to a vector measure. We will also introduce the Fourier transform of a vector measure. We will see the importance of the operator spaces when one moves from a compact abelian group to a compact nonabelian group.

Tuesday, December 17, 2019 (Day 2)

Analogs of certain quasi-analiticity results on Riemannian symmetric spaces of noncompact type

Mithun Bhowmik

(Indian Institute of Technology Bombay, India)

An L^2 version of the celebrated Denjoy-Carleman theorem regarding quasi- analytic functions was proved by Chernoff on $\mathbb{R}^d$ using iterates of the Laplacian. In 1934 Ingham used the classical Denjoy-Carleman theorem to relate the decay of Fourier transform and quasi-analyticity of integrable functions on $\mathbb{R}$. In this talk, we will talk about analogs of both these theorems to Riemannian symmetric spaces of noncompact type.

On the Taylor coefficients of functions in the Hardy space over the bidisc

Oscar Blasco

(Universidad de Valencia, Spain)

In this paper we analyze the Taylor coefficients of functions in Hardy spaces on the bidisc. We present the two-dimensional versions of some classical Hardy and Paley inequalities and we find certain conditions on the Taylor coefficients for the converse of these inequalities to hold.

Bibliography:

[1] Pavlovic, M., *Analytic functions with decresing coefficients and Hardy and Bloch spaces,* Proc. Edinburgh Math. Soc. **56** (2013), 623–625.

[2] Rudin, W., *F*unction theory in polydiscs, W. A. Benjamin, Inc., New York-Amsterdam, 1969.

Weighted inequalities for Rubio de Francia's square function

Luz Roncal

(Basque Center for Applied Mathematics, Spain)

Weighted L^p estimates for the dyadic Littlewood–Paley square functions were first proved by D. Kurtz in 1980. Later on, for the square functions associated with an arbitrary family of intervals, J. L. Rubio de Francia established in 1985 the corresponding weighted boundedness with $A_{p/2}$ weights, when $p > 2$. The validity of the A_1-weighted L^2-estimate was conjectured by Rubio de Francia in the same paper, and the problem remains open.

We will survey the results on the topic, and we will show another approach to get the weighted estimates for the Rubio de Francia's square function based on time-frequency analysis and sparse domination.

(Joint work with Rahul Garg and Saurabh Shrivastava, IISER Bhopal, India)

Almost everywhere convergence of Bochner–Riesz means on groups of Heisenberg type

Alessio Martini

(University of Birmingham, United Kingdom)

The study of Bochner–Riesz means is a classical, challenging research area of harmonic analysis, both in the Euclidean case and for more general eigenfunction expansions. In joint work with Adam D. Horwich (`arXiv:1908.04049`), we consider the case of Bochner–Riesz means for sub-Laplacians on Heisenberg-type groups, and prove the existence of a $p > 2$ for which Bochner–Riesz means of arbitrarily small order of any given L^p function converge a.e. to the function. This appears to be the first result of this kind for nonelliptic sub-Laplacians beyond the case of Heisenberg groups. The proof is based on a reduction of weighted L^2 estimates for the maximal operator to analogous estimates for the corresponding 'nonmaximal' operator and on suitable 'dual Sobolev trace lemmas', which ultimately rely on precise estimates for Jacobi polynomials.

The lacunary spherical maximal function on the Heisenberg group

Sayan Bagchi

(Indian Institute of Science Education and Research Kolkata, India)

In this talk, we investigate the L^p boundedness of the lacunary maximal function $A_r f$ associated to the spherical means on the Heisenberg group. By suitable adaptation of an approach of M. Lacey in the Euclidean case, we obtain sparse bounds for these maximal functions, which lead to new unweighted and weighted estimates. In order to prove the result, several properties of the spherical means have to be accomplished, namely, the L^p improving property of the operator $A_r f$ and a continuity property of the difference $A_r f - \tau_y A_r f$, where $\tau_y f(x) = f(xy^{-1})$ is the right translation operator.

Boundedness and Compactness of linear combination of composition operators

D. Venku Naidu

(Indian Institute of Technology Hyderabad, India)

Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be complex Hilbert spaces and $\phi : \mathcal{E}_1 \to \mathcal{E}_2$ and $\psi : \mathcal{E}_1 \to \mathcal{E}_2$ are two mappings. We show that for $a, b \in \mathbb{C} \setminus \{0\}$ and $\phi \neq \psi$, the operator $aC_\phi + bC_\psi : \mathcal{H}(\mathcal{E}_1) \to \mathcal{H}(\mathcal{E}_2)$ is bounded (compact, resp.) if and only if both the composition operators C_ϕ and C_ψ are bounded (compact, resp.). We also characterize the boundedness and compactness of these operators in terms of the function theoretic properties of the inducing maps ϕ and ψ.

Weighted boundedness of bilinear Bochner-Riesz operators

Kalachand Shuin
(Indian Institute of Science Education and Research Bhopal, India)

The bilinear Bochner-Riesz operator is defined as,

$$\mathcal{B}^{\alpha}(f,g)(x) := \int_{\mathbb{R}^n}\int_{\mathbb{R}^n} (1-|\xi|^2-|\eta|^2)_+^{\alpha}\hat{f}(\xi)\hat{g}(\eta)e^{-2\pi\iota x\cdot(\xi+\eta)}d\xi d\eta,$$

where α is a complex number with non-negative real part and f, g are Schwartz class functions and $r_+ = r$, if $r > 0$ and 0 elsewhere.
One can observe that the critical index of this operator is $n-\frac{1}{2}$. i.e. the operator $\mathcal{B}^{\alpha}$ maps $L^{p_1}(\mathbb{R}^n)\times L^{p_2}(\mathbb{R}^n)$ to $L^p(\mathbb{R}^n)$, for all $1\leq p_1, p_2\leq\infty$ with $\frac{1}{p_1}+\frac{1}{p_2}=\frac{1}{p}$, if $Re(\alpha) > n-\frac{1}{2}$. It was known that for $Re(\alpha) > n-\frac{1}{2}$, the operator $\mathcal{B}^{\alpha}$ is pointwise dominated by the product of Hardy-Littlewood maximal operators and from there one can get the weighted boundedness for the product type Muckenhoupt weight classes $A_{p_1}\times A_{p_2}$. In this talk, I shall talk about the weighted boundedness of the operator $\mathcal{B}^{n-\frac{1}{2}}$, for bilinear weight class $A_{\vec{P}}$, which is a bigger class than $A_{p_1}\times A_{p_2}$ and $\vec{P}=(p_1,p_2)$, $1 < p_1, p_2\leq\infty$.

Bibliography:

[1] F. Bernicot; P. Germain, *Boundedness of bilinear multipliers whose symbols have a narrow support.* J. Anal. Math. 119, 165–212 (2013).
[2] F. Bernicot; L. Grafakos; L. Song; L. Yan, *The bilinear Bochner-Riesz problem.* J. Anal. Math. 127, 179–217 (2015).
[3] X. Shi; Q. Sun, *Weighted norm inequalities for Bochner-Riesz operators and singular integral operators.* Proc. Amer. Math. Soc. 116, no. 3, 665–673. MR 1136237.

Wednesday, December 18, 2019 (Day 3)

Maximal estimates for the fractional Schrödinger equation

Chuhee Cho

(Seoul National University, South Korea)

In this talk we consider the pointwise convergence problem for the solutions of generalized Schrödinger equations. We establish the associated maximal estimatesfor a general class of phase functions, which give the pointwise convergencefor $f \in H^s(\mathbb{R}^d)$ whenever $s > \frac{d}{2(d+1)}$. Our arguments are based on recent works of Du, Guth, and Li [1] and Du and Zhang [2].
This is a joint work with Hyerim Ko.

Bibliography:
[1] X. Du, L. Guth and X. Li, *A sharp Schrödinger maximal estimate in* $\mathbb{R}^2$, Annals of Math. **186** (2017), no. 2, 607–640.
[2] X. Du and R. Zhang, *Sharp* L^2 *estimate of Schrödinger maximal function in higher dimensions*, Annals of Math. **189** (2019), no. 3, 837–861.

Some aspects of Knapp-Stein operators

Bent Ørsted

(Aarhus University, Denmark)

The intertwining operators first found and studied by Knapp and Stein have since been basic in representation theory of semisimple Lie groups. In this lecture we give some explicit examples, in particular for differential forms on Euclidean space, transforming under the conformal group, and some ways to calculate the spectrum of such operators. This relates to studying singular integral operators.

Strichartz estimates for orthonormal families of initial data and weighted oscillatory integral estimates

Sanghyuk Lee
(Seoul National University, South Korea)

In this talk we establish new Strichartz estimates for orthonormal families of initial data in the case of the wave, Klein–Gordon and fractional Schrödinger equations. Our results extend those of Frank–Sabin in the case of the wave and Klein–Gordon equations, and generalize work of Frank–Lewin–Lieb–Seiringer and Frank–Sabin for the Schrödinger equation. Due to a certain technical barrier, except for the classical Schrödinger equation, the Strichartz estimates for orthonormal families of initial data have not previously been established up to the sharp summability exponents in the full range of admissible pairs. We obtain the optimal estimates in various notable cases and improve the previous results. The main novelty of this paper is the use of estimates for weighted oscillatory integrals which we combine with an approach due to Frank and Sabin. This strategy also leads us to proving new estimates for weighted oscillatory integrals with optimal decay exponents which we believe to be of wider independent interest. This is joint work with Neal Bez and Shohei Nakamura.

On an isomorphism theorem for the Feichtinger's Segal algebra on locally compact groups

R Lakshmi Lavanya
(Indian Institute of Science Education and Research Tirupati, India)

In this talk, we observe that any locally compact group is completely determined by the algebraic properties of its Feichtinger's Segal algebra. More precisely, any linear (not necessarily continuous) bijection between the Feichtinger's Segal algebra of two locally compact groups which preserves the convolution and pointwise products is essentially a composition with a homeomorphic isomorphism of the underlying groups.

Heisenberg uniqueness pairs for the hyperbola

Deb Kumar Giri
(Indian Institute of Technology Guwahati, India)

Let Γ be a finite disjoint union of smooth curves in $\mathbb{R}^2$ and Λ be any subset of $\mathbb{R}^2$. Let $X(\Gamma)$ be the space of all finite complex-valued Borel measures μ in $\mathbb{R}^2$ which are supported on Γ and absolutely continuous with respect to the arc length measure on Γ. For $(\xi, \eta) \in \mathbb{R}^2$, the Fourier transform of μ is defined by

$$\hat{\mu}(\xi, \eta) = \int_\Gamma e^{\pi i(x\cdot\xi + y\cdot\eta)} d\mu(x, y).$$

The pair (Γ, Λ) is said to be a Heisenberg uniqueness pair (HUP), if for any $\mu \in X(\Gamma)$ satisfying $\hat{\mu}(\xi, \eta) = 0$ for all $(\xi, \eta) \in \Lambda$, μ is identically zero. In particular, let Γ be the hyperbola $\{(x, y) \in \mathbb{R}^2 : xy = 1\}$ and Λ_β be the lattice-cross defined by $\Lambda_\beta = (\mathbb{Z} \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z})$ in $\mathbb{R}^2$, where β is a positive real. Then Hedenmalm and Montes-Rodríguez has shown that (Γ, Λ_β) is a Heisenberg uniqueness pair if and only if $\beta \le 1$. In this talk, we present that for a rational perturbation of Λ_β namely,

$$\Lambda_\beta^\theta = ((\mathbb{Z} + \{\theta\}) \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z}),$$

where β is a positive real and $\theta = 1/p$, for some $p \in \mathbb{N}$, the pair $\left(\Gamma, \Lambda_\beta^\theta\right)$ is a Heisenberg uniqueness pair if and only if $\beta \le p$.

Dynamics of semigroups generated by analytic functions of the Laplacian on homogeneous trees

Sumit Kumar Rano

(Indian Institute of Technology Guwahati, India)

Let f be a non-constant complex holomorphic function defined on a connected open set containing the L^p-spectrum of Laplacian L on a homogeneous tree. In this talk we give a necessary and sufficient condition for the semigroup $T(t) = e^t f(L)$ to be chaotic on L^p-spaces. We also include some of the important semigroups such as the heat semigroup and the Schrödinger semigroup.

Mean Value property in limit for eigenfunctions of the Laplace–Beltrami operator

Muna Naik

(Indian Statistical Institute Kolkata, India)

Thursday, December 19, 2019 (Day 4)

Pointwise convergence of noncommutative Fourier series

Guixiang Hong
(Wuhan University, China)

In this talk, I shall talk about my recently finished joint work with Simeng Wang and Xumin Wang, which is the first progress on the pointwise convergence of noncommutative Fourier series, solving an open problem since Junge-Xu's remarkable ergodic maximal inequality in noncommutative analysis. Going back harmonic analysis on Euclidean space, one of our results suggests a new class of maximal inequalities which is quite interesting but challenging and deserves to be investigated. For more information, see the following abstract of the paper: This paper is devoted to the study of convergence of Fourier series for nonabelian groups and quantum groups. It is well-known that a number of approximation properties of groups can be interpreted as some summation methods and mean convergence of associated noncommutative Fourier series. Based on this framework, this work studies the refined counterpart of pointwise convergence of these Fourier series. We establish a general criterion of maximal inequalities for approximative identities of noncommutative Fourier multipliers. As a result we prove that for any countable discrete amenable group, there exists a sequence of finitely supported positive definite functions tending to 1 pointwise, so that the associated Fourier multipliers on noncommutative L^p-spaces satisfy the pointwise convergence for all $1 < p < \infty$. In a similar fashion, we also obtain results for a large subclass of groups (as well as discrete quantum groups) with the Haagerup property and weak amenability. We also consider the analogues of Fejer means and Bochner-Riesz means in the noncommutative setting. Our results in particular apply to the almost everywhere convergence of Fourier series of L^p-functions on non-abelian compact groups. On the other hand, we obtain as a byproduct the dimension free bounds of noncommutative Hardy-Littlewood maximal inequalities associated with convex bodies. As an ingredient, our proof also provides a refined version of Junge-Le Merdy-Xus square function estimates $H_p(M) \simeq L_p(M)$ when $p \to 1$.

A Laplacian on a totally disconnected metric measure space

Sharvari Neetin Tikekar
(Indian Institute of Science Education and Research Thiruvananthapuram, India)

Symbolic spaces are a mainstay in the dynamics community. These are totally disconnected compact metric measure spaces, whose proper subsets are used to model various fractal subsets of Euclidean spaces. However, the natural metric defined on these spaces is not equivalent to the Euclidean one, thus giving rise to new problems. In this joint work with Shrihari Sridharan, we will define and analyse the Laplacian on the full symbolic space and obtain solutions to a Dirichlet-type boundary value problem.

One-sided Hardy-Littlewood maximal function on generalized Lorentz spaces

Abhishek Ghosh

(Indian Institute of Technology Kanpur, India)

For a locally integrable function f on $\mathbb{R}$, the one-sided Hardy-Littlewood maximal function M^+f is defined by

$$M^+f(x) = \sup_{h>0} \frac{1}{h} \int_x^{x+h} |f(y)|dy.$$

E. Sawyer characterized the pair of weights (u, v) on $\mathbb{R}$, for which the operator M^+ is bounded from $L^p(u)$ to $L^p(v)$ for $1 < p < \infty$ and $L^1(u)$ to $L^{1,\infty}(v)$. Then P. Ortega Salvador has studied M^+ and characterized weights such that M^+ is bounded on weighted Lorentz ($L^{p,q}(u)$) spaces. In this talk we will discuss one-sided Hardy-Littlewood Maximal function on generalized weighted Lorentz spaces $\Lambda_u^{p,q}(w)$. We will also provide a direct proof of equivalence between weak-type and strong-type boundedness of M^+ on generalized weighted Lorentz spaces. As a consequence, we characterize weights for which the one-sided maximal function M^+ is bounded on $\Lambda_u^{p,q}(w)$ spaces. This is a joint work with Prof. Parasar Mohanty.
