

PROGRAM FOR ICCGNFRT-2018

The goal of this conference was to bring various experts on the subject at one place and provide young number theorists an opportunity to learn the techniques in the subject. This four day conference was begun with a keynote talk on October 08, 2018 and end with a valedictory function on October 11, 2018. There were 15 sessions consisting of 1 keynote, 7 plenary, 13 invited and 8 young scholar's talks. The details about this conference is available on the conference webpage "<https://sites.google.com/site/iccgfnrt2018/home>" the conference web page. We have received very good feedback from speakers as well as participants, and as a request from experts we are going to organize the third version of this conference which will be held at the same place during October 16–19, 2019. We are very much thankful to CDC (IMU) and HRI for supporting us.

DAY-01 (October 08, 2018, MONDAY)

TIME	SCHEDULE
8:50–9:40	REGISTRATION
9:50–10:00	INAUGURAL PROGRAM
	MORNING SESSION CHAIR PERSON: Prof. Sudesh Kaur Khanduja
10:00–11:00	KEYNOTE TALK: On the Landau–Ramanujan constant SPEAKER: Prof. Michel Waldschmidt ABSTRACT: The Landau–Ramanujan constant α is defined as follows: for $N \rightarrow \infty$, the number of positive integers $\leq N$ which are sums of two squares is asymptotically $\alpha \frac{N}{\sqrt{\log N}} \cdot p$. In a joint work with Etienne Fouvry and Claude Levesque, we replace the quadratic form $\Phi_4(X, Y) = X^2 + Y^2$, which is the homogeneous version of the cyclotomic polynomial $\phi_4(t) = t^2 + 1$, with other binary forms.
11:00–11:20	Tea & Discussion
	PRE-LUNCH SESSION CHAIR PERSON: Prof. Yasuhiro Kishi
11:20–12:00	INVITED TALK: Hypergeometric series in Arithmetic Geometry SPEAKER: Prof. Rupam Barman

TIME	SCHEDULE
	<p>ABSTRACT: In 1987, John Greene introduced the notion of hypergeometric series over finite fields analogous to classical hypergeometric series (Trans. Amer. Math. Soc. 301 (1987)). Finite field hypergeometric series were developed mainly to simplify character sum evaluations. After 10 years, Ken Ono found interesting relations between Greene's finite field hypergeometric series and L-functions of elliptic curves (Trans. Amer. Math. Soc. 350 (1998)). In 2013, Dermot McCarthy extended the finite field hypergeometric series to the p-adic setting (Pacific J. Math. 261 (2013)). In this talk, I will introduce finite field and p-adic hypergeometric series, and show how their values are related to L-functions of elliptic curves.</p>
12:00–12:10	Discussion
12:10–12:50	<p>INVITED TALK: Arithmetic Dynamics of Polynomials SPEAKER: Prof. Chatchawan Panraksa</p> <p>ABSTRACT: Arithmetic dynamics is a combination of dynamical systems and number theory. In this talk, we discuss the rational periodic points of polynomials with rational coefficients. We also discuss Morton and Silvermans uniform boundedness conjecture. It states that the number of periodic points of any rational function with rational coefficients is bounded by a constant depending only on the degree of the function. The conjecture is still unsolved even for quadratic polynomials.</p>
13:00–14:30	LUNCH
	<p>POST-LUNCH SESSION CHAIR PERSON: Prof. Stephane Louboutin</p>
14:30–15:30	<p>PLENARY TALK: On the integer ring of compositum of algebraic number fields. SPEAKER: Prof. Sudesh Kaur Khanduja</p> <p>ABSTRACT: For an algebraic number field K, let \mathbb{A}_K denote the ring of algebraic integers of K. It is well known that if K_1, K_2 are algebraic number fields with coprime discriminants, then $\mathbb{A}_{K_1K_2} = \mathbb{A}_{K_1} \mathbb{A}_{K_2}$ and K_1, K_2 are linearly disjoint over the field \mathbb{Q} of rational numbers. In this lecture we discuss the converse of this result and show that if K_1, K_2 are algebraic number fields which are linearly disjoint over $K = K_1 \cap K_2$ such that $\mathbb{A}_{K_1K_2} = \mathbb{A}_{K_1} \mathbb{A}_{K_2}$, then the relative discriminants of the extensions K_1/K and K_2/K are coprime. We also give some necessary and sufficient conditions for the equality $\mathbb{A}_{K_1K_2} = \mathbb{A}_{K_1} \mathbb{A}_{K_2}$ to hold.</p>
15:30–15:50	Tea & Discussion
	EVENING SESSION

TIME	SCHEDULE
	CHAIR PERSON: Prof. Toru Komatsu
15:50–16:30	<p>INVITED TALK: Class number formula for certain imaginary quadratic fields</p> <p>SPEAKER: Prof. Nianliang Wang</p> <p>ABSTRACT: In this talk, we shall follow Carlitz's work in 1954 which could have reached an analogue of the Voronoi congruence in the more difficult case of $p \equiv 1 \pmod{4}$: $h(-4p) \equiv B_{\frac{p+1}{2}}(\chi_4) \pmod{p}$, where $B_{\frac{p+1}{2}}(\chi_4)$ is the generalized Bernoulli number with χ_4 being the Kronecker symbol associated to the Gaussian field $\mathbb{Q}(\sqrt{-4})$.</p>
16:30–16:40	Discussion
16:40–17:00	<p>YOUNG SCHOLAR TALK: Primes dividing the index of a trinomial</p> <p>SPEAKER: Dr. Anuj Jakhar</p> <p>ABSTRACT: Let A_K denote the ring of algebraic integers of an algebraic number field $K = \mathbb{Q}(\theta)$, where θ is a root of an irreducible trinomial $F(x) = x^n + ax^m + b$ belonging to $\mathbb{Z}[x]$. We give necessary and sufficient conditions involving only a, b, m, n for a given prime p to divide the index of the subgroup $\mathbb{Z}[\theta]$ in A_K. In particular, we deduce necessary and sufficient conditions for A_K to be equal to $\mathbb{Z}[\theta]$.</p>
17:00–17:20	<p>YOUNG SCHOLAR TALK: On Quadratic non residue non primitive roots</p> <p>SPEAKER: Ms. Bidisha Roy</p> <p>ABSTRACT: Let $q \geq 1$ be any integer and let $\epsilon \in [\frac{1}{11}, \frac{1}{2})$ be a given real number. In this talk, we consider all primes p satisfying</p> $p \equiv 1 \pmod{q}, \quad \log \log p > \frac{\log 6.83}{\frac{1}{2} - \epsilon} \quad \text{and} \quad \frac{\phi(p-1)}{p-1} \leq \frac{1}{2} - \epsilon.$ <p>We will discuss that for those primes there exists a quadratic non-residue g which is not a primitive root modulo p such that $\gcd\left(g, \frac{p-1}{q}\right) = 1$. This is a joint work with Mr. Jaitra Chattopadhyay, Mr. Subha Sarkar and Dr. R. Thangadurai.</p>
17:20–17:30	Tea
20:00–21:30	DINNER at guest house

DAY-02 (October 09, 2018, TUESDAY)

TIME	SCHEDULE
	MORNING SESSION CHAIR PERSON: Prof. Renate Scheidler
9:30–10:30	PLENARY TALK: On the Continued Fraction of \sqrt{p} SPEAKER: Prof. Anupam Saikia ABSTRACT: We show that the length of the period of the continued fraction of \sqrt{p} is divisible by 4 when p is a prime congruent to 7 modulo 8, and the length is of the form $4k + 2$ when p is a prime congruent to 3 modulo 8. We further show that for any prime p congruent to 3 modulo 4, the central term in the palindromic part of the period of \sqrt{p} is the largest odd integer not exceeding \sqrt{p} . As a consequence of our approach, we show that Mordell's conjecture concerning the fundamental unit of $\mathbb{Q}(\sqrt{p})$ holds if \sqrt{p} has period of length 4 or 6.
10:30–10:50	Tea & Discussion
	PRE-LUNCH SESSION CHAIR PERSON: Prof. Michael J. Jacobson Jr.
10:50–11:30	INVITED TALK: Finite sequences of ELE type and a lower bound for the class number of certain real quadratic fields SPEAKER: Prof. Yasuhiro Kishi ABSTRACT: The first goal of this talk is to introduce a notion of extremely large end (ELE) for a finite sequence and to give a way to construct it. The second goal is to give a lower bound for the class number of real quadratic fields $\mathbb{Q}(\sqrt{d})$ such that the primary symmetric part of the simple continued fraction expansion of \sqrt{d} is of ELE type. By applying this lower bound to a sequence $\langle 2, \dots, 2, 2, 1 \rangle$ of pre-ELE type, we get a family of real quadratic fields with non-trivial class number.
11:30–11:40	Discussion
11:40–12:20	INVITED TALK: An height pairing on split tori SPEAKER: Prof. Valerio Talamanca ABSTRACT: Let G be a split tori defined over a number field K . We construct an height pairing between the algebraic points of G and the monoid of rational representations of G defined over the algebraic closure of K and discuss some of its property.
12:20–12:30	Discussion
12:30–13:10	INVITED TALK: On a Conjecture of Erdős on Squares in Arithmetic Progression SPEAKER: Prof. Shanta Laishram

TIME	SCHEDULE
	<p>ABSTRACT: A remarkable result of Erdos and Selfridge states that a product of a two or more consecutive integers is never a perfect power. Erdős conjectured that if a product of k consecutive terms of an arithmetic progression is a perfect power, then k is bounded explicitly. In this talk, I will give an overview of the problem with emphasis on the squares case and present some new results.</p>
12:50–14:30	LUNCH
	<p>POST-LUNCH SESSION CHAIR PERSON: Prof. Anupam Saikia</p>
14:30–15:30	<p>PLENARY TALK: Numerical investigation of fake real quadratic orders SPEAKER: Prof. Renate Scheidler</p> <p>ABSTRACT: In an unpublished note from 2014, Henri Cohen coined the term "fake real quadratic order" for the ring obtained by adjoining the inverse of a prime ideal above a split rational prime to an imaginary quadratic order. He demonstrated that these objects behave very much like real quadratic orders which can in some sense be interpreted as fake real quadratic orders with respect to the infinite prime. This invites the question of whether certain well-known conjectures formulated for actual real quadratic orders also hold in fake real quadratic orders. Two such conjectures include the widely believed Cohen-Lenstra (CL) heuristics and the more controversial Ankeny-Artin-Chowla (AAC) conjecture. The CL heuristics assert in particular that approximately 75% of all real quadratic fields have class number 1. The AAC conjecture alleges that if $e = a + b\sqrt{q}$ is the fundamental unit of a real quadratic field of prime discriminant q, then q never divides b. Both these conjectures have undergone extensive numerical testing: there is ample computational evidence supporting CL, and no counterexample to AAC has been found to date. In this talk, we present convincing numerical data that speak to these two conjectures in the setting of fake real quadratic orders. This is joint work with Mike Jacobson and Hongyan Wang.</p>
15:30–15:50	Photo Session, Tea & Discussion
	<p>EVENING SESSION CHAIR PERSON: Prof. Nianliang Wang</p>
15:50–16:30	<p>INVITED TALK: A family of pairs of imaginary cyclic fields with both class numbers divisible by p SPEAKER: Prof. Miho Aoki</p>

TIME	SCHEDULE
	<p>ABSTRACT: We construct a new infinite family of pairs of imaginary cyclic fields of degree $(p - 1)/2$ with both class numbers divisible by a given prime number p. The pairs are explicitly given using a generalized linear recurrence sequences. For the proof, we use the fundamental unit of $\mathbb{Q}(\sqrt{p})$, certain units which are roots of a parametric quartic polynomial, the Kummer theory, the Gauss sums and the Jacobi sums, linear recurrence sequences, a consequence of the Weil conjecture and a result of Lenstra which is a generalization of Artin conjecture on primitive roots.</p>
16:30–16:40	Discussion
16:40–17:00	<p>YOUNG SCHOLAR TALK: Discriminants of pure square-free degree number fields SPEAKER: Dr. Neeraj Sangwan</p> <p>ABSTRACT: Let $K = \mathbb{Q}(\theta)$ be an algebraic number field where θ is a root of an irreducible polynomial $f(x) = x^n - a \in \mathbb{Z}[x]$ of square-free degree n and a is an n-th power-free integer. We will look at the discriminant of K and some of its applications.</p>
17:00–17:20	<p>YOUNG SCHOLAR TALK: Fixed Divisors and its Applications SPEAKER: Mr. Devendra Prasad</p> <p>ABSTRACT: In this talk, we will give a quick introduction to the fixed divisor of a polynomial and its applications. For a polynomial $f \in \mathbb{Z}[x]$, its fixed divisor over \mathbb{Z} is defined as the gcd of all values taken by f over \mathbb{Z}. This talk is a part of our survey article ‘A Survey on Fixed Divisors’. We discuss the Bharagava’s generalized factorials and their connection with fixed divisors. We will present how this can be further generalized to the case of several variables. We give applications of fixed divisors to the number fields. At the end, we will also discuss few open problems and conjectures.</p>
17:20–17:30	Tea
18:30–19:30	MUSICAL PROGRAM at auditorium
20:00–21:30	DINNER

DAY-03 (October 10, 2018, WEDNESDAY)

TIME	SCHEDULE
	MORNING SESSION CHAIR PERSON: Prof. Michel Waldschmidt
9:30–10:30	PLENARY TALK: Integrality of L-values, and the Herbrand-Ribet theorem SPEAKER: Prof. Dipendra Prasad ABSTRACT: Following the natural instinct that when a group operates on a number field then every term in the class number formula should factorize compatibly according to the representation theory (both complex and modular) of the group, we are led - in the spirit of Herbrand-Ribets theorem on the p -component of the class number of $\mathbb{Q}(\zeta_p)$ - to some natural questions about the p -part of the classgroup of any CM Galois extension of \mathbb{Q} as a module for $\text{Gal}(K/\mathbb{Q})$, and about integrality of L -values. This talk will attempt doing this in terms of precise conjectures.
10:30–10:50	Tea & Discussion
	PRE-LUNCH SESSION CHAIR PERSON: Prof. Valerio Talamanca
10:50–11:30	INVITED TALK: An algebraic approach to the Siegel-Weil average for binary quadratic forms SPEAKER: Prof. Benjamin Kane ABSTRACT: In this talk, we will consider the celebrated results of Siegel and Weil about the number of representations by the genus of a quadratic form. By restricting to the case of binary quadratic forms and investigating the question via the associated algebraic theory of quadratic fields and Gauss's composition law, we obtain a new proof that these are coefficients of certain Eisenstein series and obtain nice explicit formulas for their evaluations. This is based on joint work with Pavel Guerzhoy.
11:30–11:40	Discussion
11:40–12:20	INVITED TALK: Integral points on a hyperelliptic curve of certain type SPEAKER: Prof. Toru Komatsu ABSTRACT: In this talk, an effective method to find all the rational integral points on a hyperelliptic curve of certain type is given. The method is elementary and requires no data on the Jacobian.
12:20–12:30	Discussion
12:30–13:10	INVITED TALK: On the factorization of p -adic L-functions SPEAKER: Dr. Daniele Casazza

TIME	SCHEDULE
	<p>ABSTRACT: Ever since the introduction of p-adic L-functions in number theory, mathematicians studied their properties and created new techniques to achieve the task. In particular, one often finds an interplay between special values and formulas and factorization formulas. I will discuss about some recent development in this direction, in particular for p-adic L-functions associated with tensor products of two and three modular forms.</p>
13:10–14:30	LUNCH
	<p>POST-LUNCH SESSION CHAIR PERSON: Prof. Jean Gillibert</p>
14:30–15:30	<p>PLENARY TALK: Compact Representations: Applications and Recent Results SPEAKER: Prof. Jr. Michael J. Jacobson</p> <p>ABSTRACT: Compact representations are explicit representations of algebraic numbers or functions, with size polynomial in the logarithm of their height or, respectively, degree. These representations enable much more efficient manipulations of large algebraic numbers or functions than would be possible using a standard representation, and have proved to be useful in a variety of applications.</p> <p>In this talk, we will describe two such applications - how compact representations are essential for short certificates of the unit group and ideal class group of a number field, and how they can be used to speed the resolution of certain Diophantine equations. We will also present recent improvements that reduce the size of compact representations, efforts to generalize these to hyperelliptic function fields, and applications of the latter to speeding the computation of bilinear pairings.</p>
15:30–15:50	Tea & Discussion
	<p>EVENING SESSION CHAIR PERSON: Prof. Rupam Barman</p>
15:50–16:30	<p>INVITED TALK: Recent progress on the theory of Egyptian fractions SPEAKER: Prof. Francesco Pappalardi</p> <p>ABSTRACT: We consider the function $A_k(n)$ which is defined as the number of positive integers a such that $a/n = 1/x_1 + \dots + 1/x_k$ admits a solution in positive integers x_1, \dots, x_k. After having reviewed earlier results we shall present new ones recently obtained in collaboration with F. Luca and C. Banderier.</p>
16:30–16:40	Discussion

TIME	SCHEDULE
16:40–17:00	<p>YOUNG SCHOLAR TALK: Generalized Lambert series, Raabe’s cosine transform and a two-parameter generalization of Ramanujan’s formula for $\zeta(2m + 1)$.</p> <p>SPEAKER: Mr. Rajat Gupta</p> <p>ABSTRACT: A comprehensive study of the generalized Lambert series is undertaken; transformations of this series are derived by investigating Raabe’s cosine transform. Using this, we will obtain a two-parameter generalization of Ramanujan’s formula for $\zeta(2m + 1)$; where $\zeta(s)$ is Riemann zeta function. This involves a delicate analysis of an infinite sum of Raabe’s cosine transform, which is an interesting result in itself. This is joint work with Prof. Atul Dixit, Rahul Kumar and Bibekananda Maji.</p>
17:00–17:20	<p>YOUNG SCHOLAR TALK: Geometric progressions in syndetic sets.</p> <p>SPEAKER: Mr. Bhuwanesh Rao Patil</p> <p>ABSTRACT: In order to investigate multiplicative structure in additively large sets, Beiglböck et al. raised a significant open question as to whether or not every subset of the natural numbers with bounded gaps (syndetic set) contains arbitrarily long geometric progressions. Here we prove that for each $k \in \mathbb{N}$, a syndetic set contains geometric progressions of length $2k$ with common ratios $n^k r_1$ and $p^k r_2$, where $r_1, r_2 \in \mathbb{N}$, $p \in \mathbb{P}$ (the set of primes), $n \in \mathbb{N} \setminus \mathbb{P}$, $r_1 \equiv 1 \pmod{n}$ and $r_2 \equiv 1 \pmod{p}$. We also show that syndetic sets with bounded gap 2 contain infinitely many geometric progressions of length 2 with their respective common ratios being perfect squares.</p>
17:20-17:30	Tea
20:00-21:30	CONFERENCE BANQUET DINNER

DAY-04 (October 11, 2018, THURSDAY)

TIME	SCHEDULE
	MORNING SESSION CHAIR PERSON: Prof. Dipendra Prasad
10:00–11:00	PLENARY TALK: From Picard groups of hyperelliptic curves to class groups of quadratic fields SPEAKER: Prof. Jean Gillibert ABSTRACT: Let C be a hyperelliptic curve defined over \mathbb{Q} . Under some mild hypotheses, we prove that any line bundle of degree 0 on C which is not torsion can be specialised into ideal classes of imaginary quadratic fields whose order can be made arbitrarily large. This generalises a result of Soleng, who treated the case of elliptic curves.
11:00–11:20	Tea & Discussion
	PRE-LUNCH SESSION CHAIR PERSON: Prof. Francesco Pappalardi
11:20–12:00	INVITED TALK: Discrepancy estimate for generalized polynomials SPEAKER: Prof. Anirban Mukhopadhyay ABSTRACT: We obtain an upper bound for discrepancy of sequence given by generalized polynomials. This is a joint work with G. K. Viswanadham and O. Ramare.
12:00–12:10	Discussion
12:10–12:50	INVITED TALK: Class numbers of imaginary quadratic fields SPEAKER: Dr. Prem Prakash Pandey ABSTRACT: For a given odd integer $n > 1$, we will discuss some families of imaginary quadratic number fields of the form $\mathbb{Q}(\sqrt{x^2 - t^n})$ whose ideal class group has a subgroup isomorphic to $\mathbb{Z}/n\mathbb{Z}$. This talk is based on a joint work with K. Chakraborty, A. Hoque, and Y. Kishi.
13:00–14:30	LUNCH
	POST-LUNCH SESSION CHAIR PERSON: Prof. Benjamin Kane
14:30–15:30	PLENARY TALK: \mathbb{Z} -basis for Galois-invariants orders and applications SPEAKER: Prof. Stephane Louboutin

TIME	SCHEDULE
	<p>ABSTRACT: Let α be an algebraic integer of degree $n \geq 2$. Assume that the extension $\mathbb{Q}(\alpha)/\mathbb{Q}$ is Galois. Let \mathbb{M}_α be the order of $\mathbb{Q}(\alpha)$ generated by the n complex conjugates of α. Apart from the case that $\text{Gal}(\mathbb{Q}(\alpha)/\mathbb{Q})$ is the symmetric group \mathfrak{S}_n, only for $n = 3$ are an explicit \mathbb{Z}-basis and the discriminant of \mathbb{M}_α known. In the talk, (i) we prove that for any $n \geq 2$ there always exists a \mathbb{Z}-basis of \mathbb{M}_α of the form $\{1, \alpha, \omega_2, \dots, \omega_n\}$ starting with 1 and α. Then, (ii) bearing on (i) we give a new proof of the foregoing known result for $n = 3$ in the hope that (i) could be helpful to settle the unsolved case $n = 4$. Finally, (iii) for $n = 3$, bearing on numerical computations based on our formula for the discriminants of these orders, we propose an unsolved problem related to the apparent behavior of the sequence of orders \mathbb{M}_{α^k}, $k \geq 1$, and (iv) for ε an algebraic unit of any degree $n \geq 2$ we include a proof on the behavior of the orders $\mathbb{Z}[\varepsilon^k]$, $k \geq 1$, (without assuming that $\mathbb{Q}(\varepsilon)/\mathbb{Q}$ is Galois). In particular, at least in the Galois cubic case, it seems that the orders \mathbb{M}_{α^k}, $k \geq 1$, behave completely differently from the orders $\mathbb{Z}[\alpha^k]$, $k \geq 1$.</p>
15:30-15:40	Discussion
15:40–16:00	<p>YOUNG SCHOLAR TALK: Zeros of combinations of the Riemann Ξ-function and the confluent hypergeometric function on bounded vertical shifts</p> <p>SPEAKER: Mr. Rahul Kumar</p> <p>ABSTRACT: In 1914, Hardy proved that infinitely many non-trivial zeros of the Riemann zeta function lie on the critical line using the transformation formula of the Jacobi theta function. Recently Dixit obtained an integral representation involving the Riemann Ξ-function and the confluent hypergeometric function linked to the general theta transformation. Using this result, we show that a series consisting of bounded vertical shifts of a product of the Riemann Ξ-function and the real part of a confluent hypergeometric function has infinitely many zeros on the critical line, thereby generalizing a previous result due to Dixit, Roy, Robles and Zaharescu. The latter itself is a generalization of Hardy's theorem. This is the joint work with Dixit, Maji and Zaharescu.</p>
16:00–16:20	<p>YOUNG SCHOLAR TALK: Biquadratic fields having a non-principal Euclidean ideal class</p> <p>SPEAKER: Mr. Jaitra Chattopadhyay</p>

TIME	SCHEDULE
	<p>ABSTRACT: In 1979, H. W. Lenstra defined the notion of an Euclidean ideal class and proved that if a number field K has a non-principal Euclidean ideal class then the ideal class group Cl_K of K is cyclic. Except for the imaginary quadratic fields, he was able to prove the converse, under the assumption of GRH. Later, H. Graves constructed an explicit biquadratic field having an Euclidean ideal class and after a few years C. Hsu provided a family of such fields. In this talk, we shall give a new class of biquadratic fields other than the ones given by Graves and Hsu. This is a joint work with M. Subramani.</p>
16:25–16:40	VALIDICTORY FUNCTION
16:40-17:00	HIGH TEA
20:00–21:30	DINNER