Research Report by Prof. P.P. Ntumba

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IMU-Simons African Fellowship Program

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3. <u>Place of Visit</u>: Paris XIII University, Mathematics Department- LAGA, 99 Avenue Jean-Baptiste Clément, 93430 Villetaneuse, France

4. <u>Dates of Visit</u>: 25 May- 04 July 2018

5. <u>Duration of Visit</u>: 41 Days

6. <u>Project title</u>: **Exceptional isomorphisms between groups of similitudes of Azumaya algebras with involution**

7. <u>Conditions of Research Visit</u>: During my research visit at Paris XIII University, I was accommodated on the university campus, which was very convenient as from my apartment to the office, it took me about 10 minutes on foot. Because of this proximity, I could afford to work till as late as I wanted. My host, Prof. A. Quéguiner arranged for all necessary facilities, such as office space in the department, access to the department's photocopying machine and library, etc... I attended seminar talks in the Algebraic Topology research group and interacted with the group faculty.

8. <u>Description</u>: It is known (cf. [1]) for central simple algebras over a field, there exist isomorphisms, dubbed exceptional isomorphisms, between certain categories. The project at hand consists of investigating counterparts of these exceptional isomorphisms in the event that we substitute the base field with a ring, and in the place of central simple algebras, we consider Azumaya algebras. The isomorphisms expected would be between the following categories:

i) $A_1 \equiv B_1 \equiv C_1$

ii) $A_1 \times A_1 \equiv D_2$

iii) $B_2 \equiv C_2$

iv) $A_3 \equiv D_3$,

where

- A_1 is the category of Azumaya algebras of degree 2
- B_1 is the category of quadratic spaces of dimension 3
- C_1 is the category of algebras with symplectic involution of degree 2
- D_2 is the category of algebras of degree 4 endowed with a quadratic pair
- B_2 is the category of quadratic spaces of dimension 5
- C_2 is the category of algebras with symplectic involution of degree 4
- A_3 is the category of algebras with unitary involution of degree 4
- D_3 is the category of quadratic spaces of dimension 6.

All along my research visit at Paris XIII, with the assistance of Profs A. Quéguiner-Mathieu and B. Calmès, I focused on the background of classical results pertaining to exceptional isomorphisms, that relate central simple algebras with orthogonal involutions of degree $n = 3, 4, 5, 6$ with their Clifford algebras. The background work is yet to be finished as it requires a good grasp of the theory of central simple algebras over a field.

The people involved in this research project include B. Calmès (Université d' Artois), P.P. Ntumba (University of Pretoria), A. Quéguiner-Mathieu (Université Paris XIII), and my Ph.D. student R. N'Gambi (University of Pretoria).

9. Sources of Funding for my travel, accommodation and meals: **IMU-Simons African Fellowship Program**

References

[1] Knus M-A., Merkurjev A., Rost M., Tignol J-P., *The Book of Involutions,* American Mathematical Society, Colloquium Publications, Volume **44, 1998.**