



# **The International Conference**

## **QUANTUM TOPOLOGY & HYPERBOLIC GEOMETRY**

*Da Nang, Vietnam, May 27<sup>th</sup>-31<sup>st</sup>, 2019*

# Quantum Topology and Hyperbolic Geometry Conference

Da Nang, May 27–31, 2019

## Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00–8:50	<b>Registration</b>	Bar-Natan	Liu	Porti	Kalfagiani
9:10–10:00	Sikora	Beliakova	Tian	Purcell	Tran
10:30–11:20	Kofman	Queffelec	Chen	Jeon	Wong
11:40–12:30	Nawata	Wedrich	Karuo (30')	Irmer	Bonahon
12:30–14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00–14:50	Kania-Bartoszynska	Discussion/free	Excursion	Discussion/free	Discussion/free
afternoon	free	free	Excursion	free	free
18:00–20:00				banquet	

Excursion: Wednesday 14:00–19:00.

Banquet: Thursday 6-8pm. Rooftop bar (22nd floor) Haian Beach Resort Hotel, 278 Võ Nguyên Giáp.

## Abstracts

- **Bar-Natan, Dror** (University of Toronto, Canada)

### Everything around $sl^{\epsilon}2+$ is D-PeGDO. So what?

I'll explain what "everything around" means: classical and quantum  $m, ?, S, tr, R, C$ , and  $\theta$ , as well as  $P, \Phi, J, D$ , and more, and all of their compositions. What D-PeGDO means: the category of Docile Perturbed Gaussian Differential Operators. And what  $sl^{\epsilon}2+$  means: a solvable approximation of the semi-simple Lie algebra  $sl_2$ . Knot theorists should rejoice because all this leads to very powerful and well-behaved poly-time-computable knot invariants. Quantum algebraists should rejoice because it's a realistic playground for testing complicated equations and theories. This is joint work with Roland van der Veen and continues work by Rozansky and Overbay.

Link to handout: <http://www.math.toronto.edu/~drorbn/Talks/DaNang-1905/>.

- **Beliakova, Anna**

### On Functoriality of the Khovanov homology

We provide an elegant solution of the functoriality problem for Khovanov homology and any other link homology theory that factorizes through the Bar-Natan category. Among them are the Chen-Khovanov tangle homology, author's quantum annular homology and a categorification of the colored Jones polynomial in characteristic zero. We solve the problem not by replacing the original homology theory by a (more complicated) new one, but rather by showing explicitly how to correct signs in the original construction. The proofs are based on a simple geometric idea: replacing a foam with two intersecting surfaces and analyzing how deformations of these surfaces affect the foam.

This is joint work with Kris Putyra, Matt Hogancamp and Stephan Wehrli.

- **Bonahon, Francis** (University of Southern California, USA)

### How to multiply matrices? From hyperbolic geometry to quantum topology

We of course know how to multiply matrices when their entries commute with each other. However, extending constructions from classical geometry (such as 2- and 3-dimensional hyperbolic geometry) to quantum topology (such as the Jones polynomial of a knot, or the Kauffman bracket skein algebra of a surface) requires the consistent multiplication of matrices with non-commuting entries. I will explain how to do this in the context of the quantum group  $U_q(sl_2)$ .

- **Chen, Qingtao**

### Recent progress of various Volume Conjectures for links as well as 3-manifolds

The original Volume Conjecture of Kashaev-Murakami-Murakami predicts a precise relation between the asymptotics of the colored Jones polynomials of a knot in  $S^3$  and the hyperbolic volume of its complement.

I will discuss two different directions that lead to generalizations of this conjecture. The first direction concerns different quantum invariants of knots, arising from the colored  $SU(n)$  (with the colored Jones polynomial corresponding to the case  $n = 2$ ). I will first display subtle relations between congruence relations, cyclotomic expansions and the original Volume Conjecture for colored Jones polynomials of knots. I will then generalize this point of view to the colored  $SU(n)$  invariant of knots. Certain congruence relations for colored

$SU(n)$  invariants, discovered in joint work with K. Liu, P. Peng and S. Zhu, lead us to formulate cyclotomic expansions and a Volume Conjecture for these colored  $SU(n)$  invariants. I will also discuss similar ideas for the superpolynomials that arise in HOMFLY-PT homology. In fact, I proposed cyclotomic expansion conjectures and Volume conjectures for superpolynomials.

Another direction for generalization involves the Witten-Reshetikhin-Turaev and (modified) Turaev-Viro quantum invariants of 3-manifolds. In a joint work with T. Yang, we formulated a new Volume Conjecture for the asymptotics of these 3-manifolds invariants evaluated at certain roots of unity, and numerically checked it for many examples. Interestingly, this conjecture uses roots of unity that are different from the one usually considered in literature. This may indicate that the understanding of this new phenomenon requires new physical and geometric interpretations that go beyond the usual quantum Chern-Simons theory. I will also introduce a work on Krillov-Reshetikhin quantum  $6j$ -symbols done by J. Murakami & me.

- **Jeon, Bogwang** (Postech, Korea)

**On the number of hyperbolic Dehn fillings of a given volume**

Let  $M$  be an  $n$ -cusped hyperbolic 3-manifold. By Thurston's Dehn filling theorem, it is well-known that the number  $N_M(v)$  of hyperbolic Dehn fillings of  $M$  with volume  $v$  is finite, and thus it is natural to ask whether there exists  $c$  (depending only on  $M$ ) such that  $N_M(v) < c$  for any  $v$ . In this talk, I will discuss possible approaches answering this question as well as other related questions.

- **Kalfagiani, Effie**

**Asymptotic behavior of quantum representations**

this talk I will discuss some progress on understanding the asymptotic behavior of certain representations of surface mapping class groups. I will also discuss some geometric properties of surface bundles detected by these asymptotics.

- **Kania-Bartoszynska, Joanna** (National Science Foundation, USA)

**Skein algebras and modules**

We will recall some joint results with C. Frohman and T. Le that describe the structure of the Kauffman bracket skein algebra of a surface. We will also compute the skein module of a connected sum of products of circles and 2-spheres, and use it to get a new derivation of the trace in an analogy to the construction of the Yang-Mills measure.

- **Karuo, Hiroaki** (RIMS, Kyoto, Japan)

**The reduced Dijkgraaf–Witten invariant of twist knots in the Bloch group of  $\mathbb{F}_p$**

For a closed 3-manifold  $M$ , a finite group  $G$ , and a representation  $\pi_1(M) \rightarrow G$ , (an invariant which is equivalent to) the image of the fundamental class of  $M$  by a map  $H_3(M) \rightarrow H_3(G)$  induced by the representation and a 3-cocycle is called the Dijkgraaf–Witten invariant. In the case that  $G = \mathrm{SL}_2\mathbb{C}$ , Neumann described the Dijkgraaf–Witten invariant by using the Bloch group of  $\mathbb{C}$  in 2004.

In this talk, in the case that  $G = \mathrm{SL}_2\mathbb{F}_p$  ( $\mathbb{F}_p$  denotes a finite field of prime order) I describe the Dijkgraaf–Witten invariant of the complements of twist knots by using the Bloch group of  $\mathbb{F}_p$ .

- **Kofman, Ilya** (CUNY, USA)

### Mahler measure and the Vol-Det Conjecture

The Vol-Det Conjecture relates the volume and determinant of alternating links. We prove the Vol-Det Conjecture for infinite families of alternating links using the dimer model, the Mahler measure of 2-variable polynomials, and the hyperbolic geometry of biperiodic alternating links. This is joint work with Abhijit Champanerkar and Matilde Lalin.

- **Irmer, Indrid** **Fibering rigidity of 3-manifolds with Torelli monodromy**

If a 3-manifold  $M$  fibers over the circle, Thurston norm theory completely describes all the different ways this can happen. In particular, if the first betti number of  $M$  is greater than one,  $M$  fibers in infinitely many ways if it fibers at all.

The first betti number of  $M$  determines a lower bound on the genus of a fiber surface of  $M$ . The Torelli group is the normal subgroup of the mapping class group that acts trivially on the homology of the surface. The genus of the fiber surface realises the lower bound coming from the betti number iff the monodromy is in the Torelli group. In this talk, I will give an outline of the proof of the following: If  $M$  is hyperbolic and fibers over the circle with Torelli monodromy, then there is only one such fibering, up to isotopy of the fiber surface.

- **Liu, Yi**

#### Learning mapping classes through their finite quotient actions

Given a closed orientable surface and any mapping class of the surface, one may look at the outer automorphism induced on any finite quotient of the surface fundamental group. How much about the mapping class can we learn through these induced outer automorphisms? If two mapping classes induce conjugate outer automorphic actions on all finite quotients, what properties do they have in common? In this talk, I will discuss some recent results on and around the above topic.

- **Nawata, Satoshi** (Fudan University, China)

#### Positivity in knots

I will talk about several ways to obtain positivity properties in knot invariants.

- **Porti, Joan** (Universitat Autònoma de Barcelona, Spain)

#### Twisted Alexander polynomials and hyperbolic volume for knots

Given a hyperbolic knot exterior  $S^3 - K$ , we consider the composition of a lift of its holonomy in  $SL(2, \mathbb{C})$  with the irreducible representation on  $SL(n, \mathbb{C})$ , that yields a twisted Alexander polynomial  $A_n(t)$ , for each natural  $n$ . We prove that, for a complex number  $\zeta$  of norm one,

$$\lim_{n \rightarrow \infty} \frac{\log |A_n(\zeta)|}{n^2} = \frac{1}{4\pi} \text{vol}(S^3 - K)$$

This generalizes and uses a theorem of W. Mueller for closed manifolds, previous joint results with P. Menal-Ferrer, and work of H. Goda. This is a collaboration in progress with J. Dubois, M. Heusener (Clermont-Ferrand) and L. Bénard (Genève).

- **Purcell, Jessica**

#### Effective geometry change under Dehn surgery

Hyperbolic Dehn surgery is a way to obtain one hyperbolic manifold from another. Many results, going back to Thurston in the late 1970s, imply that for ‘most’ Dehn surgeries, the hyperbolic geometry changes ‘very little.’ However, it has been difficult to apply these theorems to concrete examples because ‘most’ and ‘very little’ have not been quantified in

general. In this talk, I will describe results giving effective bounds on change in hyperbolic metric under Dehn surgery, and present some consequences. One consequence is that the verification of the cosmetic surgery conjecture for any given hyperbolic knot can be reduced to a finite computer search. Another consequence is to Margulis numbers of closed hyperbolic manifolds. This work is joint with David Futer and Saul Schleimer.

- **Queffelec, Hoel**

**Khovanov homology for thickened surfaces**

(joint works with David Rose and Paul Wedrich)

Khovanov homology was, when first defined, an invariant for links in the 3-space. It was extended by Asaeda, Przytycki and Sikora to links embedded in thickened surfaces, which can be thought of as a first step towards an extension to general 3-manifolds via Heegaard decomposition. Roberts' approach to the annulus version and its reinterpretation by Grigsby, Licata and Wehrli highlighted the fact that the annular homology arises as the first page a spectral sequence associated to a non-negative grading coming from the topology of the annulus, and converging towards usual Khovanov homology.

After discussing the origin of this non-negative grading, I'll explain how it extends to the case of general surfaces and allows to re-interpret the APS construction in terms of truncation functors.

- **Sikora, Adam**

**Quantum Toric Degenerations of Skein Algebras**

We explore the properties of skein algebras of surfaces by applying the theory of pseudo-Anosov diffeomorphisms and of measured foliations. In particular, we prove that every sufficiently generic measured foliation of a surface defines a quantum toric degeneration of the skein algebra. One of the consequences of this result is a novel proof of skein algebras being Ore domains, for both orientable and non-orientable surfaces. It also implies that measured foliations define toric degenerations of character varieties of surfaces.

- **Wong, Helen**

**Roger-Yang's Arc Algebra**

In the case of a closed surface, there is a rich body of work describing how the Kauffman bracket skein algebra can be regarded as a quantization of the Teichmuller space. In order to generalize to a surface with punctures, Roger and Yang defined an arc algebra  $\mathcal{A}(\Sigma)$  that they proposed to be the analogous quantization of Penner's decorated Teichmuller space. We will discuss their approach, and in particular, we will explain how certain multiplicative properties of  $\mathcal{A}(\Sigma)$  from joint work with Han-Bom Moon fits into the picture.

- **Yang, Tian** (Texas A&M University, USA)

**Some progress on the volume conjecture for the Turaev-Viro invariants**

In 2015, Qingtao Chen and I conjectured that at the root of unity  $\exp(2\pi\sqrt{-1}/r)$  instead of the usually considered root  $\exp(\pi\sqrt{-1}/r)$ , the Turaev-Viro and the Reshetikhin-Turaev invariants of a hyperbolic 3-manifold grow exponentially with growth rates respectively the hyperbolic and the complex volume of the manifold. In this talk, I will present a recent joint work with Giulio Belletti, Renaud Detcherry and Effie Kalfagianni on an infinite family of cusped hyperbolic 3-manifolds, the fundamental shadow links complement, for which the conjecture is true.

- **Tran, Anh**

**Left-orderability and L-space conjecture**

Heegaard Floer homology is a package of 3-manifold invariants introduced by Ozsvath and Szabo. Manifolds with minimal Heegaard Floer homology are called L-spaces. The L-space conjecture of Boyer, Gordon and Watson states that an irreducible rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable. In this talk, we will discuss this conjecture for 3-manifolds obtained by Dehn surgeries on twisted torus knots and pretzel knots.

- **Wedrich, Paul**

**Categorical invariants of annular links**

A classical result of Turaev identifies the positive HOMFLYPT skein algebra of the annulus with the algebra of symmetric functions. Queffelec and Rose categorified this using annular webs and foams. I will recall their construction and compute explicit symmetric functions and their categorical analogues for some links. This is joint work with Eugene Gorsky.



# MINI-COURSES

## QUANTUM TOPOLOGY & HYPERBOLIC GEOMETRY

*Quy Nhon, Vietnam, June 02<sup>nd</sup>-05<sup>th</sup>, 2019*



# MINI-COURSES

## Quantum Topology and Hyperbolic Geometry Conference Quy Nhon, June 2 - Jun 5 , 2019

### Schedule

	13:00-13:45	14:00-14:45	15:00 - 15:45	16:30-17:30
Sunday	Thăng Lê	Thăng Lê	Khôi Vũ	Paul Wedrich

	8:00-9:00	9:30-10:30	11:00 - 11:45	13:00-14:00	14:30-15:30
Monday	Wedrich	Wedrich	discussion	Sikora	Porti
Tuesday	Wedrich	Sikora	discussion	Porti	Porti
Wednesday	Sikora	Sikora	discussion	Porti	Porti

### Introductory talks

- **Le, Thang** (Georgia Institute of Technology, USA)

#### Introduction to knot theory

Knots, Reidemeister moves. Composite and satellite knots. Jones polynomials. Skein module, skein algebra. We will do a lot of examples.

- **Vu, Khoi** (Institute of Mathematics, Ha Noi)

#### Character variety, Alexander polynomials

Knot groups, Wirtinger presentations. Examples of character varieties and Alexander polynomials.

### Mini-courses

- **Sikora, Adam** (University at Buffalo, USA)

#### Character varieties in low-dimensional topology

For a matrix group  $G$ , the  $G$ -character variety  $X_G(Y)$  of a topological space  $Y$  the set all representations of  $\pi_1(Y)$  into  $G$  up to conjugation. We will discuss its algebraic properties as well as some of its important applications to low-dimensional topology through the Culler-Shalen theory, the A-polynomial invariant of knots and skein modules.

- **Porti, Joan** (Universitat Autònoma de Barcelona, Spain)

#### Hyperbolic knot theory

I plan to start with the basics of hyperbolic geometry and I will go to the main results of the theory, in particular Thurston's hyperbolization. I will not prove this theorem, but construct several examples. I also plan to cover important results as Margulis lemma, Mostow rigidity and hyperbolic Dehn surgery. As a consequence of Mostow rigidity, metric invariants are also topological invariants. A particular attention will be devoted to the volume. I will also talk about invariants like Chern-Simons and Reidemeister torsion.

- **Wedrich, Paul** (The Australian National University, Australia)

#### Skein theory quantum $gl_n$

The famous Temperley-Lieb algebra or, in other words, the Kauffman bracket skein relations give a diagrammatic way to describe the representation theory of quantum  $sl_2$  and to compute the Jones polynomial and its colored versions. In this mini-course, I will give an introduction to the less well-known skein theory for quantum  $gl_n$ . In the first

lecture, I will define the diagrammatic calculus of  $gl_n$  webs and show how it can be used to compute (colored)  $gl_n$  and HOMFLY knot polynomials. In the second lecture, I will explain how webs faithfully encode intertwiners for representations of quantum  $gl_n$ . In the third lecture, I will introduce  $gl_n$  skein modules and skein algebras and compare the  $gl_2$  case to the Kauffman bracket skein modules and skein algebras. Possible topics for the final lecture, depending on the interests of the audience, include quantum  $gl(m|n)$  skein theory and categorified skein theory.