Volunteer Lecturer Program: Exit Survey

Name of Volunteer: *Friedrich Hubalek*
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Home Institution: *TU Wien*
Position at home institution: *Associate Professor (Universitätsdozent)*
Host Institution: *National University of Mongolia (NUM)*
Arrival and departure date in host country: *June 7 - 27, 2015*

Who was your main contact in the host country (name, affiliation and email address)?

Associate Professor Tsogzolmaa Saizmaa
National University of Mongolia, Department of Mathematics
tsogzolmaa@smcs.num.edu.mn

Please answer the following questions:

Note: In my answers I refer to the "ensuing summer school", by which I mean:

> *Stochastic Processes and Applications, CIMPA - DAAD Research School, National University of Mongolia July 27 - August 07, 2015, Ulaanbaatar, Mongolia.*

1) Location (country, city, institution) of your lecture:

   Mongolia, Ulaanbaatar, National University of Mongolia, Department of Mathematics

2) Dates of your lecture:

   June 8 - 26, 2015 (Monday to Friday, daily)

3) Subject and title of the course:

   Subject: Probability Theory;

Title: "Introduction to measure-theoretic probability theory"

4) How often did you teach a course?

One course, three weeks, five days per week, each unit four hours; in total 15 units of lectures including some time for exercises.

5) How many students took part in the course(s)?

Initially 70 participants registered. Many were interested in any kind of lecture given in English language, but soon it became clear that this was an intensive and focused course with mathematical content and then the number dropped to 15 serious students of mathematics, who followed the course and actively participated in the exercises and discussions for the whole three weeks.

6) Background of students: Undergraduate/ Master/ PhD Students?

Mixed both undergraduate and master students. The main focus was to teach on a level appropriate for master students, and the course was component of a regular degree program for a Master of Science, majoring Mathematics, but several advanced bachelor students also participated successfully.

7) Please provide (if possible) any schedule of activities/list of topics covered during your visit.

Unit 1: Short introduction of myself, my university TU Wien, my home country Austria; Probability theory - What is it about? Classical, frequentistic, and subjective probability theory; axiomatic, measure-theoretic probability theory; measure theory; probability space; sigma-algebras and information; measurable spaces; sigma-algebras generated by systems of sets;

Unit 2: Pi-systems; Borel sets; the uniqueness theorem for pi-systems; Caratheodory's extension theorem; Lebesgue measure; elementary inequalities; monotone convergence of measures; null-sets;

Unit 3: Limit superior and limit inferior for sequences and events; the reverse Fatou lemma; the first Borel-Cantelli lemma;

Unit 4: Measurable functions and Borel functions; level sets; rules and tools for measurable functions; general definition of measurability; random variables, their law, and their distribution function;

Unit 5: The intuitive interpretation of measurability; the sigma-algebra generated by a family of functions; unions and sigma-algebras; properties of distribution functions; existence of a random variable with given CDF; Lebesgue-Stieltjes measure and Skorokhod representation; quantile functions;

Unit 6: The Doob-Dynkin lemma; monotone class theorem; independence of sigma-algebras; independence and pi-systems; independenc and CDFs; the second Borel-Cantelli lemma;

Unit 7: Construction of an iid sequence with given distribution; Bernoulli, binomial, and Dirac distribution; integration of non-negative functions; summation and integration; properties of integration; monotone convergence; the Fatou lemma and the reverse Fatou lemma;

Unit 8: Integrable functions; positive and negative parts, linearity, dominated convergence; the "standard machine"; integrals and subsets; measures with densities; absolutely continuous and equivalent measures; expectation; convergence theorems for expectations; the semi-colon notation;

Unit 9: Markov inequality; sums of non-negative random variables; Jensen's inequality; the truncation technique; L^2, Pythagoras, variance and covariance; orthogonality; quotient spaces; essential supremum; convergence in probability; the "standard machine" at work; Hölder and Minkowski inequalities;

Unit 10: "Independence means multiply"; an easy strong law; Chebychev's inequality; binomial distribution; probabilistic proof of the Weierstrass approximation theorem; product sigma-algebras;

using and practicing the monotone class theorem; product measure and Fubini's Theorem;

Unit 11: Joint laws, distribution functions, and densities; basic notions from the Caballero-Notes; metric and topological spaces; Polish spaces;

Unit 12: Conditional expectations and conditional probabilities; evaluated conditional expectation; the candidate method; first properties of conditional expectation;

Unit 13: Second group of properties of conditional expectation; characteristic functions;

Unit 14: Complements on conditional expectations; elementary conditional expectation and probabilities; conditional densities; Levy's inversion theorem for characteristic functions; the Gil-Pelaez inversion formula; Laplace transform and inversion in the complex domain;

Unit 15: Additional topics; Moments and moment generating functions, convolution; The central limit theorem; weak convergence; subsequences and convergence in probability; introduction to martingales in discrete time;

8) Did you develop or follow a prescribed syllabus or did you write your own? Was it available to the students before the course or when the course began?

Please also mention the references you used or any textbooks that were referred to:

I did not follow a particular prescribed syllabus, but prepared the course based on the book "Probability with martingales" by David Williams, Cambridge University Press, Cambridge, 1991. The author himself describes the book in the preface as "a set of lecture notes" for a course at Cambridge. I used also some notes entitled "Introduction to conditional expectation" by María Emilia Caballero from Instituto de Matemáticas, Universidad Nacional Autónoma de

México (UNAM), who wrote the notes as preparation and background for her course at the ensuing summer school, plus some spontaneous material (mainly in the introductory overview, Laplace transforms, additional material section) from my teaching experience. The Williams book and the Caballero notes were made available to the students before the course began. I referred also briefly to other books, see next question.

9) Did you use any books, classroom material, AV, or other technology-based materials?

I used mostly my favorite book on probability theory,

David Williams, "Probability with martingales", Cambridge University Press, Cambridge, 1991

and some notes entitled "Introduction to conditional expectation" by María Emilia Caballero from Instituto de Matemáticas, Universidad Nacional Autónoma de México (UNAM), that Prof. Caballero distributed as background and preparation for her course on stochastic processes at the ensuing summer school. All this material was available to the students before the course.

Exercise sheets have been prepared and distributed during the course.

I used a short beamer presentation to introduce myself, my university TU Wien, and my home country Austria. Otherwise I used mostly the blackboard, which is in my opinion still the best way to teach mathematics.

A few lectures have been recorded on video by staff from the National University of Mongolia, but I did not get those recordings.

At the beginning of each lecture I mentioned important and classical books on probability theory with additional or complementary material. I focused on European and American authors, as I believe they are less known than Russian and Soviet authors in Mongolian mathematics.

Feller, William "An introduction to probability theory and its applications. Vol. I". Third edition John Wiley & Sons, Inc., New York-London-Sydney 1968.

Feller, William "An introduction to probability theory and its applications. Vol. II". Second edition John Wiley & Sons, Inc., New York-London-Sydney 1971.

Chung, Kai Lai "A course in probability theory". Third edition. Academic Press, Inc., San Diego, CA, 2001

Loève, Michel "Probability theory. I." Fourth edition. Springer-Verlag, New York-Heidelberg, 1977.

Loève, Michel "Probability theory. II." Fourth edition. Springer-Verlag, New York-Heidelberg, 1978.

Billingsley, Patrick "Probability and measure." Third edition. John Wiley & Sons, Inc., New York, 1995.

Breiman, Leo "Probability." Corrected reprint of the 1968 original. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992.

Chow, Yuan Shih; Teicher, Henry "Probability theory. Independence, interchangeability, martingales." Third edition. Springer-Verlag, New York, 1997.

Bauer, Heinz "Probability theory." Walter de Gruyter & Co., Berlin, 1996.

Bauer, Heinz "Measure and integration theory." Walter de Gruyter & Co., Berlin, 2001.

Kallenberg, Olav "Foundations of modern probability." Second edition. Springer-Verlag, New York, 2002.

Hoffmann-Jørgensen, J. "Probability with a view toward statistics. Vol. I." Chapman & Hall, New York, 1994.

Hoffmann-Jørgensen, J. "Probability with a view toward statistics. Vol. II." Chapman & Hall, New York, 1994.

Ross, Sheldon M. "Introduction to probability models." Seventh edition. Harcourt/Academic Press, Burlington, MA, 2000.

Jacod, Jean; Protter, Philip "Probability essentials." Second edition. Universitext. Springer-Verlag, Berlin, 2003.

Durrett, Rick "Probability: theory and examples." Fourth edition. Cambridge University Press, Cambridge, 2010.

Pollard, David "A user's guide to measure theoretic probability." Cambridge University Press, Cambridge, 2002.

Chaumont, Loïc; Yor, Marc "Exercises in probability. A guided tour from measure theory to random processes, via conditioning. " Second edition. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, 2012.

10) What type of assessment tools did you use? Attach if available, any notes or exams/quizzes that were distributed to students.

I did not use any particular tools. Assessment was obtained from the students' participation and performance during the exercise sessions in combination with an oral examination after the course.

11) In which language was the course given:

In English.

12) Was the course language the native language of the students?

No. In fact, is was one of the major goals of the course to teach and practice "doing mathematics in English" to the Mongolian students to prepare them for the ensuing summer school and possible future studies abroad.*

** Stochastic Processes and Applications, CIMPA - DAAD Research School, National University of Mongolia July 27 - August 07, 2015, Ulaanbaatar, Mongolia*

13) Did you give any public lectures, and did you discuss with local staff issues regarding the curriculum?

Apart from the course I gave two research presentations about my own research, which can be considered as public lectures, but were addressed mainly to colleagues working in mathematics.

Among the many people I met and discussed with about curriculum, possible exchanges and collaborations are at the NUM Department of mathematics, Prof. Purevsuren, Prof. Itel, and Assoc. Prof. Bayarmagnai, Assoc. Prof. Tsogzolmaa Saizmaa, B. Bayarjargal, Prof. Amarzaya A. (also Dean of Natural Science Division of School of Arts and Sciences); then Prof. B. Barsbold (Department of Applied Mathematics), Prof. R. Enkhbat (Director of the Institute of Mathematics), Dr. B. Boldgiv (Vice President for Research and International Relations) and Sh. Mendbayr (Department of International Relations and Cooperation)
Prof. T. Srantuya (Mongolian University of Science and Technology and National Coordinator for Eurasia-Pacific Uninet); Oyun Batkhuyag (Director of The Society of Actuaries of Mongolia) about activities related to insurance mathematics.

14) Where did you live? (e.g. hotel, hostel, on campus, in city e.g.)

I lived in a privately rented apartment (as requested by myself), within walking distance from the university main building.

15) Do you have any recommendations/suggestions to the professor who will visit the university in the future (also regarding accommodation, health and visa issues)?

Exercises: Prepare exercises in a very structured way and in detail. Due to difficulties with language there is less time for discussion and spontaneous development of ideas. I found out that Mongolian students do not like to make errors or show uncertainty on the blackboard. So they will volunteer to solve an exercise only when quite sure to have the complete and correct solution.

Accommodation: I preferred to stay in a self-catering accommodation. Sometimes there are problems with hot-water supply in Ulaanbaatar due to frequent repairs of hot-water pipes in the city. If you want to make sure to avoid this, you may prefer a hotel.

Practical matters: Many roads are rough in Ulaanbaatar, bring strong shoes for walking. Go to the ATM for money in the morning, they may be empty in the afternoon.

Visa and vaccinations: For me as an Austrian citizen no special vaccinations and visa arrangements were necessary.

16) Would you like to share anything else, such as a particular experience, testimonial etc.?

Be relaxed. Learn about the traditional Mongolian attitude towards time and appointments.

Older people including colleagues may not speak English very well, rather Russian. I found always a younger colleague translating.

17) Do you have any pictures which you would allow us to put on the IMU-CDC website?

Yes, you can watch and download them from:

http://www.fam.tuwien.ac.at/~fhubalek/imu-mongolia-hubalek/

18) Are we allowed to publish the answers you provided above?

Yes, you can publish my answers.