Report on the course of B. Rousselet at Royal University of Phnom Penh, master of Mathematics.

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As for other courses offered in previous years, students are very hard-working, inquiring, self-willed. I have been dealing with roughly 20% of the syllabus that was posted. This points out that my program was too much advanced; by reading other syllabus, I found that this is often the case. In my opinion we have to stick to L2 to L3 level; however the teaching should be open minded, problem solving oriented.

Syllabus actually covered

1. Straight lines (and planes in dimension 3) equations in an affine euclidean space of dimension 2 or 3; this includes explicit, implicit and parametric equations; I wanted to link with barycentric equations but barycenter coordinates were forgotten so I skipped as this was useless for my purpose. I have emphasized the use of director and normal vectors and drawing figures. I should say that they have clearly understood the topic in dimension 2 but that was already painstaking in dimension 3; for example the number of equations involved with explicit, implicit and parametric equations, the use of vector notations or the use of components and rectangular matrices were difficult for them. They draw very well straight lines equations, director and normal vectors but drawing planes and straight lines in a 3 dimensional space seems to be entirely new and difficult for them. Then the computation of the intersection of planes ans straight lines raises troubles but they understand the use of solving linear systems for that, including a Wims tool for this. Similar comment for the orthogonal projection. For this lecture, the main point was to link the level sets of a linear function of two variables with a straight line and the use of the gradient to split the two dimensional space of the variables into a half space in which it increases and an other one in which it decreases.

2. I tried to carry similar points related to level sets of quadratic functions of 2 variables; the new main point being to distinguish the shape of the conics from an implicit equation; I emphasized the use of a matrix and the case of positive definite case. I stated various Taylor formulas; their use with several variables was disturbing starting from the definition of second derivative being a bilinear form (with links to partial derivatives). The link between derivative and linear approximation or tangent space was not obvious. The fact that depending of the nature of the equation (explicit, implicit, parametric), the derivative provides a tangent or a normal space was upsetting. It has been difficult for them to study the restriction of a function to, a straight line ($x = x_0 - \rho u$) which is a crucial point for optimization algorithm.

3. The previous paragraph was supposed to be only a motivation to do it in spaces of large dimensions and using matrices for linear mapping and bilinear forms; manipulations were to be performed with Scilab and in some cases Wims tools. Most of the students were familiar with Scilab but this ability should be improved, in particular the use of rectangular matrices. As a small step toward general algorithms, we have found the minimum of the restriction to a straight line of a quadratic function defined with a positive definite matrix with a flavor of the its use in a gradient method to minimize such a function. I have proved that this is equivalent to solve a linear system.

Difficulties

1. During the lectures, I have often told them that the real challenge was to routinely study functions with as many as thousands of variables and to have in mind that minimization problems involve constraints; unfortunately, I had no time to consider this last chapter.

2. I had trouble to make clear what is an implicit versus an explicit equation; this difficulty has been somewhat cleared to me when I asked a student to write a translation found in a dictionary; it was a sentence; in other words this distinction is not in their culture!

Suggested improvements

1. Syllabus that should be mastered at the end of bachelor:
   (a) Tangent subspace, linear approximation for functions of 2 variables;
   (b) study of parametric curves, including radius of curvature;
   (c) to distinguish a geometric object and an equation to describe it; in particular distinguish a curve and the graph of a function;
   (d) explicit and implicit equations.
   (e) Use of graphic and numerical software.

2. More restricted choice of mathematical topics.

3. Offer to students options of lectures of other masters (physics, computer science, economics, management, civil engineering, electronic engineering) and a track in engineering in second year.

4. English certification required at the entrance of students in this master track.

5. Offer open topics at bachelor level (in the spirit of what is used in France in "math en jeans", hypocampe etc).

6. A local assistant should be involved; I had this opportunity in 2007 and this was working fine.

7. I have met no mathematician from URPP; at least one local colleague should be involved in a module taught by a foreign professor.

8. Start actions in tracks preparing high school teachers such as Kemara University and Pedagogical Faculty.