# Report on the ICHM Co-Sponsored 28th Novembertagung on the History of Mathematics

Youth Hostel Jaques Brel, Brussels, Belgium
November 2-4, 2017

The 2017 Novembertagung on the History of Mathematics had 46 participants; 44 doctoral students or post-doctoral researchers and two invited speakers, Prof. Liesbeth de Mol (University of Lille) and Prof. Ralf Krömer (Bergische Universität Wuppertal). The participants came from various different countries including Belgium, Czech Republic, Denmark, France, Germany, Italy, Netherlands, Romania, Spain, the United Kingdom and the United States of America.

The programme lists 17 parallel sessions (34 short talks) and three joint sessions (opening session plus the two talks by the invited speakers). However, two speakers for shot talks failed to attend without notice. All talks were on the history and philosophy of mathematics. The programme and abstracts are provided below.

In addition to the generous funding from the ICHM, we received funding from the Centre for Logic and Philosophy of Science (CLWF) at the Vrije Universiteit Brussel (VUB); the Belgian Society for Logic and Philosophy of Science (BSLPS); the Centre for History of Sciences and Techniques of the University of Brittany (CFV, Brest); the GDR 3398 “Histoire des mathématiques”.

We used the grant from ICHM to subsidize the cost of accommodation for the junior researchers. Participation in the Novembertagung was free of charge; the organisers covered accommodation and all meals for the duration of the conference. The participants paid for their own travel to Brussels, but we were able to give out four travel grants of 100 Euro each for participants with limited (or non-existing) travel budgets.

As mentioned in our application for funding, we had expected 40 participants. We received many more requests for participation and were able to provide further spaces due to the generous financial help of the CLWF.

The 28th Novembertagung successfully provided a forum for young researchers to meet and discuss their work in a friendly environment. The topic of this year's Novembertagung was well received and stimulated the discussions. In this sense then was the 28th Novembertagung a success.

Maria de Paz from the University of Seville will be the main organizer of the 29th Novembertagung.

On behalf of the organising committee and myself,

Colin Rittberg, Centre for Logic and Philosophy of Science, VUB

**Main Organiser** Colin Rittberg (VUB)

**Organising Committee** Joachim Frans (VUB), Lisa Rougetet (CFV), Lorenz Demey (KU Leuven), Nicola Oswald (Universität Wuppertal), Nigel Vinckier (VUB), Sven Delariviére (VUB)

# NOVEMBERTAGUNG 2017 | PROGRAM

| | WEDNESDAY, 1.11 | |
|---|---|---|
| 15:00 | Check-in opens | |
| 18:00 | *Informal hellos* | |
| 19:00 | Communal Dinner | |

| | THURSDAY, 2.11 | |
|---|---|---|
| 09:00 | | |
| 09:30 | *Opening talk* | |
| 10:00 | Harald Kümmerle<br>***The development of Japanese mathematics at the end of the 19th and the beginning of the 20th century as observed from inside the Mathematico-Physical Society - using proceedings as a primary source*** | Gatien Ricotier<br>***Trends in and around the Bourbaki group through quantitative data extracted from the report of the group's meetings*** |
| 10:30 | *Break* | |
| 11:00 | Britgitte Stenhouse<br>***The Mathematics of Mary Somerville*** | Maria de Paz<br>***Why conventions? Exploring the historical roots of a new epistemic category*** |
| 11:30 | Thomas Perfettini<br>***Mathematics and mathematicians in the Russian emigration in Paris*** | Nicolas Michel<br>***What's in a conic? Ontological and epistemological shifts in the development of enumerative geometry*** |
| 12:00 | Antina Scholz<br>***(Re-)Internationalization of Mathematics in Germany after World War II*** | Anna Kiel Steensen<br>***Transformations of reference between representations in mathematical practice. A case study.*** |
| 12:30 - 14:30 | *Lunch* | |
| 14:30 | Aurelien Jarry<br>***On the history of the notion of scheme*** | Sylvain Moraillon<br>***Mathematical understanding according to Poincaré : topology as a case study*** |
| 15:00 | Lisa Rougetet<br>***Folding During the 17th-18th Centuries in Recreational Mathematics: Between Geometry and Wonder*** | Michael Tobin<br>***Advances in Transcendental Number Theory Since the Proof of the Gelfond-Schneider Theorem*** |
| 15:30 | Michael Friedman<br>***Diagrams and sketches: On curves in the Italian school of algebraic geometry*** | Martin Muffato<br>***Quest for practical arithmetics II: Wonder Women in disguise*** |
| 16:00 | *Break* | |
| 16:30 - 18:00 | *Invited Speaker*<br>Liesbeth de Mol<br>***When logic meets engineering or why histories of computing should not be reductionist*** | |

# NOVEMBERTAGUNG 2017 | PROGRAM

| | FRIDAY, 3.11 | |
|---|---|---|
| 09:00 | | |
| 09:30 | George Florin Calian<br>*Plato's generation of numbers and Brouwer's intuitionism* | Jabel Alejandro Ramírez Naranjo<br>*A vision of numerical methods from the perspective of the mathematical practice. From equations to coding* |
| 10:00 | Gail Brekke & Jakob Giraud<br>*Fighting with Infinity: A Proposal for the Addition of New Terminology* | Daniel Rompf<br>*Ernst Cassirer's philosophy of mathematics - A Structuralist Approach?* |
| 10:30 | *Break* | |
| 11:00 | Spencer Johnston<br>*The Use of Formalisation in the History of Logic* | Flavio Baracco & Davide Quadrellaro<br>*Rational Reconstructions in the History of Mathematics* |
| 11:30 | Jan Zeman<br>*Hilbert's arithmetisation of geometry* | Robert Middeke-Conlin<br>*Predictive Modelling and Brick Deliveries* |
| 12:00 | Deborah Kant<br>*The Practice of Forcing in Set Theory*<br>*Is forcing used as a philosophically neutral tool in set theory?* | |
| 12:30 - 14:30 | *Lunch* | |
| 14:30 | Benjamin Wilck<br>*Mathematical Definitions and a New Problem for Pyrrhonian Scepticism* | Tony Royle<br>*Spinning, stalling, and falling apart* |
| 15:00 | Shafie Shokrani<br>*Nelson's Socratic Method* | Ana Jimena Lemes<br>*Potentialities of the History of Mathematics in the training of mathematics teachers* |
| 15:30 | Line Andersen<br>*Proof as Dialogue* | Giuditta Parolini<br>*Investigating statistical tools in the life sciences: An opportunity to reconnect the history of mathematics to the history of science and technology* |
| 16:00 | *Break* | |
| 16:30 - 18:00 | *Invited Speaker (BSLPS lecture)*<br>Ralf Krömer<br>*The distinction of tool and object in conceptual history of mathematics: epistemology and examples* | |

# NOVEMBERTAGUNG 2017 | PROGRAM

| | SATURDAY, 4.11 | |
|---|---|---|
| *09:00* | | |
| *09:30* | Patrick Walsh<br>***The Right Level of Abstraction: Category theory and methodological frames*** | Sven Delariviere<br>***The Extended Mathematician: Does mathematical understanding ever extend to include one's tools?*** |
| *10:00* | Jio Jeong<br>***Burgess and Maddy on Naturalist Philosophy of Mathematics and Benacerraf's Dilemma*** | Manuel Gracia Perez<br>***History of mathematics as a tool for research in studies of geometric cognition*** |
| *10:30* | *Break* | |
| *11:00* | *Discussing the Future of NT* | |
| *11:30* | | |
| *12:00* | End of the Novembertagung 2017 | |

# NOVEMBERTAGUNG 2017

## Workshop on the history and philosophy of mathematics

*2 - 4 November*
*Brussels, Belgium*

# ***Book of Abstracts***

THURSDAY 2 NOVEMBER

**10:00 – 10:30 (Session 1)**

**Harald Kümmerle** *The Development of Japanese Mathematics at the End of the 19th and the Beginning of the 20th century as Observed from Inside the Mathematico-Physical Society - Using Proceedings as a Primary Source*

When studying the case of Japan, one has to abstain from using many heuristics that are useful for analyzing the development of mathematics in Western countries, in order to avoid misjudgings. When a modernization policy was adopted in the midth of the 19th century and -while mainly aiming for a transfer of technological knowledge-Western mathematical knowledge was imported, too, an indigenous tradition of mathematics was still in full bloom. This complicates simple narratives of knowledge transfer: While most practitioners were not interested in its applicability to the natural sciences, the fact that this tradition was marginalized in just a few decades was neither a socially necessary process, nor was it the result of a multigenerational paradigm shift. Rather, mathematicians and physicists who had been educated in the West on government scholarships sometimes schemingly usurped power in institutions that had been nonpartisan up to that point, a process which has been called a "coup d'état" by the research literature in some cases. The main example where this dynamic played out is the Tokyo Mathematical Society. It had been founded in 1877 by a diverse group of people studying mathematics, most of them traditionalists, but due to efforts of the first Japanese mathematics professor at the first Japanese university became a society which was centered on Western mathematics after just a few years. Then, in 1884, it was extended to include physics and renamed to Tokyo Mathematico-Physical Society.

While the Tokyo Mathematical Society has been rather well-studied in Japan, the development of its successor, the Mathematico-Physical Society, has not been the topic of systematic research inside or outside of Japan.

This is understandable as my Ph.D. research is the first decidedly historical project to study the institutionalization of mathematics as a science in Japan during that time in a comprehensive manner. From my results it follows that while on the one hand neither locality nor hierarchy of educational institutions have been taken into account sufficiently yet, on the other hand several parallel developments at the different research centers must be made explicit as such. That being said, the society, especially after being renamed to Physico-Mathematical Society of Japan in 1918, indeed became a focal point for the development of mathematics in modern Japan. When combined with knowledge about the other institutions, an investigation of the academic activities at its meetings gives insight into the cooperation processes inside and outside of the universities.

Information on these is contained in the monthly proceedings which were sent out to academic institutions all over the world. While the research articles were mostly written in Western languages and addressed to the international community, the Japanese-language protocols were also included and are easily accessible to the present day in the library of the Mathematisches Institut in Göttingen, for example.

The subject of the talk is to assess correctly what information can be obtained from the proceedings and what pitfalls must be avoided if sufficient knowledge about other institutions is not available.

**10:00 – 10:30 (Session 2)**

**Gautien Ricotier** *Trends in and around Bourbaki Group through Quantitative Data Extracted from the Report of the Group's Meetings*

Already at the first proto-bourbaki meeting, André Weil explained that the group had to be self-managed. Combined with the voluntary work and the anonymity of the members, these are some of the major characteristics of the group. In this presentation I explain how certain quantitative data (for example the presence at the meetings, the contribution in the write-up of the treatise, the organisation of the seminars) extracted from the report of the group's meetings (between 1934 and 1952) can show the implication and determination of the various members. Through these data we can also glean some clear roles which can in fact be attributed to some of the members. Furthermore, I will explain how these data can then be used to compare the group, or its members, with other communities or personal projects, within the mathematical world, and also outside of it.

**11:00 – 11:30 (Session 1)**

**Britgitte Stenhouse** *The Mathematics of Mary Somerville*

Mary Somerville was known as one of the most distinguished scientists in the 19th Century, and has since been almost exclusively written about as a populariser of science or an astronomer. However, Somerville was an expert in French mathematics, specifically the calculus, at a time when many English mathematicians believed they had fallen behind their European counterparts. Furthermore, in 1831 Somerville published *Mechanism of the Heavens*, a translation of Laplace's *Mecanique Celeste* with added introductory material intended to make the work accessible to an English audience. Used as a textbook at Cambridge University immediately after its publication, it heavily influenced the dissemination of French analysis to Great Britain.

The contributions Somerville made to the development of the calculus in Britain have frequently been overlooked in historical literature. Described on its publication as the "most complete account of the discoveries of continental mathematics in physical astronomy which exists in our language [English]", *Mechanism of the Heavens* is mostly remembered in the 20th century and later for its 'Preliminary Dissertation', which contains no mathematics whatsoever. Somerville and her works provide a powerful demonstration of the severity with which gender and wealth can influence the work of a mathematician, and indeed the recognition they are accorded for their work. While the impact of Somerville's gender on her work and life has been treated before, this does little to re-introduce Somerville's work into the historical narrative of the calculus where it rightly belongs.

In addition, Somerville wrote a second work on the calculus in 1834, titled *On the Theory of Differences*, which was never published. *On the Theory of Differences* is an introductory calculus text, which, unlike her previous book, is stripped of applications to astronomy and deals with pure mathematics. This manuscript has never been thoroughly researched before, and is at most a passing comment in most academic works on Somerville's life. By studying 19th century calculus, both British and French, I will investigate what contribution her work could have made had it been published, as well as further investigating why it was not published when it was written.

**11:30 – 12:00 (Session 1)**

**Thomas Perfettini** *Mathematics and Mathematicians in the Russian Emigration in Paris*

In my talk, I focus on the mathematics in the Russian emigration in Paris in the aftermath of the Russian revolutions. Many aspects of this history have already been studied, mostly by Russian academics, but scarcely using the documentation available in France on this question. I try therefore to offer new perspectives and complements based on various documents I found during my research. I present the activities of the Russian Academic Group and examine how it was involved in the mathematical life of Russian scientists who emigrated in Paris: let me mention, for instance, the creation of Russian sections at Paris university. I describe the trajectories of three individuals, Serguei Savitch, Ernest Kogbetliantz and Vladimir Kostitzin, emphasizing on their works, their links with French scientists and French laboratories, and how they manage to recreate propitious conditions for the continuation of their research in this particular context.

**12:00 – 12:30 (Session 1)**

**Antina Scholz** *(Re-)Internationalization of Mathematics in Germany after World War II*

After World War I, academia in Germany was excluded from academic communities internationally. By contrast, there is no evidence of such a boycott after World War II. The academics in Germany, including mathematicians, were soon reintegrated into the international scientific community, due to the general political context during the Cold War.

In my Ph.D. project I analyze the process of internationalization of mathematics in Germany and the reintegration of mathematicians in/from Germany into the international mathematical community after World War II until 1960. My research concerns different aspects such as the role of mathematicians in Germany inside the International Mathematical Union (IMU) and their participation in the International Congresses of Mathematics (ICM) in the 1950s. Furthermore, I investigate other international meetings of mathematicians, such as the conferences held by the *Mathematisches Forschungsinstitut Oberwolfac*h (MFO). This serves as an example of how mathematicians in Germany planned conferences with international participants after World War II. Moreover, my project aims to analyze the influence of the remigration of mathematicians forced to leave Germany during the Nazi era as well as the general exchange of mathematicians during the 1950s. My research is supplemented by case studies on mathematicians who were supported by scholarship programs such as the *Fulbright Program* (for German-American exchange) and those of the *Alexander von Humboldt-Stiftung* (for foreign academics in Germany). The individual cases will show the effect of this exchange and the influence of the individual mathematicians on the internationalization of mathematics. To show the development of German mathematical journals inside the international community, I study different journals such as *Archiv der Mathematik*, *Journal für die reine und angewandte Mathematik*, *Mathematische Annalen, Mathematische Nachrichten*, *Mathematische Zeitschrift* and *ZAMM*. Empirical analysis of those journals will reveal the extent of their internationalization and the cooperation with mathematicians from outside Germany.

In my talk I will present first results of my research with a focus on the integration of mathematicians in Germany into the IMU and their participation in the ICMs in the 1950s. In this context the following questions are of interest: How was it possible for the German mathematical community to get involved in the foundation process of the IMU so shortly after World War II? Which actors inside the international mathematical community supported the (re-)integration of mathematicians in/from Germany? And which actors had a negative attitude towards the (re-)integration of the Germans? Both the background of the attitude of mathematicians from outside Germany and the attitude of the mathematicians in Germany towards international contacts are of interest for my talk.

**11:00 - 11:30 (Session 2)**

**Maria de Paz** *Why Conventions? Exploring the Historical Roots of a New Epistemic Category*

In the winter semester of 1847/1848 Carl Gustav Jacobi gave a course on Analytical Mechanics at the University of Berlin. In that course, he characterized the principles of mechanics as 'conventions' introducing a new epistemic category not applied before to the mathematical domain. In 1852, a French naval engineer, Fréderic Ferdinand Reech published a course on mechanics that he gave at the Ëcole du Génie Maritime in the small town of Lorient. In that course, he used precisely that very same word, i.e. 'convention' to characterize the principle of inertia. Half a century later the word was to become famous by the prestigious hand of Poincaré. But by then, the use of the category convention was already widespread in science as is shown in many works of the time such as Lange's, Duhem's, Hertz's, Milhaud's and others.

With the only exception of Diderot's "On the interpretation of nature" (1754) who says that mathematics is like a game and thus a "matter of convention", Jacobi is the first to introduce conventions to the mathematical domain and as an epistemic category. The aim of this talk is to find and present tools to explore historically the genesis of this introduction. Thus, our main question is - what should be the historiographical procedure to study the roots of this concept?

The first natural move is to understand the concept as transferred from a field alien to mathematics, that is, from the field of jurisprudence and law. It is in jurisprudence where we can find agreements ruled by conventions and this aspect of legal regulations was particularly present in the 19th century, given the context of changing political systems and the strong debate about them. But we cannot understand this transference from the field of jurisprudence to mathematics as a simple metaphor, since in the realm of mathematics this transference has more than linguistic consequences changing the status of the principles which qualify as conventions.

So, how did Jacobi come to the use of the concept? How did Reech arrive to it, given that Jacobi's lectures were not published until 1996? How did the term become common at the end of the century? Which were the channels of transmission?

It is clear that to give an adequate answer to these questions we cannot explore only the single histories of the protagonists, given, in the first place that the two earliest figures have quite an unequal relevance in the history of mathematics (Jacobi being a first-rate one and Reech almost forgotten). We have to take into account the networks, the education context, and particularly, the historical contexts in which both lived, given the strong impact that the Revolutions played in Europe along the 19th century and knowing that Jacobi was politically involved (particularly in the 1848 Revolution).

**11:30 – 12:00 (Session 2)**

**Nicholas Michel** *What's in a Conic? Ontological and Epistemological SHifts in the Development of Enumerative Geometry*

In the second half of the XIXth century, a few geometers in the so-called synthetic tradition started to develop tools to tackle a new sort of enumerative problems such as finding the number of conics tangent to five given conics. While Jakob Steiner was arguably the first to raise these questions, his answers would shortly be refuted by Michel Chasles, who published in 1864 arguably the first systematic approach to what is now called enumerative geometry. At the heart of Chasles' theory lies a new set of concepts. His primary objects were systems of conics (i.e. the sets of all conics satisfying simultaneously four given geometrical conditions), of which he considered $\mu$, the number of elements passing through an arbitrary fixed point, and $v$, the number of elements touching an arbitrary fixed line. His central claim would then be that these two numbers, which he called the characteristics of a system, encoded the solution to each and every enumerative problem in the geometry of conics.

While this claim, expressed in more detailed terms, would shortly be known as Chasles' theorem, it was never the object of any proof by the French geometer: he had merely based it on a strong induction. Subsequently, it attracted a great deal of attention. In 1873, Clebsch, Lindemann, and Halphen found three different proofs simultaneously and independently. However, three years later, Halphen changed his mind : he had found a counter example to the theorem using an analytical approach to the general equation of conics, which in turn had led him to a more refined classification of degenerate conics. This refutation did not go unnoticed, and gave rise to some debates. Most notably, in 1885, Eduard Study reopened the case in his doctoral dissertation. Using modern algebraic methods inspired by Clebsch and Gordan, he had managed to save Chasles' theorem from Halphen's analysis; but doing so, he had surreptitiously redefined the very notion of conics.

We wish to investigate the ontological shifts that set in motion these successive falsification and defense of Chasles' theorem, which, we argue, must be correlated with an evolution of the status of algebra within geometrical practice. Our emphasis will be on the specific methodological insights that can be acquired through the study of mathematical controversies, and on the epistemological weight carried by a recourse to contemporary mathematical knowledge in order to untangle debates of the past.

**12:00 – 12:30 (Session 2)**

**Anna Kiel Steensen** *Transformations of Reference Between Representations in Mathematical Practice. A Case Study.*

Shifting between different representations is a common method in both the exploratory and communicatory aspects of mathematical research. In this talk I will present a case study, which highlights the importance of these shifts to the development of mathematics. Specifically, I will describe the use and development of representations of permutations in selected texts by Lagrange and Galois, focusing on local transformations of reference from one representation to another. Inspired by Bruno Latour?s theory of reference in the empirical sciences, which he develops in the essay *Circulating Reference*, I will account for the referential development as a series of representations playing alternating functional roles relative to each other. The analysis suggests that changing the representations affords a referential development from permutations as practice to groups of permutations, which in turn suggests how such a powerful notion as the group emerges from the relatively simple practice of permuting.

**14:30 – 15:00 (Session 1)**

**Aurelien Jarry** *On the History of the Notion of Scheme*

I would like to present elements from my PhD work-in-progress, developed in the context of the DFG-project "Duality - an archetype of mathematical thinking" lead by Prof. R. Krömer and Prof. K. Volkert at the University of Wuppertal.

Since decades, no one anymore doubts that analogy plays an important role in mathematics, especially when it comes to the development of new theories and concepts. However, it is still a matter of debate, what kind of analogy plays a role and at which level exactly (see e.g. [1], [2], [3], [6] and [7]).

Starting fom Schlimm's distinction between the "structure-mapping" and the "axiomatic" models of analogy ([5]), the purpose of my work is to study the historical development of some of Grothendieck's mathematical theories under the scope of analogy and to try to find out what kind of analogy played a role in Grothendieck's work and for what purpose. I'm interested in particular in the role played by duality, as it seems that Grothendieck used this principle in order to make analogies between different domains of mathematics complete (cf. [4]). To illustrate this point, I will present some first results of my inquiry on the history of the notion of scheme.

**References**

[1] Corfield, David : *Towards a philosophy of real mathematics, Cambridge*: Cambridge Univ. Press, 2004

[2] Durand-Richard, Marie-José [Ed.] : *L'analogie dans la démarche scientifique*, Paris : L'Harmattan, 2008

[3] Hesse, Mary : *Models and analogie in science*, Notre Dame, IN : Notre Dame Univ. Press, 1970

[4] Krömer, Ralf & Corfield, David : "The duality of space and function, and category-theoretic dualities", in *Siegener Beitrge zur Geschichte und Philosophie der Mathematik* 1 (2013), p. 125-144

[5] Schlimm, Dirk : "Two ways of analogy: Extending the study of analogies to mathematical domains", in *Philosophy of Science*, 75 (2), 2008, p. 178-200

[6] Schlimm, Dirk : "*Conceptual metaphors and mathematical practice: On cognitive studies of historical developments in mathematics*", in Topics in Cognitive Science, 5(2), 2013, p. 283-298.

[7] Schlimm, Dirk : "*Metaphors for mathematics from Pasch to Hilbert*", in Philosophia Mathematica, 24(3), 2016, p. 308-329

**15:00 – 15:30 (Session 1)**

**Lisa Rougetet** *Folding During the 17th-18th Centuries in Recreational Mathematics: Between Geometry and Wonder*

This contribution, in collaboration with Michael Friedman (Humboldt University, Berlin), aims to present how paper-folding activities were integrated into recreational mathematics during the 17th and the 18th century. Recreational mathematics was conceived during these centuries as a way not only to pique one's curiosity, but also to communicate mathematical knowledge to the literate classes of the population. Starting with Leurechon's 1624 *Récréation mathématique*, which did not contain any exercise concerning paper folding, we show how two other traditions - Dürer's folded nets on the one hand and napkin folding on the other hand - prompted and influenced the integration of folding within subsequent books and manuscripts, especially those of Georg Philipp Harsdörffer and Daniel Schwenter. In Germany, but also to a lesser extent in France, folding was henceforth re-conceptualised within recreational mathematics as a way to transmit geometrical knowledge. Following Harsdörffer, the paper will claim that practicing folding activities enabled the acquiring of a geometrical knowledge, which was haptic rather than symbolical or merely visual. This tactility reflects the Baconian conception of science and scientific experiment; and the paper will try to illuminate how folding, by advancing practice and tactility via experiments, was representing these traditions and conceptions.

**15:30 – 16:00 (Session 1)**

**Michael Friedman** *Diagrams and Sketches: On Curves in the Italian School of Algebraic Geometry*

Mathematics in the early 20th century is usually characterized via the great narrative of the crisis of intuition, where a turn towards a more rigorous, formalized account of mathematics was taking place. Diagrams, sketches and drawings as tools of research in mathematics were regarded either as secondary or as misleading; thus for example, the 1890 space filling curve of Peano and the 1872 Weierstrass function were not drawn even once in Peano's or Weierstrass's papers. Indeed, a counter movement can be noted with Felix Klein, Alexander Brill and Walther Dyck, advocating a more visual approach to mathematics, seen for example with the construction of physical models of mathematical objects (such as curves and surfaces) from strings, plaster or cardboard. However, also this tradition declined starting from the 20s of the 20th century. Focusing on the Italian school of algebraic geometry, which thrived starting from the end of the 19th century, physical models of surfaces were hardly manufactured in Italy. Concerning the secondary role of these visual tools, Livia Giacardi notes: "[The] members [of the school] attributed great importance to intuition and visualization [...] [but] they did not use physical models in their research work, but preferred to employ the *Gedankenexperiment.*"[1] A more poignant description suggests that the algebraic "objects [of the Italian school], whose existence is finally established in an algebraic way, are typically absent from [...] drawings; [hence] it is plausible to interpret drawings [that do appear in several manuscripts] as a spontaneous reflex when setting up an investigation, rather than viewing them as a key element of the argument."[2] These accounts might suggest that diagrams were neither considered as a tool that stimulates mathematical understanding nor as what prompts the discovery of mathematical theorems.

However, a closer look at the history of algebraic geometry in Italy suggests a more complex picture. Focusing on the case study of branch curves, one may note that various diagrams and sketches were nevertheless integrated in various papers of Oscar Zariski, Federigo Enriques and Oscar Chisini, to name only a few mathematicians[3]. These diagrams and drawings were not at all illustrations of the curve itself; that is, there was no attempt to illustrate how this curve "actually" looked like. Rather they were conceptual diagrams: of loops, braids and paths in the complex plane, which were tools for - at the very least - assisting the visualization of the involved procedures. Moreover, as I will claim, they were indispensable for the explanation of steps of the proofs as well as for the construction of concepts. In my paper I will survey several examples of these diagrams, emphasizing their role as well as their epistemological implications.

**14:30 – 15:00 (Session 2)**

**Sylvain Moraillon** *Mathematical Understanding According to Poincaré: Topology as a Case Study*

[1]Giacardi, Livia M. (2015), "Models in Mathematics Teaching in Italy (1850-1950)", in: P*roceedings of Second ESMA Conference*, Mathematics and Art III, (ed.: Bruter, C.), Paris: Cassini, pp. 9-33, here p. 12.

[2]Schappacher, Norbert, (2015), "Remarks about Intuition in Italian Algebraic Geometry", in: *Oberwolfach Report* 47/2015 of the workshop: History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences, pp. 2805-2807, here: p. 2807.

[3] Enriques, Federigo (1923), "Sulla construzione delle funzioni algebriche di due variabili possedenti una data curva diramazione", in: *Ann. Nat. Pure Appl.* 1, pp. 185-198; Zariski, Oscar (1929), "On the Problem of Existence of Algebraic Functions of Two Variables Possessing a Given Branch Curve", in: *Amer. J. Math.* 51(2), pp. 305-328; Chisini, Oscar (1944), "Sulla identita birazionale delle funzioni algebriche di due variabili dotate di una medesima curva di diramazione", in: *Rend. Ist. Lombardo* 77, pp. 339-356.

After the ‘practical turn’ in the philosophy of mathematics, several authors, including Kenneth Manders und Micheal Detlefsen, have promoted a renewed conception of mathematical rigor, associated with an emphasis on certain aspects of mathematical knowledge, in which understanding is at least as important as reliability. This conception can in turn, according to Detlefsen, be traced back to Poincaré: a very famous text of Poincaré has led him to characterize mathematical intuition as a faculty to grasp what he calls a ‘mathematical architecture’, which gives us a genuine understanding of the matter, as opposed to a merely logical competence.

It is then a natural question to ask whether the very mathematical practice of Poincaré is consistent with this view. There are good reasons, originating in writings from Poincaré himself, to choose his work on topology as a case study in this respect. In the introduction of his first paper ‘On analysis situs’, Poincaré makes the case for a geometrical language, better equipped to give comprehension as the analytical one. Nevertheless, and much to the surprise of some commentators such as Alain Herreman, what Poincaré seems to create in this paper would be better described as an algebraic language : the introduction of the homologies, and their combination as equations to replace the more geometrical content of Bettis’s lemma, is perhaps the most striking example in this regard.

This leads us to three questions: 1) To what extent can the algebraic tool introduced by Poincaré serve as this support for a better mathematical comprehension, and in which sense can it be said to involve some ‘mathematical architecture’? We will answer this question by examining some basic algebra, drawing on one paper from Danielle Macbeth, and relating it to the way the algebraic tool functions in Poincaré’s topology. 2) Is nonetheless something like a geometrical tool, or language, to be found in the topological work of Poincaré? We will try to answer this problem by studying in depth the content and the proof of Poincaré’s theorem of duality. 3) How is one to characterize the relation between these two approaches of topology that seem to coexist in the same text? We will consider this question in relation to a more general one, namely the role of formalism in mathematics.

**15:00-15:30 (Session 2)**

**Michael Tobin** *Advances in Transcendental Number Theory Since the Proof of the Gelfond-Schneider Theorem*

A survey of the literature of transcendence theory since the proof of the Gelfond-Schnieder Theorem (1934) reveals two divergent tendencies manifested in the work of Alan Baker and Boris Zilber and presaged by the work of Hermite and Cantor, respectively. The first "school," embodied in the work of Baker, uses auxiliary functions to approximate transcendental numbers to natural numbers. The second, represented by Zilber and, distantly, Cantor, seeks to approximate transcendental numbers to complex numbers. The evolution of these somewhat divergent techniques is observed in the ensuing literature and analyzed. The synthetic ramifications of a turn toward computer science modeling and thinking are accounted for and evaluated, especially the research that was sparked by the appearance of Daniel Richardson's (1968) "Some Undecidable Elementary Functions of a Real Variable." An ambient assessment of the current state of transcendence theory is made on this basis, specifically in regard to the synthetic implications of computer science applications in the field.

**15:30 – 16:00 (Session 2)**

**Martin Muffato** *Quest for Practical Arithmetics II: Wonder Women in Disguise*

Among the about two hundreds and fifty French authors having published mathematical treatises during the seventeenth century, only two are women, namely Marguerite Bramereau and Marie Crous. Both of their three texts are dealing with practical arihtmetic.

Firstly, Marguerite Bramereau was just twelve when she published her treaty. The latter is a perfect testimony of the syllabus composed for girls at that time. It is also the occasion to discover the educational institution that existed for girls. Secondly, Marie Crous' work is strikingly different from the work of her contemporaries in a few regards. For instance, she brings to France the concept of positional decimal as a writing system for numbers, brought in Europe by the Flemish scholar Simon Stevin in La Disme. Then, she introduces the use of a separator between the integral and decimal parts of a number. These two examples are nowadays commonly used mathematical tools.

In this presentation, I would like to present the work of these two mathematicians, in order to understand what they can bring to us, from a historical point of view.

**16:30 – 18:00 (Plenary)**

**Liesbeth de Mol** *When Logic Meets Engineering or Why Histories of Computing Should Not Be Reductionist.*

Today, it is widely acknowledged amongst historians, philosophers and computer scientists that the computer and, with it, modern computing, has multiple roots. This is echoed in the shaping of computing both as a discipline and as a practice. Indeed, in the literature one can find numerous statements about the nature of computing that emphasize its interdisciplinary character and so, implicitly or explicitly, call for a method which takes into account the diversity of computing. For historians there is an important challenge here methodologically wise. The only way to render transparent a history of computing which somehow accounts for that interdisciplinarity is to consider different "histories of" and understand how they are intertwined to develop a history of computing which is non-reductionist and contextual. In this talk I will sketch the developments in the historiography of computing focusing mostly on its problematic relation with mathematics and the history of science in general and argue for the need of integrating history of technology with the history of science in this particular context. This methodological frame will be exemplified by a particular case study which focuses on how, in the 1950s, logical insights get intertwined with engineering practices illustrating how a detailed study of a related set of computing practices unveils a complex set of interactions around the machine which is neither purely technological, social or scientific. It also shows how the history of computing cannot be seen in isolation from other histories: the use of logical insights into computing reshapes also those very same logical insights and so has an effect not just on computing but also on logic.

FRIDAY 3 NOVEMBER

**09:30 – 10:00 (Session 1)**

**George Florin Calian** *Plato's Generation of Numbers and Brouwer's Intuitionism*

In the *Parmenides* 142b-144b Plato argues that the difference between *one* and *being* is done by virtue of *difference*, and from these three entities numbers are generated; firstly by obtaining the *first even* and the *first odd* number, and then, by the process of multiplication all numbers. This Platonic argument is seldom considered and discussed by the philosophers or the historians of mathematics. The argument rises several questions which could put a new perspective on Plato's philosophy of mathematics and Greek mathematics. Similar to Plato's view on the generation of number which starts from twoness (143c-d5), from pair to two, and from two to one, L.E.J. Brouwer postulates the structure of duality as basis.

This paper explores any possible resemblance between Plato's conception on the generation of numbers and the intuitionism of Brouwer. It provides an analytical commentary of the stages of Plato's arguments: the types of dualities used by Plato in order to generate number two are discussed: one is always two, it is never one; From three distinct entities pairs can be distinguished ($\tau\iota\upsilon\epsilon$) (143c3); A pair is called both ($\alpha\mu\varphi o\tau\rho\omega$) (143c4); What is called both is two ($\delta\upsilon o$) (143d2). I try to venture on why the mathematical one ($\epsilon\nu$ $\epsilon\iota\nu\alpha\iota$) for counting is obtained via $\delta\upsilon o$, and not as a given from the very beginning, from an initial ontological unity ($\epsilon\nu$). The common ground for this comparative reading is the conception of two-oneness shared by Plato and Brouwer - the two-oneness, "the basal intuition of mathematics", creates all finite cardinal numbers. I am also considering the possible reason for which Plato used an apparently specific formula for obtaining number 3. What the argument seems to be doing is to display that 3 is not 1+1+1, but it is essentially 1 added to 2. I argue that Plato's formula for the generation of numbers could be understood in the following manner. We need only 2 and 2+1, and by multiplication, 2x2, 2x(2+1), (2+1)x(2+1) and so on, we obtain the remaining numbers. The rest of the numbers after two and three are products of multiplication: 2x2 ($\delta\upsilon o$ $\delta\iota\sigma$), 3x3 ($\delta\upsilon o$ $\tau\rho\iota\sigma$), 2x3($\tau\rho\iota\alpha$ $\tau\rho\iota\sigma$), and 3x2 ($\tau\rho\iota\alpha$ $\delta\iota\sigma$). The paper discusses also possible answers concerning the reason for which Plato's argument does not provide a theory for the generation of prime numbers.

**10:00 – 10:30 (Session 1)**

**Gail Brekke & Jakob Giraud** *Fighting with Infinity: A Proposal for the Addition of New Terminology*

This paper proposes the addition of two new terms, "afinite" and "unfinite" to supplement the current terminology of "finite" and "infinite". The restrictions of the current terminology used in science, math, and linguistics result in inaccurate conclusions. The new terms are defined both linearly and through the medium of a Punnett Square, and explained through both theoretical and applied uses. Articles using only the traditional terms reveal the shortcomings of using two narrowly defined terms. Using four terms, instead of the traditional two, results in more accurate and truthful knowledge. This paper does not attempt to determine whether specific theories, including Cantor's set theory, Baye's Theorem, or Chomsky's Discrete Infinity Theory are correct or incorrect: it simply argues for the addition of two new terms in order to more accurately define ideas.

**09:30 – 10:00 (Session 2)**

**Jabel Alejandro Ramirez Naranjo,** *A Vision of Numerical Methods From the Perspective of the Mathematical Practice. From Equations to Coding*

This work aims to introduce in the interpretation of the field of computational methods, numerical methods or numerical simulation from the point of view of the philosophy of mathematical practice.

The numerical methods are not only one of the main exponents of the, so-called, applied mathematics, but also an important meeting point between mathematics and the computer (Goldstine, 1972). In this way it could be said that they were created in order to be able to apply the computer as a tool in the calculation of physical phenomena (Nash, 1990).

Is therefore a good scenario to elucidate, from a new perspective, the question of the applicability of mathematics to the physical real, and at the same time try to understand the mechanisms that act in the development of mathematical methodologies.

With this purpose a first and synthetic attempt to approach to numerical methods will be proposed, especially the finite element method, from a triple vision of mathematical practice (Ferreirós, 2016), that is:

- cognitive or obtaining a part of its status as objective truth from the intersubjective practice.
- historical or dependent on the development of previous theories and influenced, at some point, by the socio-economic facts.
- pragmatic, or directed by the objectives of the discipline concerning the simulation and prediction of physical reality.

The author argues that this is a suitable field of mathematics to demonstrate the relevance of the previous theses despite other idealistic, conceptualist or transcendental interpretations (Cavaillés, 1992).

These three aspects, among others, will be briefly treated in the analysis of numerical methods, from the partial differential equations to the coded algorithm, passing through the different mathematical steps of the process (Heat, 1977).

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**10:00 – 10:30 (Session 2)**

**Daniel Rompf** *Ernst Cassirer's Philosophy of Mathematics - A Structuralist Approach?*

Ernst Cassirer's (1874 - 1945) philosophy of culture is subject to a great renaissance in the philosophical research in the past three decades and therefore his studies were subject of many recent publications. In his early works Cassirer is a representative of the Neo-Kantian Marburg School and strongly influenced by his mentor Hermann Cohen. Here, one important question was the discussion of the compatibility of non-Euclidean geometry with the philosophy of Immanuel Kant. *Substance and Function* (1910) is his first study where Cassirer develops his own approach for the first time. Since one of his main influences is the development of mathematics in the 19th century he comments in this work on the development of the concept of number, space and geometry.

Cassirer's reference to the philosophy of mathematics seems to gain interest for Cassirer research in recent years. There are, for example, works from Jeremy Heis (2010, 2011, 2013, 2014) and Thomas Mormann (2008) that concentrate on Cassirer's early works. Heis (2015) himself demands that it will be fruitful to consider Cassirer's later writings in the philosophy of culture. The philosophy of symbolic forms is eponymous for Cassirers three volume main work (1923, 1925, 1929) and for his philosophical approach. In the third volume *The Phenomenology of Knowledge* Cassirer embeds his earlier studies about mathematics in his general theory of symbol and works out what distinguishes mathematics from the other sciences which constitute the symbolic form of scientific knowledge [*Erkenntnis*].

In my talk - being a first step in my PhD project - I want to highlight the main points of Cassirer's perspective on the concept of number in his early works and show that there are similarities to the position of structuralism in the philosophy of mathematics which comes up only years later. Furthermore, I will give a perspective of his views in the *philosophy of symbolic forms*.

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**11:00 – 11:30 (Session 1)**

**Spencer Johnston** *The Use of Formalisation in the History of Logic*

Applying the process of formalisation to critically assess and contribute to our understanding of various historical theories has played an important role in our reception of the history of logic. Both Ancient and Medieval logic have been studied using formalisation. From Łukasiewicz's studies of Aristotle's syllogistic, to Malink's reconstruction of Aristotle's modal logic; from Prior's distinction between the possibly true and the possible, to Duthil-Novaes analysis of *Obligationes* as a consistency-maintenance game, formalisation seems to play an interesting and, in some cases, important role in our understanding of these theories. By looking at these four instances as case studies, this talk will address the questions:

(i) What, if anything, does formalisation contribute to our understanding of historical theories?

(ii) How should we think of these formalisations?

In our answer to i) the paper will argue that formalisation acts as a kind of interpretative lens or strategy for approaching difficult or unusual aspects of a historical logical theory. In particular, by attempting to formalise a particular historical figure's logical theory, we are trying to find a way to express these ideas using modern mathematical logic as both the standard we wish to analyse the theories against, and as providing the questions that we hope the historical system can answer. By doing this, the process of formalisation requires us to ask a number of deep and important questions of these systems that, when done carefully, can guide us into a deeper understanding of the historical systems under discussion, or indeed (e.g. in the case of Malink), to new interpretations of these systems that can make sense of logical theories that were previously thought to be marred by confusion and mistake.

In answer to ii) we will argue that we should approach these formalisations as a kind of model, or reconstruction. In particular, we will suggest that these formalisations should not be thought of as identical with, or the same thing as, the logical systems that were developed by a historical figure, but should be thought of more as a model or a reconstruction of the system. To motivate this, considerations will be drawn by looking at different formal reconstructions of the same historical system that are adequate to the remarks made by that figure.

**11:30 – 12:00 (Session 1)**

**Jan Zeman** *Hilbert's Arithmetisation of Geometry*

In this address, we will strive to show how does the arithmetisation of geometry take place in the work of David Hilbert (1862-1943). At first sight, the question looks like a paradox, since we try to show continuous notions in a discrete field of study. We will concentrate on the method, presented in Hilbert's *Grundlagen der Geometrie* from year 1899. First, we introduce what preceded it, whether in Hilbert's university lectures or by his forerunners. Next, we explain in detail his proof of the consistency of his new system of axioms of geometry. Along with the advantages of the arithmetisation of geometry, this will serve us also to explain the formalism of the axiomatic method and the role of the continuity in the Euclidean geometry. We formulate our answers to the question, what was Hilbert's main purpose to write the *Grundlagen der Geometrie*.

**12:00 – 12:30 (Session 1)**

**Deborah Kant** *The Practice of Forcing in Set Theory: Is Forcing Used as a Philosophically Neutral Tool in Set Theory?*

We provide a presentation of the current practice of forcing. For this purpose, we focus on the current research in set theory, leaving historical considerations aside, and we take a perspective towards the future of set theoretic research. We address the following questions:

1. Which aims are tried to be achieved using forcing (and how is forcing used)?
2. Which philosophical interpretations of forcing do exist, and do they play a role in mathematical practice?

The answer to the first question categorises research questions connected to forcing. In addition, it includes a presentation of the different forcing techniques, and their scopes of application. Since its invention, forcing is used as a tool to prove independence results in set theory. For example, the independence of the continuum hypothesis (CH) can be proven completely by forcing, i.e., assuming the existence of a (countable transitive) model of ZFC, forcing provides us with a model of ZFC + CH as well as a model of ZFC + $\neg$ CH. But -as the research of Matteo Viale shows[4]-forcing is used as well to prove theorems.

[4]See for example: Matteo Viale, *Forcing and absoluteness as means to prove theo-*

The answer to the second question provides the present interpretations of forcing. We give three short examples: based on the wide use of forcing in set theoretic practice, Joel Hamkins argues for a realism of the models of set theory[5], Joan Bagaria argues for the naturalness of forcing axioms, suggesting their general acceptance in set theory[6], and Giorgio Venturi defends that the forcing axioms clarify the idea of arbitrary sets-in his view, the most fundamental idea of formalising set theory[7]. We analyse the role of these and similar philosophical interpretations of forcing in the set theoretic research practice. Finally, we describe a picture of the current practice of forcing, suggesting an answer to the question if the practice is influenced by philosophical thoughts, and if yes, in which way.

**11:00 – 11:30 (Session 2)**

**Flavio Baracco & Davide Quadrellaro** *Rational Reconstruction in the History of Mathematics*

Our presentation deals with the notion of "rational reconstruction" within the history of mathematics: we will explain in which sense a mathematical text from the past can be rationally reconstructed. In order to achieve this goal, we will explore this notion within the broader context of the history of ideas. We will consider the many different approaches to the history of ideas that one might pursue. Firstly, we distinguish between a historical and a non-historical way to analyze the texts from the past. The former approach is related to the proper examination of the texts within their historical context, while the latter focuses on the theoretical framework that underlies the texts and tries to rationally reconstruct them. Focusing on this second perspective we can identify some relevant distinctions that allow us to make clear what a proper non-historical approach should be. Indeed, we aim to identify the many different ways to pursue a non-historical method to the history of ideas.

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*rems.* Talk in Freiburg, 13 June 2012. url: `www.logicatorino.altervista.org/matteo_viale/germany-viale.pdf`.

[5] Joel D. Hamkins, *The set-theoretic multiverse*, Review of Symbolic Logic, vol. 5, pp. 416-449, 2012.

[6] Joan Bagaria, *Natural axioms of set theory and the continuum problem.* CRM Preprint 591: 19, 2004.

[7] Giorgio Venturi, *Forcing, multiverse and realism*, in Proceedings of the first Filmat conference (Boccuni, Sereni eds.) Boston Study in the Philosophy of Science, Springer, pp. 211-241, 2016.

These clarifications on what a rational reconstruction is within the history of ideas will shed light on this very notion within the history of mathematics. Our presentation then aims to clarify what a proper non-historical way to analyze mathematical texts from the past should be.

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**11:30 – 12:00 (Session 2)**

**Robert Middeke-Conlin** *Predictive Modelling and Brick Deliveries*

The presentation will present a new methodological tool to examine practical mathematics in a professional setting by examining an Old Babylonian brick production and delivery archive in the light of mathematical texts from this same period. Bricks played an important role in construction within ancient Mesopotamia. Bricks also plays an important role in modern scholarship because they are well represented in the archeological record, in numerous mathematical texts, and appear in the administrative record. Indeed, it's been proposed that a mathematical model existed in this period and place to plan brick production, deliveries, and construction. However, direct evidence for the employment of these models has not been forthcoming for several reasons: First, individual administrative texts are produced in order to keep accountability and responsibility and so lack mathematical transparency. For this reason, mathematical processes used to plan and run a complex economy are not stated. Instead, texts are produced using already obtained statistical data for review by a higher authority, who will use this data to administer and expand the economy. Thus, in any practical environment, including those of brick production, delivery, and construction, the economic texts would only describe already attained costs in brick construction or show responsibility for current costs. It therefore must be asked, can evidence for any mathematical models, let alone evidence for a mathematical model predicting the cost of brick deliveries, be found in the administrative tradition?

This presentation will attempt to answer this question by presenting a hitherto unpublished archive of brick deliveries. Discussion is divided in two parts. The first part will present a description of the archive as well as its bureaucratic context and will lead to the second part of this article, a numerical study which explores numericity and mathematics in the brick delivery archive. Indeed, it's proposed here that the very numbers within these economic texts, as well as other texts like them, allow the modern researcher the ability to reconstruct the mathematical knowledge of ancient actors and practitioners. The numerical values found in economic texts can help reconstruct mathematical practice, even if no mathematical statement is found in a text itself.

**14:30 – 15:00 (Session 1)**

**Benjamin Wilck** *Mathematical Definitions and a New Problem for Pyrrhonian Scepticism*

My paper raises a previously unnoticed problem for the applicability of Pyrrhonian scepticism to scientific principles and, in particular, geometrical definitions. In the *Outlines of Pyrrhonian Scepticism*, Sextus Empiricus defines his sceptical method as an ability to bring about suspension of belief by constructing pairs of opposing and equally convincing arguments about any given proposition. In *adversus Mathematicos* (= *M*) I-VI, Sextus nonetheless appears to present a series of straightforward refutations of various scientific doctrines rather than oppositions of arguments and counterarguments. Subsequently, commentators have argued that the method deployed in *M* I-VI is not Pyrrhonian scepticism, but is rather negative dogmatism (see Pappenheim 1874, 16-17; Apelt 1891, 258-259; Zeller 1923, 51n2; Janácek 1972; Dumont 1972, 164; Russo 1972, viii n2; Pellegrin et. al. 2002, 23-24) ? a view that appears to go back at least to Proclus (in *Eucl.* 199:3-14 Friedlein).

Recently, however, it has become widely accepted among scholars that the apparent lapse from Pyrrhonian scepticism into negative dogmatism that we find in M I?VI can be rectified by simply supplementing additional arguments opposing Sextus? refutational arguments (see Barnes 1988, 72-77; Blank 1998, l-lv; Morison 2004, section 5).

Against this I present a counterexample. While this strategy can account for scientific theorems, which are usually accompanied by a proof, it fails in the case of particular scientific definitions, for which there is no proof or justification of some other sort. Moreover, I show that neither the standard (Striker 1983, 100; Annas and Barnes 1985, 25; Hankinson 1995, 159) nor the most recent (Morison 2011) interpretations of Pyrrhonian scepticism give a satisfying account of Sextus' arguments against particular scientific definitions. I conclude my paper by suggesting several solutions to this problem, thereby also addressing the notorious question of who are Sextus' opponents in M III, *Against the Geometers* (see Mueller 1982; Dye and Vitrac 2009).

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**15:00 – 15:30 (Session 1)**

**Shafie Shokrani** *Nelson’s Socratic Method*

Leonard Nelson (1882-1927), a German philosopher and peda- gogue from Göttingen, has conceptualized a pedagogical method for philosophy and mathematics, which he has named after Socrates. His student, Gustav Heckmann have developed this method further. In particular in context of mathematics some work has been done by Martin Wagenschein, Harmut Spiegel and Rainer Loska. The method is still beeing used and studied in some universities in Germany and in the *Politisch-Philosophische Akademie*.

Nelson’s socratic method is based on a philosophical method, which he called *regressiv method of abstraction*. This method in turn, presupposes some epistemological premises, such as the principle of self- confi dence of intellect. He borrowed this terminus from J. F. Fries (1773-1843), who followed the Kantian philosophy. In this talk, after giving a brief history about Nelson and his method, I will introduce the *regressiv method of abstraction* and discuss one of its premises, namely the *principle of selfconfidence of intellect* with an emphasize on mathematical context.

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**15:30 – 16:00 (Session 1)**

**Line Andersen** *Proof as Dialogue*

Catharina Dutilh Novaes' (2016) conceptualizes mathematical proof as a dialogue between what she calls Prover and Skeptic. Skeptic is fair but will not be easily convinced that the proof is valid and perspicuous. In this talk I will examine how recent empirical studies of mathematical practice provide evidence for this conceptualization of proof and what these can tell us about Skeptic and the 'right' amount of inferential rigor in proofs. I will draw on interview, questionnaire, and observation data on mathematicians' proof validation practices (e.g., Mller-Hill 2011; Weber, Inglis, & Mejia-Ramos 2014; Johansen & Misfeldt 2016; Andersen 2017).

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**14:30 – 15:00 (Session 2)**

**Tony Royle** *Spinning, Stalling and Falling Apart*

The birth of fixed-wing, powered flight in the first decade of the twentieth century brought with it significant potential for pilots to return to Eart by unintended, often fatal, means.

I will discuss the nature of the contemporary mathematical and engineering debates associated with these facets of flight, and the practical steps taken to facilitate safer aircraft and more robust operating procedures.

**15:00 – 15:30 (Session 2)**

**Ana Jimena Lemes** *Potentialities of the History of Mathematics in the Training of Mathematics Teachers*

The History of Mathematics (HM) has been considered an attractive resource for teaching mathematics with arguments such as contextualizing and problematizing concepts, stimulating reading, humanizing mathematics, and so on. From the ICMI History in Mathematics Education (2000) study, a community of researchers in the field of mathematics education, history, philosophy and epistemology of math, has consolidated and become an international reference. With the strengthening of this community, we see efforts to systematize researchers of mathematics courses in which HM is used as the main teaching resource (Clark, 2006 ; Arcavi et Isoda (2007) ; Dorier (2008) ; Jankvist (2009) ; Smestad (2012) ; Matthews (2014) ; Barnett, Lodder et Pengelley (2015); Smestad (2017)).

Likewise, it is possible to identify certain works (Guacaneme (2016); Fried, Guillemette et Jahnke (2016); Jankvist, Mosvold et Clark (2016)) who seek to define conceptual and theoretical frameworks that support the study of the integration of MH in mathematics education. Among the researchers mentioned above, we found only a small amount of works that integrates HM as a didactic resource in teacher trainings. Indeed, most focus lays on secondary education. This is one of the main reasons why it was chosen to conduct the research in the specific context of teacher training. Our research therefore corresponds to a need. It is therefore necessary to identify and study the occupational skills essential to the practice of the job in order to better understand the potential of HM in the training of a mathematics teacher.

In this paper, a sub-domain of Shulman's PCK (1986) is presented, defined by Ball (2009) as Mathematic Horizon, trying to establish a connection between such a sub-domain, HM and the professional competences necessary for the teaching task.

**15:30 – 16:00 (Session 2)**

**Giuditta Parolini** *Invistigating Statistical Tools in the Life Sciences: An Opportunity to Reconnect the History of Mathematics to the History of Science and Technology*

An increasing dissatisfaction for the marginal role that the history of mathematics plays in the history of science has emerged in recent years (e.g. Alexander, 2011b). To counteract this situation, historians of mathematics have increasingly broadened the focus of their research. They have shifted their attention from specialised studies of mathematical theories and methods to investigations which interest also historians of science because they highlight how mathematics has changed over time and is shaped by social and institutional factors (Alexander, 2011a; Robson & Stedall, 2009).

This cultural and social turn, which aims to reconnect the history of mathematics to the history of science, can certainly benefit from an in-depth examination of the mathematical tools used in the sciences. By investigating how mathematical tools are co-constructed by professional mathematicians and scientists and how these tools shape scientific practices and are in turn shaped by them, it is possible to address research questions that matters to both historians of mathematics and historians of science. This tools-oriented approach is worth pursuing in relation to all the sciences and in particular to the natural sciences that have a long-term relationship with mathematics (Alexander, 2011b, p. 478).

Among the natural sciences, mathematical tools are not only widespread in physics, chemistry or engineering, which have already attracted interest from historians of mathematics. Mathematical tools are also crucial in the life sciences where statistical methods, models, algorithms, etc. have become dominant during the twentieth century, as argued by many historians of biology (e.g. Fox Keller, 2003; Stevens, 2013).

My paper will examine the use of mathematical tools in the life sciences by relying on my long-term investigation of statistics in twentieth-century agricultural research. The case studies I will discuss span a wide range of experimental and observational practices in agricultural science, ranging from field experimentation to weather forecasting. In these areas, statistical tools proved crucial in the redefinition of epistemological goals and in restructuring social relationships between mathematicians and experimental scientists. The examination of these statistical tools is of interest to both the historian of mathematics and the historian of science because these mathematical tools were not only developed at the statistician's desk, but also in the experimental fields and at the lab bench and their application required both the technical knowledge of the experimentalists and the mathematical competence of the statisticians.

Furthermore, statistical tools required suitable computing equipment. Mathematicians and experimental scientists used at first mathematical tables and desk calculators, later replaced by digital computers and statistical software. How did technologies affect the work of statisticians and experimental scientists? How relevant was the mathematical training of statisticians in facilitating their approach to computing tools? These are just two examples of the questions that can interest both the historian of mathematics and the historian of technology.

The beginnings of mathematical statistics are humble and relatively recent -

mainly dating back to the nineteenth century -, but statistics has now turned into "one of the most massive parts of mathematics, while often functioning separate from it" (Grattan-Guinness, 2004, p. 174). The popularity and widespread use of statistical tools in the life sciences has certainly contributed to this achievement. Therefore, by investigating statistical tools in life science research we have an opportunity to re-evaluate the history of statistics within the broader history of twentieth-century mathematics and reconnect the disciplinary history of mathematics not only to the history of science, but also to the history of technology.

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**16:30 – 18:00 (Plenary)**

**Ralf Krömer** *The Distinction of Tool and Object in Conceptual History of Mathematics: Epistemology and Examples*

**09:30 – 10:00 (Session 1)**

**Patrick Walsh** *Title?*

I claim that category theory provides a fruitful level of abstraction that allows us to see patterns that are philosophically interesting and deep. Mathematical progress is often effected by finding the right level of abstraction with which to view a section of mathematics. Examples include Dedekind's axiomatization of the natural numbers, Klein's Erlangen program, and, I argue, category theory. This is not to say that category theory solves all problems or is some abstract panacea. It is simply the right level for some interesting and important questions.

I rehearse the history of category theory and its applications, emphasizing the cases when category theory unified previously separate fields, results, concepts, etc. Sometimes the diverse uses were thought to be the same, but only imprecisely, and sometimes they weren't recognized as having the common features that the category theory exhibited.

My main example will be about the construction of universes in foundations of mathematics. More precisely, the stock of objects that logicians, philosophers, and mathematicians take as 'basic' can be characterized quite simply in category theory and importantly, *we learn what the common structure of these universes is* by such a characterization. We learn something important by using the very abstract level category theory affords. These structures share some properties that are simple to specify by example, but not axiomatized in a structural way. Universes are inductively defined and each iteration is usually deterministic. Wilfried Sieg calls such structures *accessible domains*. I will give the characterization, and suggest it as an example of fruitful abstraction using category theory.

This follows a suggestion of Poincaré:

*"I think I have already said somewhere that mathematics is the art of giving the same name to different things. It is enough that these things, though differing in matter, should be similar in form, to permit of their being, so to speak, run in the same mould. When language has been well chosen, one*

*is astonished to find that all demonstrations made for a known object apply immediately to many new objects: nothing requires to be changed, not even the terms, since the names have become the same. A well-chosen term is very often sufficient to remove the exceptions permitted by the rules as stated in the old phraseology. This accounts for the invention of negative quantities, imaginary quantities, decimals to infinity, and I know not what else. And we must never forget that exceptions are pernicious, because they conceal laws. This is one of the characteristics by which we recognize facts which give a great return: they are the facts which permit of these happy innovations of language. The bare fact, then, has sometimes no great interest: it may have been noted many times without rendering any great service to science; it only acquires a value when some more careful thinker perceives the connection it brings out, and symbolizes it by a term. "(The future of mathematics 1914)*

**10:00– 10:30 (Session 1)**

**Jio Jeong** *Burgess and Maddy on Naturalist Philosophy of Mathematics and Benecerraf's Dilemma*

In his paper on "Mathematical Truth," Paul Benacerraf persuasively articulated the dilemma of providing a satisfactory semantics for mathematical language while at the same time giving some reasonable epistemology of mathematics as such. More recently, John P. Burgess and Penelope Maddy, two well-known advocates of naturalism in philosophy of mathematics, have attempted to defuse the epistemological horn of Benacerraf's problem that philosophers with realist inclinations are burdened to address. In her book *Second Philosophy*, Maddy takes a similar approach by expanding on a possible cognitive instrument for mathematical apprehension previously outlined in an earlier paper, "Perception and Mathematical Intuition." Maddy believes that set-theoretic methods already suggest an answer to problems like Benacerraf's, arguing that sets should be construed as causally interactable objects, akin in that respect to physical objects; the cognitive basis of that interaction, when fully grasped by scientific means, will provide the grounds for a definitive mathematical epistemology. In this way, on Maddy's view, the real source of the Benacerraf problem can be seen to lie not in philosophy, but rather in presumably temporary deficits in the current state of cognitive science. Burgess also espouses, *mutatis mutandis*, an anti-philosophical methodology of answering what he sees as speculative philosophical questions by appeal to existing mathematical practices. Denying that there is any profound epistemological mystery over mathematics, Burgess insists that standard criteria for evaluating scientific theories, criteria which ultimately led to our contemporary scientific beliefs over the long history of scientific progress, are indeed correct and lead to truly justified beliefs. That virtually all of our scientific theories are formulated with mathematics leads Burgess to conclude that belief in the existence of mathematical objects is strongly warranted. This paper will show that shortcomings in both Maddy's and Burgess' arguments are similar in nature, one being unable to establish how belief scientific theory leads to the verification in the existence of mathematical objects, the other being the difficulty of proving how knowledge of sets leads to knowledge of mathematical objects. The failure of yet another round of naturalist critiques of Benacerraf's argument strongly suggests that the associated problem remains open.

**09:30 – 10:00 (Session 2)**

**Sven Delarivière** *The Extended Mathematician: Does Mathematical Understanding Ever Extend to Include One's Tools?*

The epistemic concept of "understanding" has only recently started to gain ground in epistemology, philosophy of science and, to a much lesser extent, philosophy of mathematics. What has not received an equal amount of attention is how to conceive of who understands. The aim of this presentation is to contribute to a fruitful explicitation on the notion of an epistemic (in particular, an understanding) subject. To do so, I draw on work from philosophy of mind and cognitive science.

Recent developments in philosophy of mind (most notably Clark & Chalmers, 1998) have questioned the idea that cognition is a process that ends at the skin or skull. If certain parts of the environment (i.e. tools) play an active role in contributing to the overall cognitive result (in such a way that, were to happen inside the skull, we would readily conceive of it as cognitive), then we should be unbiased in considering its inclusion in the cognitive process. This entailed that certain cognitive properties, like beliefs, can be said to reside outside of an individual. Traditionally, epistemologists have taken for granted that individual humans should be the relevant epistemic subjects under consideration, but it seems fair to ask whether understanding is also a property that can extend beyond the individual or whether there is a reason to keep its subject-demarcation as traditionally individualistic.

To start, I briefly argue why the property of understanding is best characterised as the possession of appropriate abilities. It is tempting to think that the possession of abilities needs to be attributed to the human individuals, leaving the role of tools as mere environmental conditions. However, the ability-oriented conception of understanding is equally open to Clark and Chalmers's parity principle which urges us to consider any process as cognitive if we would readily do so if it took place inside a head. Mathematicians make use of several tools, be they paper or interactive theorem provers. If the abilities are realized by a process that criss-crosses brain, body and tool in such a way that the tool plays an active role in implementing an ability, then it its the human-tool couple that together possess the ability and thus the understanding.

Note that the claim of extended understanding is stronger than trivial summation. If a mathematician can prove Fermat's Last Theorem and an automated theorem prover can prove the Four Color Theorem, then it is trivially true that the couple can do both. However, the criss-crossing interaction between an individual and its tool (as may be the case in interactive theorem proving) makes the ability dependent on the interaction between the individual and the tool. This means that we couldn't merely reduce the ability to either seperately and need to attribute it to the extended subject instead.

**10:00 – 10:30 (Session 2)**

**Manuel Gracia Perez** *History of Mathematics as a Tool for Research in Studies of Geometric Cognition*

Cognitive sciences are considered as an interdisciplinary approach to the study of our cognitive capacities. In particular, to acquire a good understanding of the genesis and development of our mathematical abilities a joint work from neurosciences, philosophy, anthropology, and history is needed. In this talk I will show that history of mathematics can be used as a reliable source to gain a comprehensive understanding about some key concepts in geometric cognition studies.

I will place the emphasis in a famous theory in cognitive sciences where the existence of a "Natural geometry" (Spelke et al., 2010) that can be equated with Euclidean geometry is stated. Spelke and Lee (2012,2785) assert that

Euclidean concepts possess three surprising properties, such as i) they are extremely simple; ii) exceedingly useful; and iii) they go beyond the limits of perception and action. Several critiques from the history of geometry can be established towards this cognitive scientists' proposal. The focus will be in two of them.

On the one hand, a comparative study about two contemporary mathematical traditions that, in some sense, differ in their methods and underlying concepts. These are Greek geometry, and especially the results from Euclidean geometry presented in "The Elements"; and, on the other side, ancient Chinese geometry, with attention to the developments presented in "The nine chapters on mathematical procedures". The point is to see what are the main properties that Spelke and her colleagues have in mind when they use the label 'Euclidean' applied to natural geometry; and then, to show that it cannot be used to define these two geometrical traditions, or at least not in an accurate mathematical or historical sense.

On the other hand, I will show that Euclidean and ancient Chinese geometries share some important and useful results; especially, results linked with right-angled triangles in China, known as the Gou-gu theorem, and the Pythagorean theorem in the western tradition. However, the emergence of these results does not depend so much on certain innate cognitive abilities as on the use of certain tools -ruler and compas-, and their development for specific objectives -with emphasis in the astronomical needs.

Therefore, using the knowledge acquired by the history of geometry, cognitive science studies related to geometric cognition can be complemented. In fact, to properly pursue a research about the cognitive roots of our cognition, knowledge of the historical roots of geometry is more than necessary.