ICHM Co-Sponsored *AMS-MAA Special Sessions on History of Mathematics* in memory of Ivor Grattan-Guinness (1941-2014)

San Antonio, Texas; January 10-11, 2015

By Adrian Rice

This set of lectures comprised a two-day Special Session on the History of Mathematics at the Joint Meeting of the American Mathematical Society and the Mathematical Association of America held at the Convention Center in San Antonio, Texas, USA. The session was organized by Sloan Despeaux (Western Carolina University), Patti Hunter (Westmont College), Deborah Kent (Drake University) and Adrian Rice (Randolph-Macon College) and featured 24 speakers from a total of seven countries, plus the United States. The structure of the session was amended slightly from that of previous years to feature, for the first time, three moderated panel discussions on themes arising from a selection of the talks. These themes were:

- Mathematical Communication,
- Recent Trends in the History of American Mathematics,
- Common Themes in the History of Ancient and Non-Western Mathematics.

One final, last-minute, change occurred less than one month before the meeting. In response to the sad death of Ivor Grattan-Guinness on 12 December 2014, the title of the session was amended in his honor.

The following speakers presented talks at the meeting:

Table Parameter Estimation and Inference in Historical Mathematical Tables

David R. Bellhouse, University of Western Ontario, Canada

The construction of a medieval Islamic astronomical table can depend on parameter values that are often unknown to us today. Estimation of these unknown parameters can indicate the school to which the astronomer who created the table belonged. Three estimation techniques have been used by past historians: least squares, least absolute deviations and least number of errors. These techniques are compared through an examination of the assumptions underlying each technique and through an actual case of parameter estimation in the thirteenth-century Islamic solar equation table from the Zamil Zīj. In terms of the model assumptions that underlay the estimation techniques, the dictum of the statistician George Box is followed, “Essentially, all models are wrong, some are useful.” In terms of data analysis, it is suggested that what analysts of these tables should be striving for is simplicity and ease of interpretation of the results. Either least squares or the least absolute deviations criterion is a reasonable criterion to use unless there are some unusually large calculated deviations from the given table; in that case least absolute deviations would be preferred since less weight is given to the anomalous values.

Remarks on N. Oresme's Definition of Curvitas

Bogdan D. Suceava and Isabel M. Serrano, California State University, Fullerton

In a paper published in 1952, J. L. Coolidge points out that "the first writer to give a hint of the definition of curvature was the fourteenth-century writer Nicolas Oresme". Coolidge also comments: "Oresme conceived the curvature of a circle as inversely proportional to the radius; how did he find this out?" This question is the starting point of our investigation. It is established that the *Tractatus de configurationibus qualitatum et motuum* was written by Oresme sometime between 1351 and 1355. We study N. Orseme's original work in the scholarly environment of his time.

Natures of curves in the early modern period and the emergence of transcendence

Bruce J. Petrie, University of Toronto, Canada

Comparing the classification rules of René Descartes and Leonhard Euler reveals the changing significance of nature to mathematical study. Early modern algebraic and transcendental classifications were intended to describe a mathematical object's nature. This nature was useful to determine which objects were appropriate for geometrical study especially when applied to curves. The development of calculus provided the tools necessary for algebraic analysis to uncouple the study of curves and geometry effectively removing the transcendental barrier. The geometrical purpose of the transcendental classification was rendered obsolete and was replaced by focusing on functional relationships between variables. The nature of mathematical objects inherited this algebraic purpose.

The representation of curves in the early Leibnizian calculus

Viktor Blasjo, Universiteit Utrecht, The Netherlands

Transcendental curves were the focal point of a profound conflict in late 17th-century mathematics. They were at the heart of remarkable advances in the new fields of infinitesimal calculus and mathematical mechanics, but they also rendered obsolete traditional conceptions of geometrical rigour and method, forcing the boundaries of mathematics as defined by classical Greek and Cartesian geometry to be redrawn. The early development of the calculus was shaped by this tension between the old and the new, and many aspects of the early calculus that seem peculiar to modern eyes are in fact very rational attempts at resolving this forgotten conflict. I illustrate this by discussing some aspects of Leibniz's view of the exponential function, including his classroom-ready, do-it-yourself recipe for how to compute logarithms using nothing but an ordinary necklace chain.

How Johann Bernoulli solved a problem whose solution was obvious (or was it?)

George W. Heine, Pueblo, Colorado

Johann Bernoulli's work on the brachistochrone problem, one of the precursors to the calculus of variations, is well known. Here we examine a more obscure work on another problem related to the calculus of variations, that of "Drawing the Shortest Line Between Two Points on an Arbitrary Curved Surface." This was the title of a 1728 communication to the Swedish mathematician Samuel Klingenstierna. After giving his main result, Bernoulli mentions that, as a corollary, the shortest curve on a sphere is a great circle. We put this problem in historical context, discuss Bernoulli's work in detail, and briefly summarize later work by Clairaut and Euler.

Euclid's *Elements* in Spanish, during the XXth century (part I): Vera's edition

Alejandro R. Garciadiego, Universidad Nacional Autonoma de Mexico

Each particular book (independently of the topic under discussion, its purpose, or the language used) has its own history. The spectrum of these accounts (concerning personal, academic, economic, political, religious and social factors, among others) is extremely wide and open to speculation and metamorphosis. It is logical to suppose, although not necessarily historically accurate, that the sophistication of these chronicles should be in direct proportion to the relevance of the work. At least, this should be the case of those rare volumes labeled as classics that have transcended barriers of times, idioms and cultures. In mathematics, perhaps, the best example is Euclid's *Elements*. At least in the last five hundred years, this particular book has been printed in a wide variety of formats, addressed to different audiences and with diverse goals. As a consequence, this multiplicity of impressions has forced, in some cultures, the editing of definitive versions as close as possible to the 'original' one. I have already discussed some of the characteristics and peculiarities of some editions of the *Elements* published in Spanish between the XVI and XVIII centuries. In this talk, I analyze the editorial efforts of Francisco Vera (1888-1967).

From left to right: Alejandro Garciadiego, George Heine, Bruce Petrie, David Bellhouse and Viktor Blasjo.

The Mathematics of the *Encyclopédie*

Lawrence A. D'Antonio, Ramapo College

One of the greatest intellectual achievements of the 18th century was the *Encyclopédie* edited by Denis Diderot and Jean d'Alembert. The work, comprising 28 volumes with over 70,000 articles written by

more than 130 contributors, attempted to give a systematic overview of human knowledge at that time. In this talk we give an overview of the writing of the *Encyclopédie* and its place in the Enlightenment. We focus on the mathematical articles in the *Encyclopédie*, many of which were written by d'Alembert, such as the well-known article on the concept of limit, and we consider the question: what is the role of mathematics in the structure and goals of the *Encyclopédie*?

Polemics in public: controversies around the principle of duality in early nineteenth-century geometry

Jemma Lorenat, Simon Fraser University, Canada and Université Pierre et Marie Curie, Paris

A plagiarism charge in 1827 sparked a public controversy centred between Jean-Victor Poncelet (1788-1867) and Joseph Diez Gergonne (1771-1859) over the origin and applications of the principle of duality in geometry. Over the next three years and through the pages of various journals, monographs, public letters, reviews, reports, and footnotes, vitriol between the antagonists increased as their potential audiences grew. While the historical literature offers valuable resources toward understanding the development, content, and effects of geometric duality, the hostile nature of the exchange seems to have deterred an in-depth textual study of the explicitly polemical writings. We argue that the necessary collective endeavour of beginning and ending this controversy constitutes a case study in the circulation of geometry. In particular, we consider how the duality controversy functioned as a medium of communicating new fundamental principles to a wider audience of practitioners.

How drawings sprang up in a particular field of celestial mechanics

Tatiana Roque, Universidade Federal do Rio de Janeiro, Brazil

By using the methodology of text networks, I have identified a specific practice of drawings in the end of the 19th and the beginning of the 20th century. Mathematicians and astronomers that made use of periodic orbits to analyze special cases of the three-body problem started to draw these curves, which was not usual in similar texts of the same period. By exploring texts published in journals like *Astronomische Nachrichten* or *Bulletin Astronomique*, I intend to show to what extent this new practice was linked to the proposals of Poincaré, who had not himself presented drawings of periodic phenomena, which astonished some of his contemporaries, like Heinrich Hertz.

This expedient of drawings is a question about communicating mathematics that also implies a reflection upon the habits and the professional skills involved in scientific research. The computations and drawings demanded in celestial mechanics became more and more mathematical. As George Darwin remarked, astronomers increasingly needed the skills of professional computers who were also mathematicians. The same problem appears in texts of other scientists. This talk discusses the role of drawings in the boundary of mathematics and celestial mechanics in the period mentioned.

Benjamin Finkel and the *Ohio Normal University Herald*

J. J. Tattersall, Providence College

In November 1888, the New York Mathematical Society held its first meeting. Earlier that year, Benjamin Finkel, an undergraduate at Ohio Normal University, began editing a weekly column devoted to mathematical problems and their solutions in the college's newspaper. A few years later, in an effort to improve high school teaching in America, Finkel founded and co-edited with John M. Colaw the *American Mathematical Monthly*, now the official journal of the Mathematical Association of America. We focus on the contributors and their contributions to the mathematical department of the *Ohio Normal*

University Herald and the influence of the London-based *Educational Times* and *Journal of the College of Preceptors* on the column.

Gaston Darboux: monster-maker par excellence

Janet Heine Barnett, Colorado State University, Pueblo

The drama of the rise of rigor in nineteenth-century mathematical analysis has now been widely rehearsed. Notable within this saga is the appearance of functions with features so unexpected (e.g., everywhere continuous but nowhere differentiable) that contemporary critics described them as "bizarre," "ridiculous," "pathological," and even "monsters." Among those who played the part of a "monster-maker," one of the most talented and influential was French mathematician Gaston Darboux (1842-1917).

In this talk, we survey Darboux's mathematical and "backstage" contributions to the development of nineteenth-century analysis. We review in particular his 1875 *Mémoire sur les fonctions discontinues*, focusing on Darboux's discussion and proof of the result now known as "Darboux's Theorem" (i.e., all derivatives have the intermediate value property). After meeting some of Darboux's own favorite pet monsters, we examine the role that functions such as these played in setting the scene for the re-shaping of analysis during the latter part of nineteenth century.

From left to right: Bill Dunham, Janet Barnett, Tatiana Roque, Jemma Lorenat and Jim Tattersall.

A Hard(y) Integral

William Dunham, University of Pennsylvania

A century ago, G. H. Hardy easily stood among the world's foremost mathematicians. In addition, he was a superb writer, a cricket enthusiast, an avowed atheist, and a Cambridge eccentric of the first rank. In this talk, after briefly meeting the man himself, we sample a tiny crumb from his vast analytic feast: the evaluation of an improper integral that should leave calculus fans cheering.

The Schilling Kinematic Models at the Smithsonian

Amy Shell-Gellasch, Montgomery College

The National Museum of American History (NMAH) houses the Smithsonian's collections of mathematical, computing and computer objects. In my role as a volunteer researcher at NMAH, I research and prepare materials for the on-line collections. One such collection contains the late 19th-century kinematic models of the German firm of Martin Schilling. The Schilling models were constructed to mechanically produce mathematical curves of use primarily in engineering. Of particular mathematical interest are the linkages used to produce linear motion and the models that produce trochoids, of which family the cycloid is the most famous member.

F. R. Moulton and his plans for a new lunar theory

Craig Alan Stephenson, Madrid, Spain

At the beginning of the 20th century Forest Ray Moulton was arguably the leading mathematical-astronomer in the United States. During his approximately 30 years at the University of Chicago, Moulton wrote several introductory books on astronomy and celestial mechanics and is nowadays remembered amongst astronomers as being the co-author of the Chamberlin-Moulton planetesimal hypothesis. However, Moulton's main interest was the three-body problem and much of his research was aimed at gaining an understanding of this problem through the study of its periodic solutions. It is on these investigations, which began with his 1899 PhD thesis and which culminated over 20 years later with the publication of his book *Periodic Orbits*, that this talk is focused.

After giving a brief introduction to Forest Ray Moulton, I use his correspondence with the Carnegie Institution of Washington to tell the story of *Periodic Orbits*' long (11-year) road to publication and to show how the research this book contains was initially motivated by his desire to construct a new lunar theory. I also attempt to throw some light on his planned theory and to say something about the mathematical techniques which he employed throughout his research.

Mathematical Abstraction from Ancient Greece to the Stone Age

Alma Steingart, Harvard University

The use of "abstract" as an adjective denoting mathematics dates predominately to the mid-eighteenth century, when it was used as a synonym for pure mathematics and hence in opposition to what at the time was known as "mixed mathematics." Up until then, abstract was primarily used as a verb to describe how mathematical entities are discovered or defined. For example, Aristotle held that that mathematical objects, such as numbers and geometrical figures, were arrived at through a process of removal or taking away. This grammatical move from verb or adverb to adjective is meaningful, as it denotes a shift from the use of abstract as an underlying philosophical conception of mathematics to its use as a distinctive marker that differentiates among various types of mathematical activities. In this talk,

I survey how the notion of abstract mathematics has transformed, starting with Aristotle and ending with American mathematicians during the Cold War. Throughout, I ask: what does the definition of abstract mathematics tell us about conceptions of mathematics as a field in each given historical moment? How does it reflect broader cultural trends from French First Republic citizenship to turn of the century modernism to 1950s abstract expressionism?

The Stratification of the American Mathematical Community: The MAA and the AMS, 1915-1925

Karen V. H. Parshall, University of Virginia

The MAA officially came into existence over the course of a two-day-long meeting held on 30-31 December 1915 in Columbus, Ohio. This talk examines the impetus behind the formation of the MAA, its founding decade of activities, and the dynamics between the MAA and the AMS over the course of that first decade as each sought more clearly to define its place in the American mathematical landscape.

From left to right: Craig Stephenson, Peggy Kidwell, Stephanie Dick, Alma Steingart, Karen Parshall, and Della Dumbaugh.

"Constructing a Mathematical Laboratory": An Early History of Computer Algebra Systems

Stephanie A. Dick, Harvard University

This talk explores how new ways of doing mathematics were made possible by the advent of modern digital computing. Many computer-based techniques now exist for visualizing, exploring, and sometimes solving mathematical problems and they yield new perspectives and practices for mathematical research. Among them are programs like Mathlab, Maxima, and Sage that offer users environments for

exploring symbolic and algebraic mathematical systems. Programs like these were made possible at a crucial moment in the history of American computing. The first American computers were made to calculate - they were developed to perform numerical calculations faster and more efficiently than their human and machine predecessors. Many early computing practitioners believed, however, that the true power of computation would instead come from the manipulation of symbolic information by computers. This talk explores how this changing conception of computers led to the development of a new set of mathematical tools by focusing on the history of MACSYMA, among the earliest large-scale symbolic manipulation systems developed at MIT beginning in the mid-1960s.

Recent Trends in the History of American Mathematics: Rethinking Politics

Della Dumbaugh, University of Richmond

Politics in mathematics? In the past century, political decisions (seemingly) far outside the scope of mathematics have influenced the development of the discipline in small and large ways. These choices have not only had an impact on the lives of individual mathematicians but also on the larger community and the creation and diffusion of ideas. Using the lives of Hua Luo-geng and Emil Artin, this talk begins to explore the far-reaching consequences of political decisions made elsewhere on the American mathematical community.

Recent Trends in the History of American Mathematics – A Digital Divide?

Peggy Aldrich Kidwell, Smithsonian Institution

During the past thirty years, rapid change, particularly in electronic technologies associated with the computer, have transformed the mathematical instruments in common use, the development and diffusion of ideas in mathematics and the history of mathematics, and both the availability of and assumptions about the availability of historical resources. These changes challenge mathematicians, historians, and a more general public. Examples from recent work on nineteenth-century printed American discussions of the slide rule and on objects in the collections of the Smithsonian's National Museum of American History illustrate these trends.

From left to right: Joe Dauben, Jeff Chen, Duncan Melville, Annette Warner and Glen Van Brummelen.

Ancient Egyptian mathematics - questions vs. evidence

Annette Warner (Imhausen), Goethe Universität, Frankfurt, Germany

Research on ancient Egyptian mathematics is based on a small number of extant sources. As a consequence, many questions a modern researcher may have cannot be answered. The talk presents questions that cannot be tackled (and explains why) and gives an example of how the available sources may be used to further expand our knowledge of Egyptian mathematics.

New Directions in Mesopotamian Mathematics

Duncan J. Melville, St. Lawrence University

We give a brief survey of the current state of knowledge of Mesopotamian mathematics and a summary of recent directions and developments in research. We also highlight some of the most interesting new questions and methodologies including close analysis of shape and size of tablets, increased sensitivity to regional variations in mathematical practice, and development in cross-cultural studies.

Trigonometric Methods in Ancient Greece, Medieval Islam, and Early India

Glen Van Brummelen, Quest University, Canada

Astronomers in Greece, Islam and India all applied what from a modern point of view is the same trigonometry to solve astronomical problems. Nevertheless their conceptions of the functions, their modes of reasoning, their methods of calculation, and the problems they found important diverged substantially from one another. Choosing several sample texts, we examine what features of trigonometry the three cultures had in common and where they differed; and we consider how these differences affected the development of mathematics and astronomy.

Ancient Chinese Methods for Determining Square Roots, the Areas of Circles, and Volumes of Pyramids and Spheres

Joseph W. Dauben, City University of New York

The history of ancient Chinese mathematics offers numerous ingenious methods for approximating square roots and determining the areas of circles and volumes of pyramids and spheres, all of which involve aspects of the infinite in various ways. As early as bamboo texts from pre-Qin times to the edition of the comprehensive mathematical classic, the *Nine Chapters*, along with its third-century commentary by Liu Hui, methods evolved from early approximation techniques to arguments offering detailed proofs of the correctness of the results obtained. Diagrams and models served as guides, and precise algorithms could be checked to verify that the results obtained were indeed correct. How these elements of the mathematician's toolkit combined to provide rigorous arguments are discussed with examples drawn from bamboo texts like the *Shu* (Numbers) and *Suan shu shu* (Book of Numbers and Computations), as well as the *Nine Chapters*.

"Symbolic Algebra" in China

J.-P. Jeff Chen, St. Cloud State University, Minnesota

Some consider the Jesuit-introduced cossic algebra in the 1690s as the first appearance of symbolic algebra in China. Some credit Jean-François Foucquet for his 1711 manuscript for the emperor, the New Method of Algebra, in which Chinese characters were used to represent variables. The emperor ultimately dismissed this work as utter nonsense in spite of initial enthusiasm. Consequently, the content is not included in the mathematical compendium, The Essence of Numbers and Principles,

commissioned by the emperor and published in 1723. Although this work contains sections on operations of the “symbolic” expressions, historians of Chinese mathematics generally consider that symbolic algebra was not part of mathematical practice in China until after the translation of Elements of Algebra in 1859. Recently, we uncovered records of usage of “symbolic algebra” in two early 19th-century treatises. In one case, the text explicitly describes expanding squares of the sum of three terms, squares of fractional expressions, and cancellation of common factors on both sides of an equality. This paper examines the practices of “symbolic algebra” recorded in Chinese mathematical treatises, showing the need to reevaluate the uses and impact of symbolism in algebra in 19th-century China.