Report on the ICHM Co-Sponsored
Fifth Joint Conference of the British Society for the History of Mathematics and the Canadian Society for History and Philosophy of Mathematic

Trinity College, Dublin, Ireland; 15-17 July 2011

Adrian Rice

Following four previous successful conferences in Oxford, Toronto, Cambridge and Montreal, the fifth joint meeting of the British Society for the History of Mathematics and the Canadian Society for History and Philosophy of Mathematics was held from 15 to 17 July 2011 at Trinity College, Dublin. Around sixty participants were in attendance, from twelve different countries (the United Kingdom, Canada, the United States, Ireland, France, Germany, Italy, Portugal, Sweden, Japan, Egypt and Zimbabwe). A total of forty-four talks were presented, in order to accommodate which, parallel sessions of two or three talks were often held simultaneously throughout the duration of the meeting. Six plenary talks were also delivered.

A wide range of topics were covered during the course of the meeting on both the history and philosophy of mathematics. Ranging from Mesopotamian clay tablets to modern computing, subject matter included recent archival discoveries, the study of medieval Arabic manuscripts, initiatives to mobilize America's mathematicians during World War II, the history of elliptic curves, the role of women in mathematics, logic and foundational concerns, the history of – and the use of history in –

mathematics education, analyses of mathematical correspondence, the use of notation in mathematics and the use of mathematics in fiction. A full list of titles and abstracts of all presentations may be found below.

Shortly after the official end of the meeting, a local delegate kindly arranged a trip to Dublin's Dunsink Observatory, where William Rowan Hamilton lived and worked as Astronomer Royal of Ireland. Participants were also able to retrace Hamilton's steps by the Royal Canal, stopping at Brougham Bridge where, on 16 October 1843, he famously carved the fundamental equation for the quaternions: $i^2 = j^2 = k^2 = ijk = -1$.

The conference was organised by Tony Mann and Adrian Rice, and in addition to the sponsorship of the ICHM, the meeting was also sponsored by the publishers Taylor and Francis.

The following talks were presented at the conference:

The Evolution of Mathematics Teaching Practices, c.1770-1970

Amy Ackerberg-Hastings, University of Maryland University College

In 1993, Alison King drew a distinction between the "sage on the stage" and the "guide on the side" approaches to the teaching-learning process that has since become so commonplace that it has passed into popular culture. As awareness of the points raised by King and other educational theorists is reduced to a simplistic "sage bad, guide good" dichotomy, casual observers may conclude that the lecture style of teaching has always been utilized in every classroom. However, efforts to foster learning in mathematics classrooms have been more varied and more complex. This paper provided an overview of the techniques employed by mathematics teachers to facilitate and to measure learning both during and after formal class time. It also charted some of the major changes in these instructional processes, such as in the structure of textbooks and in the forms established for homework and assessment. The presenter also noted the historiographical challenges of determining what actually happened during daily routines in mathematics classrooms.

The Dual Arithmetic of Oliver Byrne: "A New Art which entirely supersedes the use of logarithms"

Janet Heine Barnett, Colorado State University – Pueblo

Prior to the invention of calculators and computers, logarithms constituted an important computational tool in scientific research and its practical applications. As computational needs of physics and engineering intensified, so did demand for reliable logarithmic tables. In its 1874 report, the Committee on Mathematical Tables of the British Association for the Advancement of Science (BAAS) catalogued and described a large number of the widely scattered logarithmic and other mathematical tables then in existence. Among these were several works by Mr. Oliver Byrne (*c*.1810–1875).

A self-educated engineer, self-proclaimed mathematician and prolific technical author of Irish heritage, Byrne is best remembered today – when he is remembered – for his strikingly beautiful 1847 edition of *The Elements of Euclid by Colours*. His other publications suggest that Byrne himself would rate his contributions to computational practice to be of even greater merit. He took particular pride in his invention of the "Dual Arithmetic," a system of computation which he initially proclaimed to "entirely supersede the use of logarithms," but which he eventually developed into an elaborate system of "dual logarithms." Concerning the latter, the 1874 BAAS Report (p. 81) observed that "in spite of the somewhat extravagant claims advanced by the author for his system, dual logarithms have found little favor as yet either from mathematicians or computers."

This talk considered Byrne's work on computation and his efforts to promote its adoption within the context of the increasingly stratified community of nineteenth-century British mathematical practitioners, and explored what Byrne's dual logarithms and their reception among "mathematicians and computers" reveal about that community and the role which individuals like Byrne played within it.

An American view of Europe. Oswald Veblen's correspondence with George Birkhoff during 1913–1914

June Barrow-Green, Open University

The Princeton mathematician Oswald Veblen spent the academic year 1913–1914 in Europe. His itinerary, which began in Scandinavia, concentrated on the mathematical centres of Göttingen, Berlin, and Paris. While he was overseas Veblen wrote long letters to his Harvard colleague and friend, George Birkhoff, describing the mathematicians he had met and the places he had visited. Veblen's letters provide a rare glimpse of an outsider's experience of the pre-War European mathematical community. This talk discussed the contents of these letters and reflected on what they tell us about the individuals and communities involved.

Shared Knowledge and Parallel Insights circa 1610 in Europe

Janet Beery, University of Redlands

Matthias Schemmel has written about the "shared knowledge" of Thomas Harriot (1560?–1621) and Galileo Galilei (1564–1642) and their resulting theories of projectile motion, and Kathleen Clark and Clemency Montelle have written about the "parallel insights" of John Napier (1550–1617) and Joost Burgi (1552–1632) on logarithms. This presentation illustrated the shared knowledge and parallel insights of Harriot and Galileo, Harriot and Johann Faulhaber (1580–1635), and Harriot and Bartholomaus Pitiscus (1561–1613) and Henry Briggs (1561–1630) about, respectively, models of projectile motion, formulas for generalized triangular numbers (binomial coefficients), and constant difference interpolation techniques.

The Deification of Newton in 1711

David R. Bellhouse, University of Western Ontario

By 1711 the mathematician William Jones had collected a number of Isaac Newton's manuscripts from the papers owned by John Collins. Jones published them in 1711, along with other works of Newton in a book entitled *Analysis per quantitatum series, fluxiones, ac differentias*: *cum enumeratione linearum tertii ordinis*. It was one of the small events in the priority controversy between Newton and Leibniz over the calculus. Inserted in the book, as well as on the title page, are a number of allegorical engravings, almost certainly commissioned by Jones. Some interpretations of the engravings are given. Similar to Halley's dedicatory poem to Newton in the *Principia Mathematica*, the engravings endow Newton with a god-like status. At the same time, the engravings also show some of Newton's activities as a mortal.

Henri Poincaré’s correspondence with mathematicians

Olivier Bruneau, Archives Poincaré - Université Nancy 2
(in collaboration with Scott Walter, Philippe Nabonnand and Amirouche Moktefi)

In addition to his leading and well-known mathematical achievements, Henri Poincaré contributed actively to the shaping of the modern mathematical community at the turn of the twentieth century. A look at his correspondence with the mathematicians of his time shows the importance of his acquaintances, and the influence he had within several academic institutions and miscellaneous scientific projects, both in France and abroad. The project of editing this correspondence is currently carried out with the aim of making it appear in 2012, thus coinciding with the centenary of Poincaré’s death.

Poincaré was an active letter-writer. Our records reckon about 2000 surviving letters exchanged with about 150 correspondents. Indeed, Poincaré maintained a regular and abundant correspondence with both well-known and less-known mathematicians, the first category including such authors as Cayley, Hermite, Cremona, Hilbert, Mittag-Leffler, Klein, etc. This correspondence covers a long period from 1880 to 1912 and deals with a wide range of mathematical disciplines. It provides interesting insights into the evolution of Poincaré’s mathematical investigations and interests, and gives some useful keys on the development of his views in the philosophy of mathematics. Also, an important part of the correspondence is related to institutional issues (life of scientific societies, academic prizes, etc), private matters (visits, family, etc.), or exchanges with journals’ editors (Craig, Tucker, etc.).

The aim of this presentation was to give an overview of Poincaré’s mathematical correspondence and to highlight its historical significance.

Various Observations on Euler’s E72

Bruce Burdick, Roger Williams University

Euler’s *Variae observationes circa series infinitas* (E72) considers a variety of infinite sums and products. His first theorem,

$$\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\frac{1}{26}+\frac{1}{31}+\frac{1}{35}+\cdots=1,$$

where the denominators are the whole numbers that are one less than a non-trivial power, he attributes to Goldbach, both for its statement and its proof. He then proceeds to prove other theorems in more or less the same manner.

The method of choice for Euler (and presumably Goldbach) involves subtracting infinite quantities from infinite quantities in a way that would no longer be acceptable as a mathematical demonstration. In a recent paper, Edward Sandifer and the speaker gave a modern proof of Euler’s Theorem 1. This talk was a follow-up to that paper, and showed that other theorems from E72 can be supplied with proofs that meet the present-day standards of rigor.

“Who do you think you are?” Thomas Penyngton Kirkman (1806-95)

Tony Crilly, Middlesex University

Thomas Kirkman, the noteworthy combinatorialist and geometer, was a Trinity College Dublin graduate in the 1830s. Details of his life tend to rely on a brief obituary written by his son and a short account of his life supplied by Alexander Macfarlane in his *Lectures on Ten British Mathematicians of the Nineteenth Century* (New York, 1916). This talk investigated what may be learned about the biography of this sharp-minded nineteenth-century mathematician using web-based techniques.

Further Adventures of the Rome 1594 Arabic Redaction of Euclid's *Elements*

Gregg De Young, The American University in Cairo, Egypt

Nearly two decades ago, R. Cassinet published "L'aventure de l'édition des Éléments d'Euclide en arabe par la Société Typographique Médicis vers 1594" (*Revue française d'histoire du livre*, 88-89 (1993), pp. 5-51), in which he described the typographic history of this Arabic version of the *Elements*. Although the title page of the book attributed the work to Naṣīr al-Dīn al-Ṭūsī, this claim cannot be correct since the original treatise was completed nearly a quarter century after his death. The still unidentified author is now commonly designated Pseudo-Ṭūsī. Al-Ṭūsī's own redaction of the *Elements*, although more influential historically, only appeared in print in the 19th century (Istanbul, 1801; Calcutta, 1824; Tehran, 1880). In my paper, I carry further Cassinet's investigations into this remarkable treatise which has, over the years, produced many faulty interpretations of the Arabic Euclidean tradition.

I have been able to locate the manuscript copy from which the Rome 1594 Arabic redaction was typeset. Two manuscripts of this Pseudo-Ṭūsī redaction are extant in the Biblioteca Medicea Laurenziana in Florence: Or. 20 and Or. 50. Examination of the manuscripts reveals that Or. 20 was the primary source from which the 1594 edition was typeset. It was also the primary source for a distinctive style of diagramming numbers that we find in books VII-IX of this printed edition. The identification of this manuscript source offers a unique opportunity to observe the process by which the printers converted the manuscript into printed form. Comparison of the manuscript source with the printed text now allows us to assess, at least in relation to this treatise, the veracity of long-standing allegations that there were numerous typesetting errors in the Medicean Press's Arabic publications.

The Rome 1594 printing was not a resounding commercial success either in Europe or in the Middle East, but it did undergo some remarkable reincarnations over the succeeding centuries. These include a reversion to manuscript form (Tehran, Sipahsaler 540, copied from the printed edition in 1101 / 1690 and itself ascribed to the wrong mathematician) and a late 19th-century republication using lithograph technology (Fez, 1293 / 1876)—reminding us that even in the history of mathematics, truth is often stranger than fiction and "the best-laid schemes o' mice an' men gang aft agley."

The Influence of Euler's Calculus Treatises

Joao Caramalho Domingues, Universidade do Minho, Portugal

Leonhard Euler (1701-1783) was arguably the most influential mathematician of the 18th century. He is generally regarded as one of the main exponents of the analytical tendency that gained predominance during that century. For instance, he published a purely analytical version of the differential and integral calculus in a set of treatises (an Introduction in 1748, a treatise on the differential calculus in 1755, and a treatise on the integral calculus in 1768-1770) that are commonly seen as a turning point in the liberation of calculus from geometry.

But how influential were these treatises, and when?

The three treatises enjoyed reprints before 1800; but it is striking that while their original editions are spread through 22 years, these first reprints are concentrated in the 10 years from 1787 to 1797. The first translations also appeared around this period, starting in 1786.

An analysis of common textbooks of the second half of the 18th century also suggests that the influence of Euler's calculus treatises grew considerably in the 1790s. In the 1760s and 1770s, only isolated passages show clear influence from Euler; while in the 1790s, important textbooks and treatises appeared that followed Euler's model in a coherent way. Pietro Paoli's *Elementi d'Algebra* (1794) and S.F. Lacroix's *Traité du Calcul différentiel et du Calcul integral* (1797-1800) are the most obvious examples. Naturally, this was reflected in the teaching practice at new institutions such as the École Polytechnique of Paris.

Possible explanations for this change include: a change in a more general cultural/philosophical framework, towards "analytical" methods; a natural time delay between the uses of concepts and methods by "research" mathematicians and by textbook authors; the great prestige of scientific *analytical* works such as Lagrange's *Méchanique Analitique* (1788).

Sources and Resources for the History of Mathematics: Contributions of David Eugene Smith

Eileen F. Donoghue, College of Staten Island

Perhaps while reading a classic 20th-century history of mathematics textbook, you have enjoyed the portraits of noted mathematicians that accompany the text. You may not have taken notice, however, of the small-print attributions alongside the images. Many of these attributions credit the David Eugene Smith Collection as their source. Smith's collections, gathered from across the globe over a forty-year academic career, encompass mathematically related manuscripts, correspondence, books, instruments, and images. Smith (1860-1944) was not content merely to accumulate such treasures; he wished to make these sources available to a wider audience for study. This paper examined Smith's efforts to develop broadly available resources that were based upon the primary sources in the collections now housed in Columbia University. The results of his efforts constitute a rich reservoir for those who research and teach the history of mathematics. Among Smith's contributions that were examined were the annotated and illustrated catalogue *Rara Arithmetica*, optical slides of historical problems and calculating machines, and a series of articles published in the American Mathematical Society *Bulletin* that dealt with correspondence written by historical figures and obtained by Smith.

Expect the Unexpected: Pioneers who Promoted Women in Mathematics and Science

Della Fenster, University of Richmond

How did a department store magnate and a playwright advance American mathematics and science—and women in these male dominated fields in particular? This talk explored the lives of Caroline Bamberger Fuld and Clare Booth Luce, examining the surprising range of personalities that influenced the development of mathematics and science in the middle third of the twentieth century.

Mathematics in the Scientific Revolution: Competing Approaches

Hardy Grant, York University

In the heady days that witnessed the rise of modern science the protagonists tended to share certain assumptions – the value of experiment, the sterility of scholasticism – which included the vital importance of mathematics. But in this last respect not everyone espoused the path taken so successfully by Galileo and by Newton. Two other visions of the role of mathematics, both reaching the 17th century from an ostensibly improbable source, and both entirely plausible, won support. One confrontation of opposing views was especially dramatic, and remains famous.

Conformal Mapping in the 18th Century

George Heine, Pueblo, Colorado

Before Lambert's 1772 publication "On the Composition of Terrestrial and Celestial Maps", cartographers and astronomers knew of two mappings from a spherical to a plane surface with the attractive property of preserving angles: the stereographic projection, used since Ptolemy's time to construct astrolabes, and the Mercator map, by that time in universal use for the construction of nautical charts. Lambert presented both a mathematical formulation of the equal-angle property, and two previously unknown families of mappings satisfying the condition. Euler in 1777 formulated the property more elegantly and gave a more general, though still incomplete, solution. Lagrange in 1779 was able to give a complete solution for the case in which all meridians and parallels on the sphere are mapped to lines and circular arcs on the plane.

Although Lagrange's work was influential in the nineteenth-century development of conformal mapping, Euler's paper seems to have remained more obscure. An 1853 (posthumous) paper by Jacobi on conformal mappings of the ellipsoid mentions both Lambert and Lagrange, but not Euler.

For the general problem of finding all angle-preserving mappings, we compare the formulation and solution methods of Lambert, Euler, and Lagrange and assess the contribution of each both to the practical mapmaker and to mathematics.

Discovering History by Dialogue

Gavin Hitchcock, University of Zimbabwe/University of Stellenbosch

Can theatrical presentation of history of mathematics, if based closely on primary sources, yield deeper insights, even for the historian? Does the casting of concept-formation and theory-making into contextual dialogic form help to recover something of the original motivation and mental pathways? Can an appropriate combination of historical authenticity and artistic license contribute to teaching mathematics and history of mathematics? In this talk I attempted to answer these questions positively by examples (with brief enactment of excerpts) selected from the following:

Chief Scribe & Junior Scribe – the making of a mathematical papyrus; Scribe & Pupil – the world of a Mesopotamian scribal school; Philippus & Euclid – motivating a learner in Plato's Academy; Chinese beaurocrat & supplicant – the origin of old mathematical problems; Brahmagupta & Bhaskara I, Galileo & Descartes – the fruitful interaction of schools of thought; Bhaskara II & Lilavati, Pacioli & Pupil (in the famous portrait) – pedagogical influences on texts; 21st-century mathematics teacher & old mathematician (e.g. Zhu Shijie, Bombelli) – deriving insights from shock, puzzlement and cross-purpose; Girard & Stevin, Leibniz & Huygens, Leibniz & De Moivre, Euler & Bernoulli – facing up to the 'imaginaries'; Hamilton and De Morgan – the sheer novelty of imagining 'imaginaries'.

Is the Inferential Conception of Applied Mathematics Complete?

Molly Kao, University of Western Ontario

There have been several accounts of the role of applied mathematics that stress the role of structural maps between mathematics and the world. Otávio Bueno and Mark Colyvan present an account called the "inferential conception" of applied mathematics which consists of three stages: immersion of a physical setup into mathematical structures, derivations within the mathematics, and interpretation of mathematical results. The most important step with respect to explanations of physical phenomena is at the level of interpretation; that is, what matters in explanation is that we can make inferences about the physical world by assigning physical meaning to the mathematics. I argue that this conception is incomplete because explanations do not always follow the pattern suggested.

Robert Batterman has argued that often, the types of things we are trying to explain in science are not individual phenomena, but physical regularities, and the robustness such patterns demonstrate under certain perturbations. He argues that the renormalization group provides a mathematical explanation for why some classes of substances exhibit identical behaviour during phase transitions. I present a detailed case for why such an explanation cannot be made to conform to the structure suggested by the inferential conception of applied mathematics. Although it may be possible to represent different systems in a universality class in terms of mathematical equations, and interpreting those equations can yield *some* information about the physical world, it is necessary to appeal to the existence of a mathematical procedure to explain the phenomenon in question. Since this is a procedure applied to the equations that represent the physical setup and not a representation of a physical setup itself, it is impossible to 'interpret' this mathematical structure so that it says something about the world. Given that proponents of the inferential conception are concerned to accommodate Batterman-type explanations, I argue that it is problematic that the renormalization group explanation does not adhere to the structure they have proposed.

An Origin History of Computer Science in Japan: Eiichi Goto and Parametron Computer

Shunshi Koyama, Aoyama Gakuin Women's Junior College, Japan

This paper described the historical process of the formation of the discipline of computer science in Japan and discusses the role of the physicist Eiichi Goto. He invented a unique computer component (the "parametron") and contributed to the development of parametron computers in the 1950s, which shaped the early history of computers in Japan. Goto researched physical and mathematical aspect of computer, later directing the information science laboratory of RIKEN and taking a central part in establishing the Department of Information Science at the University of Tokyo.

What is a neo-Carnapian foundation for mathematics, and why do we need one?

Gregory Lavers, Concordia University

Carnap's philosophical position on the foundations of mathematics, if defensible, constitutes a solution to many of the most important problems in the area. In particular, his position gives quite nice answers to questions concerning the ontology and epistemology of mathematics. However, Carnap's views on the subject have been attacked from many angles. I argue that these criticisms point to a point of weakness in Carnap's philosophy. This weakness is his overly liberal notion of an *explication*. In essence, Carnap held that in giving an explication we are free to make any stipulations we want, and that it will be up to pragmatic considerations to decide between proposals. As central as this notion is to his later philosophy, a less liberal view of explication is fully compatible with Carnap's pluralism, and would insulate Carnap's position from many criticisms. Yet even with a more constrained view of explication, a Carnapian answer to problems concerning the ontology and epistemology of mathematics can still be given.

Seeing How It Goes: The Peculiar Role of Writing in Mathematical Reasoning

Danielle Macbeth, Haverford College

Throughout its long history, mathematics has involved the use of systems of written signs, most notably, diagrams in Euclidean geometry and formulae in the symbolic language of arithmetic and algebra in the mathematics of Descartes, Euler, and others. Such systems of signs do not merely record results; instead they serve to *embody* chains of mathematical reasoning. Having clarified what this means, I argue that properly understood Frege's *Begriffsschrift* or concept-script similarly enables one to write mathematical reasoning, to put it before one's eyes in a way that is simply impossible either in natural language or in more familiar logical notations. Much as a demonstration in Euclid or in early modern algebra does, a proof in Frege's concept-script *shows* how it goes.

From Sylvia Plath to Bad Sex: Uses of Mathematics in Fiction

Tony Mann, University of Greenwich

Many novelists have used mathematical structures and mathematics and mathematicians feature in many interesting works of fiction. This talk looked at a variety of ways in which novelists have used mathematics and its practitioners, and at how fiction has been used to present mathematics.

Axiomatizing Homotopy Theory:
Lifting mathematical concepts via the axiomatic method

Jean-Pierre Marquis, Université de Montréal

The axiomatization of homotopy theory, a process which is still undergoing important developments, is a fascinating philosophical case study. It can be said that the first attempts to provide an axiomatization of homotopy theory, following the suggestions given by Eilenberg and Steenrod in their book on the foundations of algebraic topology, were dead ends. Attempts made by Kuranishi and Milnor were, in a sense, too good: they provided a categorical — in the logical sense of that expression — axiomatization of homotopy theory. Although Kan obtained a similar result, his approach also opened the way to a new, more abstract framework. It took approximately another ten years before Quillen came along and proposed a completely different standpoint which still serves as the paradigm in the field. In this paper, we contrasted the various axiomatizations that were proposed and underlined what we take to be significant philosophical morals that can be derived from the history, in particular how mathematicians moved from a categorical axiomatization — that is a unique interpretation up to isomorphism — to an axiomatization that opened up the way to interpretations in various and conceptually different domains.

Locating Mathematics in the Scribal Culture

Duncan Melville, St. Lawrence University

Recent research has established a fairly firm picture of the Old Babylonian mathematical curriculum. Less is known, however, about the uses of the more advanced mathematics outside of the school. Using the example of a scholarly tablet collection from Meturan, I argue that mathematics had a role, albeit modest, in the wider intellectual culture of the time.

On the Intellectual Heritage of Henri Poincaré

Madeline Muntersbjorn, University of Toledo

Poincaré is portrayed often as a follower of Kant. However, Poincaré was also a post-Darwinian whose metaphysics was influenced by the theory of evolution. Like Mach, Poincaré valued the economy of thought that systems of signification made possible. The influence of Darwin and Mach escapes the notice of scholars who draw too sharp a line between the philosophy of mathematics, as a single abstract foundational inquiry, and the psychology of mathematics, as assorted concrete problem-solving practices. For Poincaré, intuition is not the Kantian dyad of space and time restricted to time because space is no longer an option after the advent of non-Euclidean geometries. Instead, Poincaré sees "mathematical intuition" as a heterogeneous assortment of inherited faculties. We cultivate a more complete account of Poincaré's heritage, and develop insight into the growth of mathematics, by attending to the different representational systems employed during distinct phases of mathematical discovery over time.

Did Hamilton and Jacobi construct the Hamilton-Jacobi theory as we know it today?

Michiyo Nakane, Rikkyo University, Japan

In the 1910s, physicists constructing quantum theory realized the importance of Hamilton-Jacobi theory. Because W. R. Hamilton derived his dynamical theory while working on geometrical optics in the 1830s, it is tempting to think that it would have applied to the quantum theory in its original form. It would not. The theory's development, begun by Jacobi, who added his name to it in the 1840s, continued throughout the remainder of the nineteenth century, during which time new ideas arose that, added to the original Hamilton-Jacobi theory, made it applicable to quantum theory. This paper identified these ideas and showed how they were developed in the field of celestial dynamics.

Mobilizing Mathematics: The American Mathematical Societies and World War II

Karen Hunger Parshall, University of Virginia

Even before the United States entered into World War II in 1941, the leaders of the American Mathematical Society and the Mathematical Association of America had formed a War Preparedness Committee in anticipation of what was viewed as the eventuality of the U.S.'s involvement in the conflict. They recognized not only that mathematicians could contribute key technical expertise to the war effort but also that the community's ultimate position in the postwar era would very much depend upon those contributions. This talk examined the initiatives spearheaded by especially AMS wartime Presidents—Griffith Evans, Marston Morse, and Marshall Stone—to mobilize America's mathematicians.

The Emigration of British Arithmetics to America

Andrew Perry, Springfield College

Eighteenth-century English arithmetics varied greatly in character and quality. On the one hand, certain books were useful, well-written and coherent texts. Among these one might include Edward Hatton's *Intire System of Arithmetic* (1721), William Webster's *Arithmetic in Epitome* (1740), and John Hill's *Arithmetic Both in the Theory and Practice* (1765). Others such as *Cocker's Arithmetick* (1678; used by Ben Franklin in 1722) and Thomas Dilworth's *Schoolmasters Assistant* (London, 1744; Philadelphia, 1769) were less coherent and tended strongly toward demanding mindless memorization. Interestingly, the latter class of texts was much more widely exported to the American colonies and reprinted there. In this presentation, we compared the books that made it across the Atlantic with those that did not.

Following Your Gut and Following the Rules: The Function of Intuition and Algorithm in 18th-Century Analysis

Bruce Petrie, University of Toronto

The style of eighteenth-century analysis practiced by Euler, Lagrange, and Lambert was unlike the nineteenth-century analysis exemplified by the work of Cauchy and Weierstrass. Eighteenth-century analysis has been characterized as algorithmic and intuitive but has also been faulted for its lack of rigor and over confidence in algebraic or formal methods. Yet mathematicians such as Euler are venerated for possessing remarkable intuition. Building on this secondary literature, the author drew upon original writings of Euler and Lambert to investigate how intuition guided the appropriate use of algebraic methods in the practice of eighteenth-century analysis.

A Tale of Two Surfaces, or Why Ellipses Are Not Elliptic Curves

Adrian Rice, Randolph-Macon College

Elliptic curves are a fascinating area of algebraic geometry with important connections to number theory, topology, and complex analysis, as illustrated by their centrality to the Mordell-Weil theorem, the Birch and Swinnerton-Dyer conjecture, and most famously to Andrew Wiles's proof of Fermat's Last Theorem. Yet they only really came to the fore in mathematics after a groundbreaking paper by Poincaré in 1901. As their current ubiquity in mathematics suggests, elliptic curves have a long and fascinating history stretching back many centuries. This paper presented a survey of key points in their development, via elliptic integrals and functions, and closed with an explanation of why no elliptically-shaped planar curved line may ever be called an elliptic curve.

Polish Logic from Warsaw to Dublin: The Life and Work of Jan Łukasiewicz

V. Frederick Rickey, U.S. Military Academy

A few years after earning his Ph.D. in Lwów under Twardowski, Jan Łukasiewicz (1878-1956) joined the faculty of the newly reopened University of Warsaw where he became, along with Leśniewski and his student Tarski, one of the founders of the Warsaw School of Logic. He did seminal research in many-valued logics, propositional calculi, modal logic, and the history of logic, especially concerning Aristotle's syllogistic. He left Warsaw toward the end of World War II and found a new home at the Royal Irish Academy in Dublin where he continued his creative work.

Two Theorems and the Manifold Nature of Probability

Paolo Rocchi, IBM and LUISS University, Rome
Leonida Gianfagna, IBM, Rome

Several statisticians are inclined to use Bayesian or classical methods according to the problem to tackle. The popularity of the dualist interpretation of probability—frequentist and subjective—is growing. Various eminent thinkers argue upon the dual view of probability, however the philosophical arguments seem unable to show in a definitive manner how the aleatory and the epistemological side of the probability can coexist.

This paper was an attempt to justify the dualist position through an analytical method. It defines the random event A_n that has n trials and A_1 that has only one trial in the physical reality. Two theorems illustrate the odd properties of $P(A_n)$ and $P(A_1)$ which do not overlap because the theorems refer to distinct hypotheses.

A Forgotten Booklet by Goldbach Now Revealed

Staffan Rodhe, Uppsala University, Sweden

In my research on 18th-century Swedish mathematics I came across a remark in Anders Gabriel Duhre's textbook on geometry (1721) that said that in Stockholm, in 1719, Christian Goldbach published a thesis on sums of series. In 1884 the Swedish historian of mathematics Gustaf Eneström wrote a short notice in *Bibliotheca Mathematica* saying that he had found a booklet with the text. After that it was lost again until 2010 when it was rediscovered in a library in Linköping. None of Goldbach's biographers has ever mentioned the Stockholm thesis. Goldbach himself mentioned it in a letter to Daniel Bernoulli in 1723 and a similar text is reprinted in an article in *Acta Eruditorum* (1720). The text describes five cases of five types of series. In Duhre's book there is a Swedish annotated translation of the first four cases. As a fifth case Duhre gives another of Goldbach's methods on series. My lecture dealt with some of these cases and more about the history of the booklet.

Definition by Induction in Modern Algebra

George Rousseau, University of Leicester

Dedekind gave the first rigorous treatment of definition by induction (recursive definition) in his *Was sind und was sollen die Zahlen* ("The Nature and Meaning of Numbers") in 1888. His main result in this connection is as follows.

Given a set A, an element a of A and a mapping g of A into A, there exists a unique mapping $f : \mathbb{N} \rightarrow A$ such that

(i) $f(0) = a$,

(ii) $f(Sn) = g(f(n))$ for all n.

(Here S is the successor function.)

Peano's treatment of the natural numbers, *Arithmetices principia, novo methodo exposita* (1899), does not prove, but merely assumes, the validity of definition by induction. Dedekind's theory was well understood by the set theorists and mathematical logicians, but not in all cases, it seems, by the writers of textbooks on modern algebra. In particular, the books by van der Waerden (*Moderne Algebra*) and Birkhoff & Mac Lane (*A Survey of Modern Algebra*) display considerable confusion in this regard. [Landau's *Grundlagen der Analysis* (1930), on the other hand, gives a correct treatment of the theory of natural numbers (as well as integral, rational, real and complex numbers).]

Algebra texts since the above-mentioned have very often assumed the natural numbers as "known", with a reference to Landau for "the details". This is an unnecessary omission and one which is somewhat surprising, given that Dedekind, one of the principal founders of modern algebra, employed in his theory of natural numbers essentially the same tools as he used elsewhere in relation to groups, fields, etc. Thus the 'chain' of a set, E, of natural numbers is the smallest set containing E and closed under successor, while in a group the subgroup 'generated' by a given subset, E, is the smallest set containing *E* and closed under the group operations (composition and inversion). In the first case we have to do with a unary operation; in the latter one binary and one unary.

If binary operations are preferred to unary, then the (additive monoid of the) natural numbers may be characterised as a cancellative cyclic monoid in which not every element has an inverse. The Dedekind-Peano theory of natural numbers then becomes the beginning part of the theory of monoids and lies at the basis of modern algebra.

In my talk we looked at the way inductive definition has been dealt with by various authors, starting with H. G. Grassmann (1861), continuing with the authors mentioned above and finishing with Jacobson's *Basic Algebra, I* (1974, 1985); on the way, we contrasted Dedekind's rigorous proof with the fallacious "justification" often proffered.

Pasch's Ideas for a Renewal of Logic

Dirk Schlimm, McGill University

The history of modern logic is often conceived as a continuous development originating with Frege (1879). However, a closer look at historical developments outside of the Frege–Hilbert–Russell tradition reveal that the path to modern logic was not as smooth and unproblematic as it is often presented. Moritz Pasch's views on logic are a case in point.

In his *Lectures on Newer Geometry* (1882) Pasch clearly formulated the demand that deductions must be independent from the meanings of the non-logical terms involved, and he continued to elaborate on this view throughout the rest of his life. His growing concern for the justification of mathematical arguments led him to investigate the notion of consistency, to distinguish non-logical from logical components of expressions, and to the view that "is part of the essence of pure deduction that every proof can be 'atomized', i.e., resolved into steps of certain kinds, or that it consists of a single such step" (1917). In this talk I presented some of Pasch's reflections on the ideals of mathematical rigor, which he hoped would lead to nothing less than a "renewal of logic" (1918).

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Constructivism, obscurities, conflicts between criteria in Descartes' *Géométrie*. Mathematical and philosophical aspects

Michel Serfati, Université Paris VII-Denis Diderot

In a letter written at the end of December 1637, only a few months after the publication of the *Essays*, Descartes explained to Mersenne that it is by his *Geometry*, much more than by the *Dioptrics* and the *Meteors*, that he considered his method as being "demonstrated". Much later, in his Foreword in the

French edition (1647) of his *Principles of Philosophy*, he also said that, by writing the *Geometry* ten years previously, he had intended “to encourage in this way all men in search of truth”; the philosophical importance of this mathematical text cannot therefore be overestimated in the very conceptions of the author himself.

Given these considerations, this presentation was devoted to some fragmentary philosophical aspects of the structure of the *Geometry*. I chose to consider the issue under two perspectives which are rather distant from one another and may even be perceived as antagonistic, namely the constructivism on the one hand, and the (relative) incoherence – or absence of order – on the other.

Johannes Lohne – The forgotten Norwegian re-discoverer of Thomas Harriot

Reinhard Siegmund-Schultze, University of Agder, Norway

This talk was devoted to the memory of maybe the most important historian of physics and mathematics that Norway has ever produced, Johannes Lohne (1908-1993). He was the first to find out that the sine-law of light refraction was already known to and experimentally proven by Thomas Harriot (1560-1621), 20 years before Snellius and Descartes. In addition to being a pioneer in several physical domains, Harriot also excelled in algebra, although his results remained unpublished and were partly traced by Lohne as well. Lohne had taken his masters degree in physics at university in Oslo in 1932. He was a teacher at the Flekkefjord high-school during the 1950s and 1970s. His discovery in 1958 of Harriot’s copy (with notes) of Alhazen’s *Optics* in the Oslo university library (the copy is today unfortunately lost) got Lohne started in his research on Harriot. In the years that followed he went to England almost every summer to investigate Harriot’s papers. The fact that Lohne never obtained an academic position in Norway commensurate with his fame abroad, and that no obituary on him has appeared, is at least partly due to political circumstances. He went to the Russian front in German service and was sentenced to seven years forced labor after the war, of which he had to serve three and a half years. Johannes Lohne was the brother of Andreas Lohne/Lone (1909-1981), a central figure in the Norwegian resistance against the German occupants. The last fact underlines that political conflicts during the German occupation often went through Norwegian families and that posthumous academic reputation is shaped by many, often quite convoluted factors.

Emmy Noether and Rosalind Franklin

Charlotte K. Simmons and John F. Barthell, University of Central Oklahoma

Emmy Noether (1882-1935) was a German mathematician known for her groundbreaking contributions to pure mathematics and theoretical physics. Rosalind Franklin (1920-1958) was a biophysicist and a key contributor to the discovery of the structure of the DNA molecule that was formally elucidated in 1953 by James Watson and Francis Crick.

While neither of these women ever received the Nobel Prize or its equivalent (the Fields Medal), both played a crucial role in research for which others received the Nobel Prize. Both are becoming historical icons akin to the mathematician Hypatia; unfortunately, like her, their gender has often been superimposed on their contributions as a scientist or mathematician in the telling of their stories. In Hermann Weyl’s eulogy of Noether, after proclaiming that she was “a great mathematician, the greatest,” he felt the need to add, “No one would contend that the Graces had stood by her cradle.”

In this talk, we explored parallels between the lives of Franklin and Noether as a means to understand a common condition among women during the period of World War II and its immediate aftermath. These included: 1) the ability to find employment in their chosen disciplines; 2) their ability to achieve professional autonomy from men within their disciplines; and 3) the means by which their historical roles were defined by men during their professional careers. We also explored how their response to these circumstances influenced their professional and historical outcomes.

The *dramatis personae* of the *Spherics* of Theodosios

Robert Thomas, University of Manitoba

Philip Kitcher has discussed mathematics philosophically in terms of what a superhuman or ideal agent can do. Brian Rotman has discussed mathematical discourse semiotically in terms of what three actors say and do. David Wells understands aspects of mathematics as analogous to abstract games like chess and go, which require competing players. These claims were tested against the Hellenistic treatise Theodosios's *Spherics* and the players identified. As the title indicates, players as in a play not as in a game.

Using the History of Mathematics in a Basic Statistics Course

Patrick Touhey, Misericordia University

The typical student in a basic statistics course is usually deficient in rudimentary mathematical skills. Simple algebraic formulas for some standard descriptive statistics are often beyond the comprehension of these beginning students. In this talk we extended an idea of Sir Edmond Halley contained in his paper of 1693, "An Estimate of the Degrees of Mortality of Mankind…" Then utilizing some simple geometrical algebra we clarified understanding of a number of elementary concepts, e.g., variance, standard deviation. This then helped to make proofs of both Chebyshev's Theorem and Markov's Inequality almost transparently obvious.

A Survey of the Mathematical Sciences in Medieval Islam, 1995 to the Present

Glen Van Brummelen, Quest University

The past fifteen years have seen a marked growth in research in the mathematical sciences in medieval Islam. Studies in traditional mathematical areas such as geometry, number theory, and algebra continue unabated. However, growth has been more pronounced in disciplines that overlap with mathematics but are not immediately consonant with modern interests, such as astrology. In addition, contextualizing studies that emphasize localization of knowledge and scientific patronage are deepening our awareness of the social and political context of the medieval exact sciences. In this talk we surveyed the most prominent achievements since 1995.

Two Sorts of Explanation in 20th-Century Foundational Work

Susan Vineberg, Wayne State University

This paper argued that considerable work in the foundations of mathematics can be understood as fitting into, and motivated by, one of two different explanatory projects. I claimed that, as in science where explanation falls into two broad categories, there are two general types of mathematical explanation, paralleling explanation in science. The first (unification view) takes explanation to consist in subsuming facts under a few basic principles. The second type is more closely aligned with causal explanation, and involves displaying the minimal conditions required for various conditions to obtain. Whereas the first kind of explanation occurs in the development of ZFC and the search for additional axioms, the second is dominant in proof theory. Beyond locating examples that distinguish these two kinds of explanation in mathematics, I argued that invoking explanation yields a naturalistic account of various developments in foundations, which fares better than other philosophical views of mathematics.

Cayley, Harley, and the Quintic

Steven Weintraub, Lehigh University

The Lehigh University library has a collection of 40 letters written from Arthur Cayley to Robert Harley between 1859 and 1863, and an unpublished manuscript "A Memoir on the Quintic Equation" that Cayley was working on at the time of his death in 1895. (This material is available at http://digital.lib.lehigh.edu/remain/con/cayley.html, as part of the library's digital archive.) In this talk, largely based on this material, we gave insights into the working relationship between Cayley and Harley during those four years, especially as they attacked the quintic from an invariant-theory point of view, and into Cayley's lifelong interest in the quintic.

Inventing Rigor in the Dialogue of Early Modern Mathematics

Travis D. Williams, University of Rhode Island

In sixteenth-century Britain, mathematics acquired its first native expression in the form of English-language texts cast as dialogues between a fictional master and scholar. Intended for non-Latinate readers who wished to teach mathematics to themselves, these dialogues also initiated an improvement in the standards of both theoretical and practical rigor in mathematics. This paper examined the paradoxical interaction of dialogue, a form historically associated with conversation and probable knowledge, with mathematical rigor, a concept associated with the monologic dictates of certain knowledge. Self-instruction rectified the insufficient rigor of earlier monologic treatises by creating a *psychomachia* in the reader, who teaches himself the necessity of rigor, rather than having it imposed by an outside authority. Ultimately, however, dialogue could not sustain the needs of newly developed canons of rigor, and mathematics returned to monologic forms. Though insufficient in the long term, dialogue was a necessary stage in the evolution of modern mathematics.

Thomas Hirst: A Victorian Mathematician in Europe

Robin Wilson, Open University and Pembroke College, Oxford

The Victorian mathematician Thomas Archer Hirst did not follow the usual Oxbridge educational route, but gained his Ph.D. at the University of Marburg (Germany). Following this he spent a year each in Berlin, Paris and Italy. His diaries include his personal reminiscences of Gauss, Dirichlet, Steiner, Liouville, Charles, Cremona and many others.

Huygens' Five Problems and the History of Probability

Maria Zack and Megan Ford, Point Loma Nazarene University

In 1656, Christian Huygens wrote a brief treatise on probability called *Van Rekeningh in Speelen van Gluck* which he sent to his friend and former teacher, Frans van Schooten, who was a professor of mathematics at Leyden University. This short paper was the result of interactions that Huygens had with a group of French mathematicians that included Giles Roberval, Claude Mylon, Pierre de Carcavi and Blaise Pascal. Van Schooten suggested the Huygens have the paper translated into Latin for publication. One year later, after correspondence with Pierre de Fermat and others, Huygens expanded his treatise and published it as *De Ratiociniis in Ludo Aleae* (1657).

De Ratiociniis in Ludo Aleae contains what have become known as "Huygens' five problems." This collection of five problems provides a useful lens through which to view the history of probability. Mathematicians such as Jakob Bernoulli, Nicholas Bernoulli, John Arbuthnot and Abraham de Moivre worked directly on solutions to the problems. Problem two poses a particularly interesting challenge because it can be interpreted in multiple ways. This presentation examined early approaches to probabilistic computations by comparing the work of Huygens, J. Bernoulli and de Moivre on the five problems with a particular emphasis on problem two.