Final Report for ICHM-Sponsored Symposium 035

The ICHM-sponsored symposium 035 for ICHST-27, "Creation and Dissemination of Mathematical Knowledge in Ancient China: Peoples, Places, Exchanges, and Circulation," was co-organized by Jian-Ping Jeff CHEN, Joseph W. Dauben, Shuchun GUO and Dahai ZOU. The symposium focused on the earliest recorded mathematical works as yet known in China, works preserved on bamboo strips and wooden boards dating as far back as the late fourth century BCE, i.e. Warring States to early Han times. By the first century BCE the first great work devoted to mathematics had been compiled, the *Jiuzhang suanshu* 九章筭術 (Nine Chapters on the Art of Mathematics), which survives with two layers of commentaries, one by the third-century scholar Liu Hui and the other by Li Chunfeng and his associates in the seventh century. Later commentaries by Jia Xian and Yang Hui of the Song dynasty on this classic as well as its earlier commentaries further document the evolution of mathematics in China. Among questions this symposium addressed: What roles did Chinese mathematics in the long tradition of the *Nine Chapters* play in precipitating the remarkable mathematical achievements during the Song dynasty, including the development of algebraic techniques for solving higher-degree polynomial equations and simultaneous equations in as many as four unknowns? The symposium also explored the nature and extent of that evolution with a focus on peoples, places, exchanges, and circulation of mathematical knowledge. In particular, symposium 035 considered how mathematical knowledge was created, taught, transmitted, and transformed in the process.

Although sixteen speakers were initially expected to participate directly in the symposium, either in-person or virtually, due to the requirement announced only after registration that virtual participants must have submitted a prerecorded MP4 version of their presentations, in the end only six participants were on hand in person to participate at the New Zealand Congress, while two presentations were made by virtual participants. The symposium was scheduled for two days, July 1 and 2, and presentations were made as follows:

July 1, 13:30-15:00 NZST (9:30-11:00 CST): **Chair: YING Jia-Ming.**
Presenters: ZOU Dahai, TAN Jingnan, ZHANG Wen

July 1, 15:30-17:00 NZST (11:30-13:00 CST): **Chair: Zhang Wen**
Presenters: CHEN Jianping, YING Jia-Ming

July 2, 13:30-15:00 NZST (9:30-11:00 CST): **Chair: TAN Jingnan**
Presenters: TAMURA Makoto, HORNG Wann-Sheng, GUO Shirong

The presentations consisted of the following, made either in-person or virtually via Zoom:

Dahai ZOU (Research Professor, Institute for the History of Natural Sciences, Chinese Academy of Sciences (CAS), Beijing), in person: "An Overview of the Mathematical Documents on the Bamboo and Wooden Slips of Qin Dynasty Preserved at Peking University: 北京大学藏秦代简牍数学文献概述."

> The mathematical documents on the bamboo and wooden slips of Qin Dynasty preserved at Peking University came from the tomb of a local official in Nanjun (南郡) of the Qin Dynasty in the late Qin Shihuang period. They include 2 *Tianshu* (Field Books) that only have data but no algorithms, 3 mathematical books that have algorithms and problems, as well as a *jiujiu* multiplication table wooden slip. Their contents include discussions on mathematical views, multiplication tables, area algorithms, volume algorithms, tax algorithms, conversions of different types of grains, measurement systems, algorithms for military problems, excess and deficit algorithms, long-distance surveying methods, and other miscellaneous problems. We can find algorithms similar to those in the chapters Fangtian, Sumi, Cuifen, Shaoguang, Shanggong, Junshu, Yingbuzu, and Gougu of the *Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算术). The three mathematical books have different forms. Among them, the *Mathematical Treatise*: First Manuscript contains three chapters with titles, indicating that the chapter form of the *Nine Chapters on Mathematical Procedures*, which is divided into 9 chapters, was probably inherited from its ancestral version in pre-Qin period, as recorded by Liu Hui. These mathematical documents provide a support for Liu Hui's description of the process of the compilation of the *Nine Chapters on Mathematical Procedures*, and also provide sources for our understanding of the society, economy and measurement system of the Qin State during the Warring States Period and the Qin Dynasty.

Jingnan TAN (Jianghan University, Wuhan), in person: "New Materials and the Early Chinese Mathematical System"

> The study of Qin-Han (221 BCE–220 CE) mathematics was once mostly limited to *The Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術), but all that has changed in the last three decades with the successive discovery of mathematical manuscripts in Zhangjiashan 張家山 tomb 247, Shuihudi 睡虎地 tomb 77 and in the Yuelu Academy 岳麓書院 and Peking University 北京大學 collections. In addition to furnishing us with new, never-before-seen materials and shedding light on the origins of those incorporated into later classics like *The Nine Chapters*, these manuscripts document a variety in the language and formulation of common procedures that stands in stark contrast with the later mathematical canon. For example, the case of 'Litian' 里田 land measurement problems in the aforementioned sources and a peculiar variation thereupon in the Peking University *Suanshu* 算書 might be a mnemonic (*kuketsu* 口訣). It suggests what we see in these manuscripts might be linked to oral practices and basic training. Another example is that π in Qin and Han mathematics is usually 3, but 16/5 appears in the mathematical manuscript of Shuihudi 睡虎地. And the expression is different, which indicates that there may have been π algorithms in early arithmetic that we do not know.

Wen ZHANG (Institute for the History of Natural Sciences (CAS), Beijing), in person: "A study on the organization and research of *Shushu Jiuzhang* by scholars of the Qianjia School"

> The *Shushu Jiuzhang* is an important mathematical work written by Qin Jiushao, a mathematician of the Southern Song Dynasty. It covers multiple fields such as astronomy, calendar, engineering, and commerce, and is one of the representative works of classical Chinese mathematics during

the peak of the Song and Yuan dynasties. During the Qing Dynasty, scholars from the Qianjia School systematically organized and studied the *Shushu Jiuzhang*. This work was not only an important part of organizing traditional mathematical classics, but also an inseparable component of Qing Dynasty academia. Scholars of the Qianjia School's research on the *Shushu Jiuzhang* is not limited to text organization, but also delves deeper into the interpretation and verification of mathematical content. The organization and study of the *Shushu Jiuzhang* by scholars of the Qianjia School have provided valuable research materials and references for future scholars, and are of great significance for understanding the development and achievements of ancient Chinese mathematics. At the same time, this study helps to reveal the unique style of research on the *Shushu Jiuzhang* in the academic environment of the Qing Dynasty, and provides useful academic references and inspirations for the subsequent study of the *Shushu Jiuzhang* and even broader research on the history of ancient Chinese mathematics, promoting the deepening and development of research in related fields.

Jianping Jeff CHEN (Professor, St. Cloud State University, MN), virtually, via Zoom: "Place Values and Geometric Reasoning in Root extraction procedure in the Song commentary to the *Nine Chapters of Mathematical Art"*

Before the 15th century, the mathematical treatises in China treating square root extraction did not describe explicitly how each digit of the root was estimated and failed to pinpoint how the digit is ascertained, not the method in the *Mathematical Procedures in the Nine Chapters* (九章算術 *Jiuzhang suanshu*), not in the Liu Hui's or Li Chunfeng's commentaries, nor can they be found in Yang Hui's discussions. Moreover, in Yang Hui's comments, the valuable concrete, transitional, numerical values that appear during the course of extraction openly contradict the place values bestowed on them by their positions on the counting board. In our studies, we discovered in Yang Hui's/Jia Xian's comments an additional way of keeping tract of the numerical values of the "divisors" throughout the course of the calculation even though that they are being moved to the right multiple times in the process. This additional way of keeping track of place values allows the root extraction process to faithfully reflect the steps in the geometric reasoning described in Liu Hui's commentary.

Jia-Ming YING (Associate Professor, National Tsinghua University, Hsinchu), in person: "How did students pass the examinations? – Official mathematics curricula in ancient and medieval Korea"

The Nine Chapters on the Art of Mathematics and its commentaries played crucial roles in ancient and medieval official mathematics curricula across East Asia. Notably, it was designated as one of the core textbooks in the official mathematics curricula and examinations of eight-century Tang China, Silla Korea, and Nara Japan. Each of these three governments adopted slightly different approaches in utilizing the *Nine Chapters* and other texts in teaching and assessments. By the 12th century, official mathematics examination systems in China and Japan had been discontinued for various reasons, leaving Korea's Koryŏ Dynasty (918–1392) as the sole preserver of an official mathematics examination system in East Asia. This paper delves into the content and possible structure of the mature mathematics examination system in 12th-century Koryŏ and discusses the implications for textbook usage within this framework.

Makoto TAMURA (Professor, Osaka Sangyo University, Osaka), in person: "A comparative study of mathematical books of the Qin-Han period focusing on the *Beida qinjian*"

Since the 1980s, the following ancient Chinese mathematical books of the Qin-Han period have been discovered: Zhangjiashan Han bamboo book "Suanshu-shu", Yuelu Academy's Qin bamboo book "Shu" (Yuelu Shu, for short), Peking University's Qin bamboo books (Beida Qinjian, for short), and Suihudi Han bamboo book "Suanshu." This paper discusses Chinese mathematics during the Qin-Han period, focusing on the Beida Qinjian, whose report books with photographs of bamboo slips were published in 2023.

1. The *Gougu* method

Both the Beida Qinjian and the Yuelu Shu contain problems for finding the fractions 15+15/31 that approximate the square root of 240, but they calculate them in different manners. In addition, both the Beida Qinjian and the Yuelu Shu contain the other problems using the Gougu method. By examining their solutions, we will confirm that the Gougu method has been already used in the Qin Dynasty.

2. The chants

The "Litian" problem of the Beida Qinjian contains the chants for memorizing the constant 375 as (124+1) times 3, which were deciphered by the author and published in 2015 in Japanese and in 2018 in English. The Yuelu Shu also has the problem which includes some phrases of the nine-times-nine table as chants in the text of the method. Thus, we can see that in the Qin Dynasty, oral practices were an important means for officials to learn how to solve problems.

3. Other findings in the Yuelu Shu from the study of the Beida Qinjian of the same period will also be shown.

Wann-Sheng HORNG (Professor emeritus, National Taiwan Normal University, Taipei City), virtually, via Zoom: "*Yuanzhui* vs. *Yuantai*: Volume Calculation Issues neglected from Zhang Qiujian (ca. 400 CE) to Wu Jing (1450 CE)"

In ancient Chinese mathematics texts concerning areas and volumes, problems usually are given parameters for applying algorithms or formulas to get the answers. Take Chapter 5 of *Jiuzhang suanshu* for example. It is devoted basically to computing volumes in various shapes. The methods for solving Problem 5.11 (on *yuantai*, a truncated cone) and 5.13 (on *yuanzhui*, cone) respectively would self-explain quite clearly. However, ancient Chinese mathematics never paid attention to the geometrical properties that interconnect the two solids, say *yuantai* and *yuanzhui* until Zhang Qiujian. There follows Yang Hui of Song dynasty and Wu Jing of Ming dynasty were also interested in the problem-solving. In this talk I will try to make sense of the format which might also attract the attention of those who are concerned about educational issues.

Shirong GUO (Professor, Inner Mongolia Normal University, Hohhot), in person: "Sharing the Same Mathematical Tradition: The Role and Significance of Zhu Shijie's *Suanxue Qimeng* to the Development of Mathematics in Korea"

The *Jiuzhang Suanshu* appeared around the 1st century BCE. formed the ancient Chinese mathematical tradition. In the Tang Dynasty (618–906), mathematician Li Chunfeng (602–670), under the imperial order and with the help of 20 scholars, edited and annotated *Ten Mathematical Classics* written by predecessors which were used as textbooks for imperial mathematics education. Chinese mathematics developed to a new height, a large number of mathematical works appeared, and some important mathematical achievements were obtained, in the Song and Yuan Dynasties (960–1368). East Asian countries shared this cultural tradition of mathematics. In the Sui and Tang Dynasties (581–906), Japan and the Korean Peninsula began to introduce the *Ten Mathematical Classics* from China. In the 17th century, Chinese mathematics played a very important role in laying the foundation of Korea and Japan. Among the mathematics works of Song and Yuan Dynasties, Chinese mathematician Zhu Shijie's *Suanxue Qimeng*

(Enlightenment in Mathematics, in 1299) played an important role in the early development of mathematics in Korea and had a far-reaching influence.
The following questions will be discussed in this presentation:
The spread history of the *Suanxue Qimeng* in Korea;
The studies by Korean mathematicians on the *Suanxue Qimeng*;
How the *Suanxue Qimeng* shaping the Korean mathematics;
The place of *Suanxue Qimeng* in the history of mathematics in Korea;
The influence of the Korean mathematicians' studies of the *Suanxue Qimeng* to Chinese mathematics.
Through the discussion of the case of *Suanxue Qimeng*, we try to show how East Asian countries share the same mathematical tradition.