# Report on the 32nd Novembertagung on the History and Philosophy of Mathematics

# Co-sponsored by the ICHM

**Dates:** 18th – 20th November 2021

**Venue:** Online conference

**Theme:** Mathematics in Times of Crisis

**Invited speakers: Amir Asghari** (Liverpool John Moores University), **Juliet Floyd** (Boston University), **Tilman Sauer** (Universität Mainz)

**Organizing Committee:** Brigitte Stenhouse (Open University, UK), Richard Lawrence (Eberhard Karls Universität Tübingen), Tuya Sa (Loughborough University), Tobias Schütz (Universität Mainz), Rosie Lev-Halutz (Tel-Aviv University)

**Webpage**: https://novembertagung.wordpress.com/

This year, due to the COVID-19 pandemic, the 32nd Novembertagung on the History and Philosophy of Mathematics was held online (CET). Inspired by contemporary global events, the theme of the conference explored the ways that times of crisis shaped the development of mathematics. Almost 30 talks given by young scholars, in two parallel sessions, invoked lively discussions and gave the attendants an opportunity to present and examine their ideas in front of a diverse and international audience of philosophers, historians, and educators of mathematics.

Various countries were represented in this year's conference, such as: China, Ireland, the US, Israel, the UK, Germany, India, Portugal, Scotland, Turkey, Iran, Mexico, France, Azerbaijan, Serbia, the Czech Republic, and Italy.

In addition to the talks given by young scholars, three inspiring lectures were given by our invited speakers, who also attended students' talks, participated in the discussions, and graciously shared their knowledge, experience, and wisdom with the students.

Even though we could not meet in person this year, the committee made efforts to provide the scholars the opportunity to establish personal connections with each other. We initiated various social activities such as: A hands on workshop— Creative Science Storytelling— with Anna Ploszajski; a History of Mathematics Quiz; a Virtual Escape Room; and aVirtual London Tour focusing on events and important figures in the history of mathematics. Interesting discussions were also conducted during designated coffee breaks between sessions.

Participation at the Novembertagung was free of charge. The costs for the social activities were generously funded by the ICHM, and other co-sponsors (the LMS and the BSHM). We are grateful to The Open University for providing access to Zoom, the platform on which all talks and social activities took place.

On the final day, the students participated in the traditional discussion on the future of the Novembertagung, with two volunteers coming forward to begin planning the next meeting. The importance of having an in-person element to future Novembertagungs was widely felt, although it is as yet unclear whether that will be possible.

Programme

## *"Mathematics in Times of Crisis"*

2021

Online conference
Time zone: Central European Time (CET)

| **Day 1 – Thursday, November 18th** | | |
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| 9h Conference opening | | |
| 9h30 **Keynote lecture: Amir Asghari, Liverpool John Moores University**<br>Title: Omar Khayyam, The Mathematician who Stood up to Crisis<br>Chair: Tuya Sa and Brigitte Stenhouse | | |
| 10h30 Coffee break (Tuya Sa and Brigitte Stenhouse) | | |
| 11h<br>Parallel sessions | Session A<br>Chair: Tobias Schütz and Tiago Hirth | Session B<br>Chair: Rosie Lev-Halutz and Kati Kish |
| | **Stephen Harrop**: Isaac Barrow on Some Paradoxes of Infinite Divisibility | **Benjamin Wilck from Humboldt-Universität, Berlin**: Was Euclid a Platonist Philosopher? |
| | **Chen Yang**: The Berkley Paradox Revisited | **Avinoam Baraness from** ***Herzog College and the Hebrew University of Jerusalem***: Abner of Burgos discussing Euclid's fifth postulate |
| 12h Lunch break | | |
| 14h<br>Parallel sessions | Session A<br>Chair: Tobias Schütz and Chen Yang | Session B<br>Chair: Rosie Lev-Halutz and Tuya Sa |
| | **Kevin Tracy**: 'Which no one who is well-versed in mathematical | **Saša Popović from** ***University of Belgrade***: 'The Great Struggle' between |

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| | teaching, or who wishes to turn his gaze to the stars, will deny': Philip O'Sullivan Beare's defence of nation, faith, and cosmos in crisis (c. 1626) | Cantor and Veronese: Historicophilosophical Considerations concerning the Immediate Reception of Veronese's Fondamenti |
| | **Rachele Rivis**: Weingarten's method for applicability problem: the applications of Bianchi and Ricci | **Rima Hussein from *Johns Hopkins University***: Lambert on Euclid's Parallel Postulate and Why it Must be Proven |
| | **Anaid Linares**: Leray, Schauder and the construction of the sheaf concept | **Nicola Bonatti from *LMU Munich*** : Extremal Axioms and the Reflective Equilibrium of Intended Models |
| 15h30 Coffee break (Rosie Lev-Halutz and Tobias Schütz) | | |
| 16h<br>Workshop | **Creative Science Storytelling, with Anna Ploszajski**<br>Chair: Brigitte Stenhouse and Tuya Sa | |
| 17h30 Coffee break | | |
| 19h30 History of Mathematics Quiz (Rosie Lev-Halutz and Brigitte Stenhouse) | | |
| 20h30 Fin day 1 | | |
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| **Day 2 – Friday, November 19th** | | |
| 9h30<br>Parallel sessions | Session A<br>Chair: Rosie Lev-Halutz | Session B<br>Chair: Tobias Schütz (and Brigitte) and Benjamin Wilck |
| | **Tuya Sa from Loughborough University**: Beauty is not 'in the eye of beholder': measuring mathematicians and undergraduate aesthetic intuition through comparative judgements | **Daniel Sierra from *California State University***: Mr. Frege, The Platonist |

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| | **Jan Zeman from *Charles University Prague***: Peano- and Hilbert curve | **Moritz Vogel from *University of Bonn***: Plato's Divided Line and the Problem of Incommensurability |
| | **Liu Zixuan from *Sun Yat-sen University***: Mathematization and the Universality of Mathematics: The Possibility of a More Universal Formal Mathematics-Logic | **Richard Lawrence (from *Eberhard Karls Universität Tübingen)***: Formalism through the eyes of Weierstrass and Thomae |
| 11h Coffee break (Rosie Lev-Halutz and Tobias Schütz) | | |
| 11h30 London Tour (Tuya Sa and Rosie) | | |
| 12h30 Lunch break | | |
| 14h30<br>Parallel sessions | Session A<br>Chair: Richard Lawrence and Deniz Sarikaya | Session B<br>Chair: Rosie Lev-Halutz and Paul Hasselkuß |
| | **Josh Lalonde**: "We don't even know how much we know that we don't know we know": mathematical knowledge management and the scale crisis | **Theodor Nenu from *University of Bristol***: The Historical Emergence of Fuzzy Mathematics |
| | **Ravi Chakraborty**: Mathematics makes metaphysics countable: Normalizing crisis in mathematics | **Tiago Hirth from *University of Lisbon with the CIUHCT Research Center:*** Problems to Sharpen Mathematical Recreations |
| 15h30 Coffee break (Brigitte Stenhouse and Tuya Sa) | | |
| 16h **Keynote lecture: Juliet Floyd, Boston University**<br>Title: Wittgenstein and Turing<br>Chair: Richard Lawrence and Rosie Lev-Halutz | | |
| 17h Coffee break (Richard Lawrence and Rosie Lev-Halutz) | | |
| 19h30 Escape room (Brigitte Stenhouse and Richard Lawrence) | | |
| 20h30 Fin day 2 | | |
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| Day 3 – Saturday, November 20th | | |
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| 10h30<br><br>Parallel sessions | Session A<br><br>Chair: Rosie Lev-Halutz and Tobias Schütz | Session B<br><br>Chair: Tuya Sa and Kevin Tracy |
| | **Kati Kish from Tel Aviv University**: The Social Construction of Mathematical Truth: Taking Intuitionism into Account | **Sylvain Demanie**: Sturm's theorems in projective geometry and their circulation at the beginning of the 19th century |
| | **Paul Hasselkuß from *Heinrich Heine University Düsseldorf***: Mathematical Knowledge as Social Knowledge? | **Fatih TAŞ**: Exploration of Preservice Mathematics Teachers' Disagreements in Mathematics |
| | **Jann Paul Engler from *University of St Andrews***: Truth-theoretic (vs.) and meaning-theoretic realism and the case of bivalence | **José Antonio Pérez-Escobar and Deniz Sarikaya**: The problem of homogenization of mathematics education: Pluralism as an epistemic virtue |
| 12h Lunch Break | | |
| 13h **Keynote lecture: Tilman Sauer, Universität Mainz**<br><br>Title: History of Mathematics versus History of Physics<br><br>Chair: Tobias Schütz and Rosie Lev-Halutz | | |
| 14h Coffee break (Tobias Schütz and Rosie Lev-Halutz) | | |
| 14h30 Discussion: the future of Novembertagung | | |
| 15h30 Fin day 3 | | |

*Book of Abstracts*

# *32nd Novembertagung on the History and Philosophy of Mathematics*

# *"Mathematics in Times of Crisis"*

*November 18th-20th, 2021*

Organizing committee members: Brigitte Stenhouse (Open University, UK), Richard Lawrence (Eberhard Karls Universität Tübingen), Tuya Sa (Loughborough University), Tobias Schütz (Universität Mainz), Rosie Lev-Halutz (Tel-Aviv University)

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**Juliet Floyd**, Boston University

Title: "Wittgenstein and Turing"

**Tilman Sauer**, Universität Mainz
Title: "History of Mathematics versus History of Physics"

## Special Events

**Workshop: Creative Science Storytelling, with Dr Anna Ploszajski**
**Time:** Thursday, November 18th, 16:30 PM.

**Description**: An interactive workshop in which Dr Anna Ploszajski will guide us through how to write engaging papers and conference talks on technical topics. Suitable for researchers at any stage of writing or presenting, this 1hr30 workshop will help you to tailor your work to specific audiences, and use language and storytelling in a more effective way in your writing. More information at https://www.annaploszajski.com/academia.

**November History of Mathematics Quiz**
**Time:** Thursday, November 18th, 19:30 PM.

**Description**: A fun quiz about history and philosophy of mathematics. To participate you need to either install Kahoot application on your mobile phone, or enter the Kahoot website on your computer (Link). We will gather in the 'Session A' room. A link to the quiz will be sent separately.

Please make sure you have **two parallel screens** open, one for the Kahoot Quiz (to answer the questions) and one for the Zoom session (to view the questions).

**Virtual London Tour**

**Time**: Friday, November 19th, 11:30 AM.

**Description**: In this virtual tour, we will explore London's history, its famous as well as hidden gems, focusing on locations linked to science and particularly to mathematics.

**Online Escape Room**

**Time**: Friday, November 19th, 19:30 PM.

**Description**: Conference participants will be divided up into small teams (via Zoom break-out rooms) where they will work together to solve puzzles and cryptic clues. First team to solve the final mystery wins eternal glory.

## Abner of Burgos discussing Euclid's fifth postulate

***Avinoam Baraness, Herzog College and the Hebrew University of Jerusalem***

For about two thousand years, Euclid's fifth postulate has deceived the best of minds. Generation after generation, attempts to prove it from the other four postulates were made. Mathematicians, philosophers and even just amateurs have tried their hands and devoted a lot of their time dealing with it, but in vain; regularly after a while - sometimes a long period - their proofs found to be wrong.

All these failures may be viewed as an awful waste of time, but actually each attempt help us to comprehend the relationships between the Euclidean concepts and theorems, revealing another layer.

The long road till the acceptance of the non-Euclidean geometries can certainly teach us something about the human nature and its difficulty of challenging truths, but also may enable us to test all the proof attempts from a stable point of view.

I would like to focus on an attempt which appears in almost unknown treatise "Rectifying the Curve" by the Middle-Ages scholar Abner of Burgos (1260-1347), in which he addresses the issue (although the main purpose of the treatise is squaring the circle "truly established, neither by way of approximation").

Abner first criticizes several former attempts to prove the postulate (as of Ibn al-Haytham and al-Nairizy), then presents his own proof attempt, claimed to be based on an alternative postulate (which he previously established). Examining his original approach to the issue - besides being interesting itself - demonstrates the curvy path in which held ideas are challenged.

## Extremal Axioms and the Reflective Equilibrium of Intended Models

***Nicola Bonatti, Munich Center for Mathematical Philosophy (LMU Munich)***

Mathematicians and philosophers generally agree that theories such as Peano Arithmetic, Hilbert Geometry and Zermelo-Fraenkel Set theory concern a specific subject matter, i.e. they have an intended model – respectively, the natural numbers, the geometric continuum and the set theoretic universe. The categoricity theorem – by which all the models of a Second-order theory are isomorphic – has been taken as a means to identify the intended models of such theories.

However, the categoricity theorem has been adopted in the literature to argue for two antithetic claims: on the one hand, bearing on the formal resources adopted, the categoricity theorem demonstrates that there is a unique structure corresponding to our practice. On the other hand, bearing on the antecedent beliefs about the intended model, the categoricity theorem demonstrates that the formalisation process has been successful.

In this talk, I will claim that the tension is solved once the role of extremal axioms is properly understood. Extremal axioms – such as the axioms of Induction, Completeness, Constructibility and Large Cardinals – determine a maximality/minimality condition for the models of the theory. I will argue that the adoption of extremal axioms provides a reflective equilibrium between the reliability of our beliefs and the suitability of the formalization. More precisely, I will explain that extremal axioms are initially adopted on intrinsic grounds – thus fixing the beliefs concerning the intended model – and then justified on extrinsic grounds – thus securing the faithfulness of the formalisation. Therefore, I will conclude that the sceptical concerns about intended models do not arise from the existence of non-standard models, but rather from the (philosophical) disagreement over the extremal axioms.

## Mathematics makes metaphysics countable: Normalizing crisis in mathematics

*Ravi Chakraborty, Indian Institute of Technology Delhi*

We often work with the presumption that the idea of crisis is a state of exception in the history of mathematics rather than the rule. We must ask if the historical conditions of mathematics allows one to perceive a crisis as one. This is because the metaphysical presumptions of some mathematics is more self-evident than others. But should we not presume metaphysics as underlying all kinds of mathematics?

Albert Lautman would agree. He offers a basis to unite mathematics (in spite of its crises) in an ultimately deeper and more systematic quest to resolve metaphysical problems. He looks at the genesis of mathematics as a way of resolving the timeless dialectical problems represented by pairs such as the discrete and the continuous, the local and the global, the intrinsic and extrinsic and even existence and essence. The phenomenologist Gilles Chatelet has criticized Lautman for not accommodating far more substantive notions and called these pairs purely philosophical. But the last pair of existence and essence reflects just how metaphysically loaded even these purely philosophical notions already are. But why is Lautman suddenly filtering out the metaphysics from mathematics in the first place?

My hypothesis is that Lautman wishes to normalize the notion of the crisis so as to comprehend the greater prevalence of mixtures of different approaches in modern mathematics driven by the desire to solve problems rather than to ask fundamentally metaphysical questions (say, in Riemann, about space) .In the latter, mathematicians are much closer to a self-consciousness of the underlying philosophies of their work and it would be logical to assume that challenges to such theories would be cognized as crises more strongly. If we are to follow Lautman's idea of the dialectic, there is not much to be made about the metaphysical crisis that was provoked by Cantor's idea of the infinite because it provides just another resolution of the problem presented by the discrete-continuous dialectic. In fact, Lautman would even accommodate the supposedly non-standard mathematics of Brouwer in a similar manner. We can probably turn around and say that perhaps mathematics is a way of making metaphysical notions more precise and not just exploiting them in the context of mathematical discovery. Mathematics makes metaphysics countable, as it were, because of its crises and more broadly because of its pluralities generated in history.

*Primary References:*

Châtelet, Gilles. "Figuring space philosophy, mathematics, and physics." Springer, 2000

Lautman, Albert. *Mathematics, ideas and the physical real*. Trans.Simon Duffy . A&C Black, 2011.

## Sturm's theorems in projective geometry and their circulation at the beginning of the 19th century

***Sylvain Demanie, University of Lorraine***

The French-Swiss mathematician Charles-François Sturm (1803-1855) was the discoverer of a little known theorem in projective geometry which was the main topic of a Mémoire dedicated on conic sections and published in two parts in 1826 and 1827 in the Gergonne's journal *Annales de mathématiques pures et appliquées*. Sturm discovered this theorem during his rst stay in Paris as tutor to the de Broglie family in 1824. At the beginning of the 19th century, a community of French mathematicians had developed the project of organising the whole corpus of geometrical propositions (including famous theorems such as Pascal's) from general principles. Sturm's original work constituted a part of this project and appeared at a time when debates on questions of rigor and good practice in geometry had animated the community of mathematicians : How to interpret the concept of duality ? How to represent it ? What credibility can be given to the controversial principle of continuity enunciated by Poncelet ? Furthermore, the new theorem discovered by Sturm occured in a context of competition and priority debates with other young mathematicians also publishing in Gergonne's journal, such as Plücker or Bobillier. The study of the circulation of Sturm's theorem, which is little known in the scientic literature, shows how knowledge and practices were formed in the particular eld of projective geometry during a period of great political instability in France.

## Truth-theoretic vs. and meaning-theoretic realism and the case of bivalence

***Jann Paul Engler, University of St Andrews***

The intuitionist's or anti-realist's attack on bivalence usually equates truth with provability: a statement is dened to be true when it is provable. However, the modality of being provable that this account incorporates has not been specied su-cientlyand it is questionable if a satisfactory specication can be found (cf. Martino 1994, Raatikainen 2008). The realist circumvents this problem by understanding truth in terms of the domain on which a true statement is said to hold. But this is criticized by the anti-realist in cases in which we cannot form an adequate conception of such a domain, whereby adequacy is understood in terms of *nite constructibility*. If this criticism has any appeal, a dilemma seems to follow.

To avoid such a dilemma, I argue that nite constructibility is insu-cient as a general criterion of adequacy because it does not manage to distinguish between cases where we can specify what the truth of a statement consists in even though we have not veried it (e.g. Goldbach's conjecture), and cases where we are not able to do so (e.g. the existence of certain sets beyond ZFC). In the rst case, there is little obstacle to ascribing such statements a truth value in the realist's sense. In the second case, however, a realistic assumption xes the truth value of a statement whose *meaning* is not entirely determined. Here, I argue, the anti-realist's criticism applies convincingly.

The resulting picture is a compromise: While the rst case turns out to not support the anti-realist's attack on bivalence, their critique has nonetheless a proper target in the second one.

## Isaac Barrow on Some Paradoxes of Infinite Divisibility

***Stephen Harrop, Yale University***

In his work "The Usefulness of Mathematical Learning", Isaac Barrow, the first Lucasian professor of mathematics at the University of Cambridge, discusses a number of classical paradoxes of the infinite divisibility of magnitude, and attempts to give them a resolution. In this paper, I argue that in this work, Barrow presents a nominalist and empiricist account of number, magnitude, and their relation, and that this philosophically interesting account allows him to offer solutions to these paradoxes. Barrow is concerned to argue in favor of the infinite divisibility of magnitude, against paradoxes presented by such classical figures as Epicurus and Lucretius. In the case of each paradox, Barrow adopts a defusing approach to the paradoxes. He identifies a presupposition of the paradox, shows how this presupposition is false on his account of number and magnitude, and thereby shows how the paradox does not arise on his account.

## Mathematical Knowledge as Social Knowledge?

***Paul Hasselkuß, Heinrich Heine University Düsseldorf***

In social epistemology, *social* values concern an agent's social, moral or political background, or they involve the agent in a crucial way. They are different from values that are *internal* to the scientific enterprise (such as consistency). In mathematics, historical and socio-empirical studies of mathematicians' practices describe how agents turn to social values to fulfil various epistemic tasks, and how an agent's community and culture influence her research. In both cases, social values seem to complement the epistemic role of internal values. But how can social values be epistemically reliable? Moreover, if they are, what are the consequences for mathematical knowledge?

To answer the first question, I argue for a weak dependency claim. If a mathematician believes that $p$ is a mathematical truth, because she believes that $p$ meets some social values $S$ (in addition to other beliefs concerning $p$), her belief that $p$ is $S$ needs to be partly grounded in the mathematical properties of the mathematical entity that corresponds with $p$. That is, only if $S$ *partly* depends on mathematical properties, and only on the right subset thereof, $S$ can be epistemically reliant.

Turning to mathematical knowledge, these findings seem to be problematic. Standard *a priori* accounts of mathematical knowledge deny that social values can have *any* positive epistemic effect, while weak dependency rules out alternatives that take mathematical knowledge to be socially constructed (and, hence, *a posteriori*). I argue that both challenges can be addressed by looking at social epistemology and adopting a social theory of (mathematical) knowledge.

## Problems to Sharpen Mathematical Recreations

***Tiago Hirth, ULisbon, CIUHCT***

While Recreational Mathematics and its objects accompany the formal subject through all History of Mathematics, some texts are more relevant than others. One such text is the Propositiones ad Acuendos Juvenes (Problems for Sharpening Youths). It is likely the first collection of problems common in the folklore of mathematics for the sake of the problems themselves, this is, without obvious instruction purposes like one would find in a textbook. Credited to Alcuin of York in the Charles Magne Court of the 800's this text comprises over fifty problems and their solutions. While some of these might hold only value for the sake of the perspective of a time past others still hold a challenge and interesting approach presently.

In this talk, following and touching on past presentations, we'll present the known information on the Propositiones and explore a selection of the problems and their known biography trying to place them in a bigger picture and working towards a genealogy of recreational Mathematics

## Lambert on Euclid's Parallel Postulate and Why it Must be Proven

***Rima Hussein, Johns Hopkins University***

Why engage in a proof of the parallel postulate at all? Much of natural philosophy in eighteenth century Germany was caught up either in the vis viva debate or in the existential crisis of metaphysics in light of the increasingly accelerated success of the so-called mathematical method. This success of mathematics expressed itself in application of analysis and calculus in various areas of mechanics and dynamics, like the theory of the vibrating string and particle dynamics. In light of these accelerated developments, Johann Lambert's foundational questions on why the parallel postulate must be proven seems decelerated, and unpragmatic - a bit odd, perhaps like the man himself. Yet, his Theorie der Parallelinien contributed to breaking geometry out of Euclid's ancient grip. And recent historical work on Christian Wolff's and Immanuel Kant's philosophy of mathematics bring into sharp relief the originality and ingenuity of his arguments. I investigate his reasoning and show that Lambert puts forward three chief arguments for why the parallel postulate must be proven. First, he argues against Wolff and Kant that arbitrary mathematical constructs do not derive their validity from their construction but must be proven. Second, he argues that parallel lines cannot be abstracted as concepts from examples, because parallel lines do not touch in infinity and infinity cannot be found in examples. And third, Lambert argues that Euclid's postulates are neither hypothetical nor programmatic. They are by their nature derivable from each other.

## The Social Construction of Mathematical Truth: Taking Intuitionism into Account

***Kati Kish, Tel-Aviv University***

Social constructionist treatments of mathematical truth were introduced in the 1970s as part of the ongoing developments in the sociology of scientific knowledge and were extensively discussed during the 1980s and 1990s. Discussions of the social construction of knowledge have dealt quite extensively with mathematical concepts but have paid far less attention to the way controversial or rejected mathematical theories address the notions of truth, objectivity, and knowledge itself. Of those abandoned theories, the most notable one is the school of intuitionism, which was seriously explored only once from a social constructivist point of view, thirty years ago by Herbert Mehrtens (1990), and has remained in the shadows ever since. In my lecture, I focus on this unaccounted aspect of mathematics and examine intuitionism's alternative definition to the concept of mathematical truth.

Intuitionists have long deliberated how the truth of a mathematical statement should be conceived, constructed, and verified. Brouwer deemed the truth of a mathematical statement to be a subjective claim deriving from mental construction. Later versions of intuitionism, such as those of Heyting and Stephen Cole Kleene, proposed a formalized definition of intuitionistic truth that entails a realistic aspect as well. Unlike Brouwer, contemporary intuitionists such as Neil Tennant take mathematics to be objective and argue that every truth is knowable. The status of truth in intuitionistic mathematics had also played a central role in theories of proof and meaning developed by philosophers such as Dag Prawitz and Michael Dummett as part of their attempt to account for the connection between truth, proof, and reality, in intuitionism.

Given this broad difference of opinion, intuitionistic approaches deserve to be at least accounted for in discussions of social construction in mathematics. However, the few current social constructionist accounts treat mathematical truth as objective and well-defined, leaving intuitionist views of truth out of the discussion. The current lecture brings these alternatives to the fore by analyzing three different intuitionistic notions of mathematical truth in light of the general discussion about the social construction of mathematical truth and knowledge. Such an analysis suggests a novel, twofold perspective on the concept of truth in mathematics: as a mentally constructed subjective claim, on the one hand, and as a socially constructed objective knowledge claim on the other.

## “We don’t even know how much we know that we don’t know we know”: mathematical knowledge management and the scale crisis

***Josh Lalonde, Independent Scholar***

Crises in mathematics have often arisen in connection with worries about the limits of mathematical knowledge, most famously in the case of Gödel’s incompleteness theorems. More recently, however, a sense of crisis has instead emerged as a result of the seemingly limitless expansion of mathematical knowledge. Estimates of the number of theorems proved each year are in the hundreds of thousands, and the zbMATH Open indexing service lists 4.2 million bibliographic entries. Due to this proliferation, which can be described as a scale crisis in mathematics, it is impossible for any individual mathematician to have a comprehensive knowledge even of a sub-sub-specialism as represented by the lowest level in the Mathematics Subject Classification (MSC). Given the scale of mathematical knowledge, finding out what is already known about a particular topic can be as difficult a task as producing that knowledge in the first place. In the memorable phrase of Michiel Hazewinkel, “We don’t even know how much we know that we don’t know we know.”

In response to this crisis, mathematicians have begun to develop the field of mathematical knowledge management (MKM), which aims to develop tools, particularly software, to make finding and using mathematical knowledge easier. I will sketch the history of MKM and some of its precursors such as the work of the Bourbaki group and the Automath project. Next, I will examine how the problem of MKM has since the 1990s been a central motivation in the project of formalizing mathematics, as expressed notably in the QED Manifesto. Finally, I will argue that historians and philosophers of mathematics should pay attention to MKM as it represents mathematicians’ own reflections on mathematical knowledge.

## Formalism through the eyes of Weierstrass and Thomae

***Richard Lawrence, Eberhard Karls Universität Tübingen***

Mathematical formalism is the view that mathematics can be seen as a 'game of symbols'. One important formulation of this view was given by Johannes Thomae, who compared our signs for numbers with chess pieces: according to Thomae's formalism, such signs are given meaning by our rules for calculating with them, just as wooden pieces are given meaning by the rules of chess. This view was attacked at length by Gottlob Frege, and also played an important role in Ludwig Wittgenstein's thought, giving it an important place in early analytic philosophy.

I will argue that in order to understand mathematical formalism and Thomae's chess analogy, we need to see them in the context of a debate about the foundations of complex analysis. In the second half of the nineteenth century, two different approaches to complex analysis grew into rival schools: Riemann's intuitive, broadly geometric approach was opposed to Weierstrass's algebraic, broadly computational approach. Thomae's formalism belongs to the Weierstrass camp, and was offered as a solution to a problem at the heart of Weierstrass' approach: how can we make sense of calculations with infinitary representations?

## Leray, Schauder and the construction of the sheaf concept

***María Anaid Linares Aviña*, *Universidad Nacional Autónoma de México***

It is well known that the publication in 1945 of the three articles by J. Leray led to the concept of sheaf that H. Cartan and the members of his seminar would transform to create the modern concept of sheaf. It is also known that those articles were the continuation of a course of algebraic topology in captivity. Less known are the ideas that Leray used to form this course.

In 1933 J. Schauder and Leray met in Paris, they wrote together an article about functional equations. In this work they created a concept of index that is an extension of the concept of fixed point from Brouwer and that can be applied to Banach spaces.

In this talk, we discuss the influence of the ideas from Schauder in the course of Leray and in the creation of the sheaf concept.

## The Historical Emergence of Fuzzy Mathematics

***Theodor Nenu, University of Bristol***

Fuzzy Mathematics is a relatively young field whose historical emergence can be traced back to approximatively fifty years ago. One of its main subfields, Fuzzy Set Theory, diverges from its classical counterpart by allowing set-membership to be a matter of degree, rather than all-ornone. We will start by tracking the general development of many-valued logics up until Zadeh's (1965) seminal publication (which introduced fuzzy sets and the standard operations on these sets).

Zadeh's contribution is unanimously agreed to be the historical locus that marks the origin of Fuzzy Mathematics (Belohlavek, Klir, Dauben (2018)). Nevertheless, we will present how inklings of fuzzy thought were independently entertained around that period also by psychologist Eleanor Rosch (1971). Rosch made an empirical case for the proposition that classical sets cannot model ordinary concepts. Both Zadeh's and Rosch's proposals challenged the orthodox assumptions of that period.

We will present a selection of mathematical conflicts that took place around that time which reveal attitudes of downright hostility against Zadeh's ideas (e.g. by figures such as Kahan or Kalman). We will then track the positive evolution of attitudes towards Fuzzy Mathematics, together with the fruitful applications that were found for fuzziness in Engineering, Computer Science and many other areas.

Lastly, we'll note that it was Petr Hajek's (1998) monograph which made Mathematical Fuzzy Logic a respectable subfield of Mathematical Logic. Like other logics, it focuses on investigating familiar aspects such as proof theory or completeness. We'll conclude by exploring whether Mathematical Fuzzy Logic allows for novel philosophical treatments of famous Semantic Paradoxes (such as The Liar's Paradox or The Sorites Paradox).

*Keywords:* Fuzzy Logic and Set Theory, Degrees of Truth, The Liar's Paradox, The Sorites Paradox

### The problem of homogenization of mathematics education: Pluralism as an epistemic virtue

*José Antonio Pérez-Escobar (ETH Zürich) and Deniz Sarikaya (University of Hamburg)*

This talk discusses a potentially problematic consequence of the recent changes of mathemathics education due to the Covid 19 pandemic. We argue that the mathematical undergraduate canon should not be overly codified and formalized. We justify this drawing from two bases. First, we base our argument on the notion of productive ambiguity, inspired by ideas of Lakatos and Grosholz. Second, we sustain our claims on a historic case study on the "tripos" in Cambridge: a plurality of research practices as a great resource for mathematical progress. This second pillar is related to debates on mathematical pluralism. Because online lectures might fuel a deeply codified canon, we argue that they may lead to harmful developments for mathematical practice.

## 'The Great Struggle' between Cantor and Veronese: Historicophilosophical Considerations concerning the Immediate Reception of Veronese's Fondamenti

***Saša Popović, University of Belgrade***

Giuseppe Veronese introduced non-Archimedean geometry in the 1890's amidst the turmoil of the so-called 'crisis of intuition' and the ensuing Weierstraßian response to it (i.e. the reformation movement towards rigor). This philosophically exciting period of *fin-de-siècle* mathematics has been extensively researched, however, Veronese's contributions to the mathematico-philosophical debates concerning continuity, infinity, infinitesimals, and the foundations of geometry remain yet to be seriously investigated. My main task in the present talk will be to elucidate certain philosophico-mathematical aspects of Veronese's approach which brought him into direct conflict with Georg Cantor and the adherents of Cantorian mathematical philosophy (such as, e.g. Vivanti and Peano). These are primarily: (i) Veronese's 'return of the visual' (reliance upon geometric or spatial intuition), (ii) his Aristotelian interval-based theory of the linear continuum, and (iii) reintroduction of infinitesimals and an alternative, non-Cantorian theory of infinity. First, I will explain how and why in light of i–iii Veronese came to be seen by the Cantorians as the key figure of a counter-reformation movement (a sort of a 'geometric turn'), and his work as being regressive[1]. Next, I will show that at the crux of the Veronese-Cantor controversy was the question concerning the structure of the linear continuum, i.e. the conflict between the novel Cantorian point-based and the orthodox Aristotelian interval-based conception. We shall see why Veronese's theory of the *continuo intuitivo* could (and should) be seen as a

[1] This is also the received view in contemporary scholarship due to an oftentimes uncritical acceptance of the so so-called 'Cantor-Dedekind academic dogma'. If time permits, I will try to shed some light on certain major misconceptions in the *Rezeptionsgeschichte* of Veronese's works.

non-Archimedean variation and an expansion of classical Aristotelian intervalism.
I will conclude by showing how Veronese's theory of the *absolute, infinitesimalenriched continuum* can serve as a means for reconciling Cantorianism and Aristotelianism regarding continua, and by contrasting this with some of the more familiar recent approaches.

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## Weingarten’s method for applicability problem: the applications of Bianchi and Ricci

***Rachele Rivis, Università degli Studi di Milano***

The development of differential geometry in Italy between the 19th and 20th centuries was certainly conditioned by the sceptical welcome of the Absolute Differential Calculus invented by Gregorio Ricci Curbastro. The figure that better represents this attitude of closure towards the new techniques is Luigi Bianchi. In this context, the study of Julius Weingarten’s work and his scientific collaboration with Bianchi provides interesting insights. Over the years 1884-1894, Weingarten devoted several memoirs to surface theory, namely to the second problem of applicability, consisting of the determination of all surfaces locally isometric to a given one. In 1884 Weingarten pointed out some critical points that affected previous attempts and gradually started to develop a new approach to the problem. This process is documented by numerous letters sent to Bianchi which also attest to a sincere friendship between the two as well as a mutual scientific esteem. In addition to emphasizing the novelty of the approach pursued by Weingarten in order to overcome the limitations he had found in the pre-existing theory, the aim of this talk is to analyse Bianchi and Ricci's applications of this new method in order to provide concrete examples that can clarify the distance of views between them.

### Beauty is not 'in the eye of beholder': measuring mathematicians and undergraduate aesthetic intuition through comparative judgements.

*Tuya Sa, Loughborough University*

Under the influence of experimental philosophy seek to empirically investigate traditional philosophers' arm-chaired assumption on the stability of folk intuition, measuring the level of consensus of mathematicians' judgements on different dimensions of mathematical objects have recently raised a growing philosophical significance; mainly in two aspects. Firstly, it empirically examines philosophical assumptions on the level of consensus within mathematical practice, which saves philosophers from analysing consensus that might not actually exist, such as the assumption on the agreement over the standards of validity (Inglis et al, 2013). Secondly, it investigates the epistemic notions that is held by mathematicians in relation to its different philosophical accounts, such as the explanatoriness (Mejia Ramos et al, 2021). With these two intentions in mind, this talks aims to present the findings on the measure of level of aesthetic agreement across three different demographic groups, to empirically investigate the ongoing philosophical controversy over the nature of mathematical beauty between Aesthetic Realism and Non-Realism. In addition to the empirical dispute over the stability of folk intuition across different demographic groups between earlier and recent works of experimental philosophy.

## Mr. Frege, The Platonist

***Daniel Sierra, California State University, Long Beach***

Although Frege is one of the prominent figureheads of analytic philosophy, it is not surprising that there are still issues surrounding his views, interpreting them, and labeling them.

Frege's view on numbers is typically termed as Platonistic or at least a type of Platonism (Reck, 2005). Still, the term 'Platonism' has views and assumptions ascribed to it that may be misleading and leads to mischaracterizations of Frege's outlook on numbers and ideas. So, clarification of the term 'Platonism' is required to portray Frege's views more accurately (Reck, 2005). This clarification gives us a better picture of what Frege is interested in and what he does not emphasize.

Moreover, in such a clarifying process, we find that Frege draws heavy influence from Rudolf Hermann Lotze, who is frequently called a Neo-Kantian (Vagnetti, 2018). In Lotze's major work, Logik, Lotze has a central focus on validity, in its most general form as he used it, that investigates various related topics, i.e., concepts, language, etc (Vagnetti, 2018) (Lotze, 1888). Furthermore, we observe that Frege's work is so similar to Lotze, that it seems questionable to call his outlook 'Platonism'. Therefore, attributing 'Platonism' to Frege may be a slight misnomer. This paper's entirety is mostly a synthesis of a variety of articles related to Frege, Lotze, and their respective outlooks and the original works of Frege and Lotze that I use to support the view that the term 'Platonism' is a slight issue when predicated to Frege. As such, I include an overview of Frege's treatment in contemporary literature that highlights the usage of the term 'Platonism' and how broad its uses tend to be utilized (Balaguer, 2006) (Burge, 1992). In sum, it is observed that the general label 'Platonism' becomes less appropriate when we consider Lotze in the picture and contrast Lotze alongside Mr. Frege. Overall, this paper is just an explanatory one of Mr. Frege, the Platonist, and the issues of applying the term ‘Platonism’ onto him as his views are seemingly more of a segue from Lotze.

## Exploration of Preservice Mathematics Teachers' Disagreements in Mathematics

*Fatih TAŞ, Bartın University*

Mathematics is a science with history and philosophy. A process that has developed cumulatively throughout history is one of the characteristics of mathematics. There have been ups and downs, proofs and illusions in this process. The history of mathematics deals with how mathematics developed from the Egyptian period to the modern mathematical period. In this process, the formation of the concept of number, its expression with functions, the emergence of limit-derivative concepts and transition to higher mathematics are the subjects. The philosophy of mathematics is the branch of philosophy. It's about nature of mathematics.. The philosophy of mathematics addresses such questions as: What is the basis for mathematical knowledge? What is the nature of mathematical truth? What characterises the truths of mathematics? What is the justification for their assertion? Why are the truths of mathematics necessary truths? In this study, the subject of pre-service mathematics teachers' discovery of disagreements in mathematics and their historical and philosophical analysis are discussed. Twelve pre-service mathematics teachers, who are in the last year of a university in Turkey, took a course in history of mathematics in the fall semester and philosophy of mathematics in the spring semester. As a result of these two lessons, they revealed the crises that occurred in mathematics. For example, the formation of number systems, the emergence of square root numbers, Goldbach's hypothesis, Fermat's Theorem, such as the examination of events affecting the history of mathematics are discussed. Data analysis is ongoing. Results will be presented at the conference.

## ‘Which no one who is well-versed in mathematical teaching, or who wishes to turn his gaze to the stars, will deny’: Philip O’Sullivan Beare’s defence of nation, faith, and cosmos in crisis (c. 1626)

***Kevin Tracey, Maynooth University***

Appended to the Irish soldier-author Philip O’Sullivan Beare’s (c.1590–1635?) natural history Zoilomastix is a brief manuscript fragment of an apparently abandoned work on astronomy, written circa 1626 amidst intertwining crises theological, cosmological, national, and personal. Exiled to Hapsburg Spain as part of a cadre of Gaelic Catholic nobility from boyhood, O’Sullivan Beare’s mature writings chiefly represent their author’s attempts to defend his—and, indeed, his embattled nation’s—identity in the face of state-sponsored processes of erasure. Yet they also present evidence of their author as the product of a continental education in letters, numbers, and arms; as a soldier at the tip of the CounterReformational military spear; and as a mathematically literate, if emotionally conflicted, respondent to developments which threatened to undermine the Thomistic Aristotelian teachings through which he and his countrymen’s temporary safety was secured.

Our understanding of Irish responses to developments in early modern science and mathematics remains problematized by lacunae in both evidence and interpretation. As such, an inquiry into this unique and previously unstudied fragment identifies several valuable gateways through which such readers may be accessed. This paper will discuss how Philip O’Sullivan Beare’s brief manuscript presents evidence of his education and engagement with aspects of the quadrivium; his reading and collation of ancient and present-day astronomical authorities; his awareness of epistemic genres of observation and their attendant instruments; and his subsequent interpretation of the books of nature and of scripture in the service of a long-held worldview increasingly under attack.

## Plato's Divided Line and the Problem of Incommensurability

***Moritz Vogel, University of Bonn***

The remarks on mathematics in the "divided line" passage in Plato's *Republic* are often interpreted without relating them to a specific mathematical background. Against this I will argue that Plato refers to the extant Pythagorean proof of the incommensurability of the side and the diagonal of the square. Plato explicitly mentions the square and the diagonal "itself". He also speaks of the "hypotheses" of the even and odd, the different kinds of angles and the geometrical figures. In fact the Pythagorean proof is based on the even-odd disjunction and the relations between the sides of a right-angled triangle as determined by the Pythagorean Theorem.

This interpretation sheds light on Plato's remark that the mathematicians do not justify their "hypotheses". The Pythagorean proof is a *reductio ad absurdum*. Therefore it must presuppose that all mathematical concepts and assumptions used in it have more logical validity than the assumption that the side and the diagonal of the square are commensurable. For this reason I assume that Plato preferred a then new proof based on *anthyphairesis* (Euclidean algorithm) to the Pythagorean one.

The anthyphairetic proof exactly determines the ratio between the side and the diagonal of the square and, in this way, avoids the aforementioned problem.

This fits well with Becker's conjecture that, in Plato's time, *anthyphairesis* was used to define proportion for incommensurable magnitudes. Recently historians of mathematics criticized this conjecture arguing that ancient mathematicians were not concerned with such foundation problems. But, as I will show, this critique does not apply to the philosopher Plato.

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## Was Euclid a Platonist Philosopher?

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In this paper, I tackle the question of whether or not the mathematician Euclid of Alexandria, author of the *Elements* (ca. 300 BCE), was a Platonist philosopher.

While Euclid's *Elements* is a purely mathematical treatise, and does not mention any philosophical terminology (safe for a few occurrences of metamathematical vocabulary), there is striking evidence for the view that an ontological theory of mathematical objects is implicitly yet systematically encoded in the *Elements* (see Wilck 2020). My paper advances this line of inquiry by exploring possible ancestries of Euclid's ontological theory.

Already in late antiquity, attempts were made to present Euclid as a philosopher, rather than as a mathematician only. Most notably, the Neoplatonist philosopher Proclus argued that the *Elements* is a cosmological treatise about the geometrical elements of the physical universe because it culminates in the construction of the five regular polyhedra (the so-called Platonic solids), which prominently figure in the cosmogony of Plato's *Timaeus*.

In order to critically examine Proclus' claim, I compare Euclid's treatment of the five regular polyhedra with Plato's. The result will be that neither the way in which Euclid defines, nor the way in which he constructs regular polyhedra, resemble Plato's corresponding treatment in any way. Together with further evidence suggesting that Euclid was more of an Aristotelian, rather than a Platonist philosopher, I conclude that Proclus' claim is unfounded.

References:

Wilck, B. (2020): Euclid's Kinds and (Their) Attributes, *History of Philosophy and Logical Analysis* 23(2):362–397. https://doi.org/10.30965/26664275-02302005

## The Berkley Paradox Revisited

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The Berkley paradox states a doubt of the validity of calculus, and it has confused mathematicians for centuries. There are mainly two approaches to justify the calculus: mathematicians either attribute unique natures to the differential so that it can be infinitesimal but strictly not equal to 0 (Newton, Leibniz, and Robinson) or use limits to approximate such an infinitesimal value (Weierstrass). Different from both approaches, Deleuze in the Difference and Repetition offers a different approach solve the Berkley paradox by revealing the ontological status of the differential. Nevertheless, only a few scholars pay attention to Deleuze's analysis of calculus, and they either reduce Deleuze's analysis to modern Weierstrassian interpretations embodied in the concept of limit (Duffy, 2006 A) or hold that Deleuze follows the pre-Weierstrassian interpretation of calculus (Somers-Hall, 2009). In this paper, I intend to argue that both interpretations miss the crucial point of Deleuze's interpretation of calculus. Deleuze neither follows Weierstrass nor does he apply a pre-Weierstrassian approach. Indeed, he resorts to the Dedekind cut and the continuity of real numbers to argue that the differential is the pure element of quantitability (Deleuze, 1994), which is undetermined in relation either to any fixed numbers or normal variables. Therefore, when combined with fixed numbers or normal variables, the differential is rigorously identical with 0. However, since the differential is not 0 but the pure element of quantitability, it can function as the denominator even if separated from fixed numbers or normal variables.

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### Peano- and Hilbert curve

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In this presentation, we give a historical survey on Hilbert's interpretation of the space-filling Peano-curve which Hilbert presented at the GDNÄ session in Bremen in 1890. Since Hilbert was systematically working in the field of number theory in that period, we try to explain the reason for his sudden interest and immediate reaction to Peano's surprising result from the same year which proved the possibility of a continuous mapping of line to the plane. We argue that by means of his address, he was willingly trying to express his positive view on Cantor's set theory, an affinity which was also present by selecting the continuum hypothesis as the first problem on his list of the mathematical problems for the 20th century and which lead his thinking also long afterwards in the 1920s while working on his so-called Hilbert's program in logic. We present also Hilbert's correspondence with H. Minkowski on this topic.

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## Mathematatization and the Universality of Mathematics: The Possibility of a More Universal Formal Mathematics-Logic

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There is a paradox in Husserl's theory of formal mathematics and logic: On the one hand, he has endowed them with universality; namely they handle "*Etwas überhaupt*". On the other hand, he sharply criticized mathematization and logification of our living world by exact sciences in *Crisis*.

The paradox probably is probably a consequence of the fact that the contemporary mathematicslogic is not universal enough, while the more universal one remains undeveloped. Therefore, with Husserl's discussions on "horizon" "background" and "doxa", I attempt to develop a more universal formal mathematics-logic, a "subjective" "relative" "inexact" one in four steps: (1) without interference of attention (2) when attention functions as *Festhalten* and *Ausschließen* (3) how idealization gives rise to exact mathematics-logics (4) why the exact mathematics-logic is covered by the more universal, subjective-relative and underdetermined mathematics-logics. The more universal mathematics-logics has two modes: the idealized and the non-idealized one. They are identical only approximately, and ignoring their differences and the possibility of non-idealized formal mathematics-logic results in mathematization. Non-idealized mathematics is indispensable for artificial intelligence. Mathematization is not a lately product of modern natural science, but a process started from ancient: the approximate identity between idealized and non-idealized mode results in Plato's imitation theory, Aristotle's substance theory, determinism in natural science and normative function of exact mathematics-logics. That the current mathematics-logics is motivated by practical interest and zeal for the idealized product is the root why a descriptive formal mathematics-logic of "*Etwas überhaupt*" is underdeveloped compared with the idealized normative one.