# ICHM Co-Sponsored Special Sessions on the History of Mathematics

## Joint Mathematics Meetings, Boston, Massachusetts (USA)

## 6-7 January 2023

This set of lectures comprised a two-day Special Session on the History of Mathematics at the Joint Mathematical Meetings (JMM) hosted by the American Mathematical Society in Boston, Massachusetts. This was the first JMM to be held in person for three years, and speakers and attendees alike expressed pleasure at finally being able to be face-to-face again. The sessions were very well attended, with the most popular talks attracting over 100 people, with an average audience size of around 50.

The session was organized by Jemma Lorenat (Pitzer College), Deborah Kent (University of St. Andrews), Daniel Otero (Xavier University) and Adrian Rice (Randolph-Macon College), and featured 22 speakers from the United States, Canada, Mexico, and the United Kingdom.

The following talks were presented at the meeting:

**In Praise of Mathematics in Ancient China:**
**Works on Bamboo Strips and their Later Commentaries**
*Joseph Dauben, City University of New York*
In one of the earliest yet-known mathematical works from ancient China, a document preserved on bamboo strips at Beijing University, an opening section presents a dialogue between a master of mathematics, Chen Qi, and a student, Lu Jiuci. What the dialogue "Lu Jiuci Asks Chen Qi About Mathematics" raises are some very intriguing questions about the status of mathematics in ancient China and the relationship of the Beida material to other mathematical works that have also only recently come to light, like bamboo slips concerning mathematics that were excavated in 1983–1984 at a Western Han Dynasty tomb site near Zhangjiashan in Hubei Province, namely the *Suanshu shu* or *Book of Numbers and Computations*. Another set of bamboo slips important for comparison's sake with the Beida slips are those acquired by the Yeulu Academy in 2007, including a mathematical work, simply entitled *Shu* or *Numbers*. Direct comparisons with the Lu-Chen dialogue may also be made with the introductions and dialogues to be found in such later texts as the *Zhoubi suanjing*, the *Jiuzhang suanshu*, and the *Sun Zi suanjing*. In the Beida text, the student laments his inability to understand numbers and computations and then asks for guidance. What sets the Beida dialogue apart from the others is that the student, Lu Jiuci, admits that he has studied both classical literature and mathematical computation, but cannot thoroughly master them both, and then asks the master Chen Qi which of the two is most important? Chen Qi's answer and its implications for the comparative contexts of ancient mathematics East and West will be the subject of this presentation.

**The Tangled Tale of the Tangent**

*Glen Van Brummelen, Trinity Western University*

The tangent function in trigonometry has a long and complicated history: depending on how one defines its invention or discovery, one might choose a birth date anywhere between antiquity and the 18th century. We shall explore the emergence of the tangent in several cultures (especially Greece, Islam, China, and the West) when something like the tangent emerged, discussing shades of meaning in the identification of moments of discovery. We conclude with a new assertion of where the journey began that led to the tangent function we use today, within mathematical astrology in Italy in the 15th century.

**Mathematical Cosmology and Observational Astronomy 1920-1940**

*Craig Fraser, University of Toronto*

In the 1920s mathematical cosmologists investigated geometric models of the universe described by Einstein's general theory of relativity. During the same period astronomers were making a series of momentous discoveries in nebular astronomy. The two lines of work were largely but not completely independent of each other. A belief in mathematical foresight was expressed by Arthur Eddington, who wrote in 1929 that "a geometer like Riemann might almost have foreseen the more important features of the actual world." Eddington seemed to suggest that the revolutionary findings in nebular astronomy developed out of the theoretical framework of contemporary cosmology. A different point of view was expressed by astronomer Edwin Hubble, who in 1936 wrote, "The conquest of the Realm of the Nebulae is an achievement of great telescopes." A key question is whether the mathematical theory itself involved or implied a preference for expansionist world models, and related to this, to what extent work in relativistic cosmology was influenced by nebular astronomy. This paper explores possible answers to these questions.

**Speaking to the Public: Mathematicians on American Radio, the 1920s through 1940s**

*Karen H. Parshall, University of Virginia*
*Albert C. Lewis, Independent historian*

A new communications medium—radio—was introduced in the United States in the 1920s. Almost immediately, its potential for enlightening the public about all things scientific, including mathematics, was appreciated and exploited. This talk will highlight mathematicians on the radio through the 1940s taking the science news clearinghouse, Science Service, as its point of departure.

**Lucien Hibbert: In the Service of Mathematics and Country**

*Louis Beaugris, Kean University*

There have been efforts to share the stories and contributions of mathematicians of African descent or of the African diaspora. Not much is written about Lucien Hibbert (1899-1964) in the literature. This work recounts the story of Lucien Hibbert, born and raised in Haiti, who earned his Ph.D. in mathematics in 1937 at the Université de Paris after having studied at Rensselaer Polytechnic Institute in Troy, New York. He was one of the few mathematicians to complete their doctoral degree in France during the interwar years, and one of the first mathematicians of Black ancestry with a Ph.D. in mathematics. This paper illuminates his body of mathematical work, his long career in governmental and international affairs, his family ties, and his interactions with French mathematicians of the 1920s and 1930s. As his 125th birthday is approaching, this work aims to add to the history of mathematics by sharing his contributions to mathematics, statistics, and academia.

**Translation of *Calculus of Probabilities* by A. A. Markov, first edition, 1900**
*Alan Levine, Franklin and Marshall College*
The great Russian mathematician A. A. Markov wrote a book, *Calculus of Probabilities*, whose first edition appeared in 1900. Three later editions appeared in 1908, 1912 and 1924. I will speak about my recent English translation of the first edition. (None of the other editions have been translated into English.) The book provides a fascinating look at the state of probability in Russia at the time. I will highlight some of its interesting features and compare them to more modern approaches.

**Book History, Robert Simson, and John Playfair**
*Amy Ackerberg-Hastings, MAA Convergence*
I have been contemplating creating an article series for *MAA Convergence* that introduces historiography and methodology in ways that are accessible to undergraduate mathematics students and instructors. After providing an overview of the principles for researching and writing history, my current plan is to write or solicit brief explanations of various approaches to historical interpretation, accompanied by examples of how these approaches have been applied in the history of mathematics. Thus, this talk will outline a possible initial installment on book history and how I have used that method to make sense of the activities and significance of two 18th-century Scottish mathematicians, Robert Simson (1687–1768) and John Playfair (1748–1819). This project is also part of an ongoing reflection on the two-pronged problem posed by the series concept: 1) How can current historians of mathematics best train the next generation for the profession? and 2) What do most mathematics instructors and students need to know about the theory and practice of history?

**Early history of compactness and its use in pedagogical design**
*Naveen Somasunderam, SUNY, Plattsburgh*
The concept of compactness is arguably one of the more abstract and difficult concepts to grasp in undergraduate mathematics. It involves two seemingly disparate ideas, sequences and open covers, that nevertheless unify with some mild constraints of the underlying set.
In this talk, we examine the works of Dirichlet, Borel and Lebesgue who were early pioneers of this concept, which led Frechet and Hausdorff to develop the concept further in the early 1900s. We also discuss how the historical evolution of this concept can be used in the pedagogical design of an advanced calculus class.

**Imagining AI: mathematical stories in a museum context**
*Ursula Martin, University of Edinburgh, UK*
*David E Dunning, University of Pennsylvania*
*Máté Szabó, University of Greenwich, UK*
Oxford's Bodleian Library is one of the oldest libraries in Europe, and the second largest in the UK, holding over 13 million printed items alongside archives of figures such Byron, Shelley and Tolkien.
In recent years it has sought to work with mathematicians and scientists, as well as its more traditional humanities partners, to create compelling displays, based on Oxford's collections, that educate and inspire. Historians of mathematics and science are key to this process, providing accurate contemporary scholarship and linking specialist items to their broader context. At the

same time, designing such displays involves imagination and creativity in bringing items to new audiences through the power of the object alone, often supported only by a brief 20-word caption. We contrast two recent such displays, looking at the principles of selection, the lessons learned and the techniques used to evaluate them. In 2015 a small display celebrated the bicentenary of Ada Lovelace, bringing to the fore her mathematical ability through a focus on archival materials that had not previously been studied by historians of mathematics. In 2022 a display entitled "Imagining AI" used a variety of materials in the collections to illustrate early work on data (numerical, logical and text), algorithms and computation that underpins modern AI. By focussing on simple examples, with calculations small enough to be done by hand, we drew attention to issues and challenges for AI that pre-date modern computation and shed new light on questions such as AI ethics. Exhibits included Florence Nightingale's rose diagram, as an early example of data visualisation; Jevon's reasoning piano, that reduced reasoning a concrete process involving wood and wire; and Christopher Strachey's early computer poetry.

**Amos Dettonville, Blaise Pascal, and a Cycloidal Challenge**
*Maria Zack, Point Loma Nazarene University*
In 1658, Blaise Pascal began to consider the cycloid. This was a departure for him because in this period in his life his main focus was the study of theology. However, thinking about the cycloid was a distraction during some sleepless nights. His interest in the cycloid produced his own solutions for how to compute the area and center of gravity of any segment of the cycloid. He also computed the volume and surface area of the solid formed by rotating the cycloid around the *x*-axis. Using the pseudonym of Amos Dettonville, Pascal published a mathematical challenge, offering prizes for solutions to these problems. His challenge yielded two submitted responses and a number of comments from such well-known mathematicians as John Wallis, Christopher Wren, Pierre Fermat, and Christiaan Huygens. These responses provide an interesting glimpse into seventeenth-century mathematics.

**Mr. Newton's Approbation: John Flamsteed and Isaac Newton**
*Lawrence D'Antonio, Ramapo College*
This talk will focus on the interesting and tempestuous relationship between Isaac Newton and John Flamsteed, the first Astronomer Royal. Early in their relationship, Flamsteed said: "Mr. Newton's approbation is more to me than the cry of all the ignorant in the world." Thirty years later, Flamsteed wrote: "I always found [Newton] insidious, ambitious, and excessively covetous of praise, and impatient of contradiction."

**The history of $\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$ and its many names**
*Carey Witkov, Embry-Riddle Aeronautical University*
Students who have completed a course in single variable calculus are often surprised when they encounter the multivariable result $\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$ for functions that satisfy the implicit equation $f(x, y, z) = 0$. This remarkable result, with its sign alternating with number of variables, is found in the literature under a variety of names, including the triple product rule, cyclic chain rule, cyclic rule, cyclic relation, and Euler's chain rule, and has important applications in thermodynamics. This talk traces the history of this interesting result that, like so many results in calculus, has connections to Euler.

**Mathematics and the Real World: The Case of Jacob Amsler and his Planimeter**
*Eisso Atzema, University of Maine*
The Swiss scientific instrument maker Jacob Amsler (later Amsler-Laffon) (1823-1912) is probably most famous for his invention of the so-called polar planimeter, a device to determine the area of a plane region by tracing its contours, in 1854. Since the invention of this type of planimeter, many papers have been published purporting to explain why this planimeter does what it claims to do. Basically, all of these explanations heavily rely on Green's Theorem. Some stand up well to the test of time, whereas others are clearly incorrect or curiously vague to varying degrees. In this talk, I want to explore the contexts of some of the earlier papers as well as their author's use of Green's Theorem. Specifically, I will talk about Amsler's own thoughts about the functioning of his device and how his academical background as a mathematical physicist who worked with Franz Neumann in Königsberg may have played a role in the invention of the device to begin with.

**A Case of Mistaken Identity:**
**Hensel's Solution to the Problem of Common Inessential Discriminant Divisors**
*Fernando Q. Gouvêa, Colby College*
*Jonathan Webster, Butler University*
Number theorists know that Kurt Hensel solved the problem of characterizing when a number field has a prime number $p$ as a common "inessential" divisor of all element discriminants. They are correct about that, but they often point to the wrong theorem as representing Hensel's solution and they connect it to Hensel's $p$-adic methods. We set the record straight by showing who proved the theorem that is usually referred to, why that theorem does not really solve the problem, and how Hensel did find a first complete solution in 1894 without using the $p$-adics, which had not yet made their appearance.

**Everyday Numeracy in the Nineteenth Century:**
**Aspects of Arithmetic in Anglo-American Society**
*Adrian Rice, Randolph-Macon College*
The nineteenth century was the period in which the current conception of the utility of basic arithmetic began to emerge. Driven by rapidly-evolving developments in technology, economics, trade and industry, the fields in which everyday numeracy could be applied increased dramatically. To understand this expansion in the applicability of numeracy, and using Britain and the United States as a geographical focus, this talk will attempt to answer a variety of questions. To what extent was education in arithmetic available at this time, and to whom? What form did this education take, and what skills were imparted by contemporary textbooks? What systems of weights, measures and currency were in place, and what efforts were made to facilitate their use? And to what extent were people exposed to data and numerical information in everyday life? By focusing on aspects of everyday numeracy in Britain and the United States in the nineteenth century, we will compare and contrast the provision, application, and manifestations of arithmetic in the daily life of both cultures.

**Quaternions at Twilight: The final mathematical work of Mary Somerville**
*Brigitte Stenhouse, University of Toronto*
In 1871, when Mary Somerville was 90 years old, she received a copy of Peter Guthrie Tait's *Elementary Treatise on Quaternions*. The book was sent to her by then-president of the London

Mathematical Society, William Spottiswoode, to whom Somerville had written when looking for book recommendations on the 'higher algebra'. During her reading of Guthrie Tait, she took extensive notes, jotted down onto loose sheets of paper which are now held in the Somerville Collection at the Bodleian Library, Oxford. Using these archival sources I will detail Somerville's first documented engagement with the study of non-commutative algebra and relate this to her other mathematical writings.

**William Kingdon Clifford's evolutionary approach to mathematics and his posthumous reputation, 1860-1880**

*Sylvia Nickerson, University of Toronto*

What drives the mathematician to pursue their subject area of interest? How does their self-concept, personal identity and metaphysical belief shape this research, if at all? This talk explores these questions in the lives, works and philosophies of nineteenth-century mathematician William Kingdon Clifford and his novelist wife Lucy. In the history of philosophy, William is remembered for writing the strident essay, "The Ethics of Belief," in which he opposed organized religion in any form. In mathematics, Clifford was an early advocate of non-Euclidean geometry and quaternion algebra. He predicted curved space could cause observed physical forces. It will be demonstrated how William's commitment to the theory of evolution and to scientific naturalism motivated all aspects of his intellectual life from his arguments against religion to his revisioning of individual and social morality as well as providing an ethic he applied to pursuing his mathematical research. In the 1860s and 1870s Clifford "had some wild ideas" as his widow, Lucy, put it. The concluding part of this talk examines Lucy's role in securing William's posthumous reputation as a brilliant mathematician with prescient ideas who died too soon.

**Russell on the foundations of mathematics, early 1901**

*Alejandro R. Garciadiego, UNAM, Mexico*

Bertrand Russell (1872–1970) enjoyed an 'intellectual honeymoon' right at the threshold of the 20th century. He skated over all difficulties and problems concerning the composition of a monograph on the principles of mathematics. By then, Russell had already mastered Peano's notation and even contributed an original essay on the logic of relations. In the first month of the new century, the British philosopher also wrote a paper on the work of Boole and Cantor, among others. Inadvertently, this piece already contains some of the seeds that would ruin his confident state of mind. The purpose of this talk is to discuss this article's motivation, purpose, and contents.

**The four fours and four nines problems revisited**

*James J. Tattersall, Providence College*
*Shawn McMurran, California State University San Bernardino*

Recreational mathematics problems, besides being popular and interesting, are often of historical interest. We investigate the origin and evolution of the familiar four fours problem and the comparable but less familiar four nines problem. Henry Dudeney included the four fours problem in his *536 Puzzles and Curious Problems* and Martin Gardner popularized it in *The Incredible Dr. Matrix*. We discuss the origin of the problems and highlight work done on the problems and related problems by Thomas Dilworth, William Whewell, William Allen Whitworth, Allan J.C. Cunningham, William H. Turton, John Sturgeon Mackay, W.W. Rouse Ball, P.A.M. Dirac, and John H. Conway.

**Educating Engineers:**
**Developing Emergency Mathematics Curricula during the Second World War**
*Brit Shields, University of Pennsylvania*
To address the shortage of engineers employed by the defense industry during the Second World War, the United States Office of Education developed a national program of tuition-free courses to expediate the training of a technical workforce. Among the broad landscape of technical curricula, mathematics was seen as a foundational subject. A range of mathematics courses were widely offered as part of the Engineering, Science, and Management War Training Program. This talk will explore the ideology behind the training program, as well as its implementation within the mathematical sciences.

**Mathematical Equations as Illustrative Tools:**
**The Hidden Social Work of Mathematics in Biology**
*Abigail Taylor-Roth, University of Chicago*
In October of 1948, Nicolas Rashevsky, the Chair of the University of Chicago's newly formed Committee on Mathematical Biology, received a letter asking for help working out a "mathematical theory of the female sexual cycle." Through a few exchanges, Rashevsky and his colleagues worked with the writer, Russel Kesselman, to develop a set of differential equations representing the agency of the hormone molecules in the cycle. Unsurprisingly, the agency of menstruating humans themselves was ignored and left out from the equations. This exchange between Kesselman and Rashevsky highlights the hidden social work inherent in crafting equations, through both making initial assumptions and deciding what to include or exclude. Drawing on work in feminist science studies and studies of mathematical illustrations, I argue that these equations are illustrative tools that do social work by lending (supposed) objectivity to the biological project through mathematization, while simultaneously molding what can or cannot be seen. The central role of mathematics in this project is characteristic of Rashevsky's mathematics-based approach to mathematical biology, which suggests that studying illustrative equations is a worthwhile and necessary means through which to understand the development of biological knowledge. I suggest that elucidating the illustrative role of mathematical equations can lead to imagining alternate ways of seeing and doing mathematics.

**Mathematics in a Children's Classic: Norton Juster's *The Phantom Tollbooth***
*Thomas Drucker, University of Wisconsin–Whitewater*
In the years 1865 and 1871 a British mathematician named Charles Dodgson wrote two books for children (*Alice's Adventures in Wonderland* and *Through the Looking-Glass and What Alice Found There*) under the pseudonym of Lewis Carroll. His use of mathematical ideas throughout both books added to the distinctive tone which has endeared them to readers through the years. Almost a century later an American architect named Norton Juster decided to switch to the writing of fantasy and produced a work that has stood the test of the decades, if not yet the centuries. *The Phantom Tollbooth*, as Juster's book was called, has mathematics woven through it even more conspicuously than Carroll's books. It has had an influence on young readers who were able to get a different view of what mathematics was about. In this talk the background of mathematics teaching in Juster's youth will be shown to contribute to the writing of the book's mathematical sections. In addition to some comparisons with Carroll, the benefits that readers have derived from *The Phantom Tollbooth* will help to explain the popularity of the book throughout the author's life and after his death in 2021.