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Mathematics meets Physics – General and local aspects

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Today the mutual influences between mathematics and physics are not questioned in general, even if these might only be perceived as expressing the facts of nature in a more simple and clear way by means of mathematical formulae. However, the interrelations between mathematics and physics are more extensive and also much deeper: On the one hand an appropriate mathematical description of physical phenomena is able to produce new insights in their structures and might facilitate the deduction of further physical connections. On the other hand the search for such a description supplies various ideas to develop mathematics further. A careful look at this since thousands of years existing interrelationship between mathematics and physics shows varied changes which are determined by the internal development of the disciplines as well as by varying external conditions.

The last two centuries are of special interest in this respect since they can be characterised by a large growth of knowledge in both disciplines with the forming of many sub-disciplines and by fundamental changes through new theories and methods such as the set-theoretic penetration of mathematics, quantum theory or the principle of general relativity. In general relativity theory and in quantum theory the formation of physical theories was inseparable from new mathematical concepts – so much so that Y. Manin said with regard to the changed interrelation between mathematics and physics that "without the mathematical language physicists couldn't even say what they were seeing".

The aim of the conference is to capture and analyse the changes of this interrelationship in the 19th and the first half of the 20th century. The attention should be put on the following aspects:

– characteristic features of the mathematisation of sub-disciplines of physics

– repercussions of this process of mathematisation of physics upon mathematics

– development of the relationship between mathematical and theoretical physics

– the shaping of the interrelationship between mathematics and physics by individual scientists in local contexts

General features of this interrelationship are going to be brought out by comparing different local developments among each other and also to the sketched larger processes.

After a welcome by Pirmin Stekeler-Weithofer, the President of the Saxon Academy of Leipzig, the conference continued in four sections and was rounded off by a panel discussion. In the following an outline of the sessions is given providing abstracts of the talks if available.

Section 1: General Developments

Examples of and reflections on the interplay between mathematics and physics in the 19th and 20th century
Jesper Lützen (*Institute for Mathematical Sciences, University of Kopenhagen)*

After some general remarks concerning the nature and development of mathematics and physics and the interaction of the two fields I shall turn to three instances of such interactions selected from my earlier research. First I shall investigate the influence from physics on the mathematical theories and results developed by Joseph Liouville including Sturm-Liouville theory and the theory of differentiation of differentiation of arbitrary order (fractional calculus). Next I shall discuss the interaction between geometry and mechanics in Heinrich Hertz's Principles of Mechanics, and lastly I shall investigate the physical theories that influenced Laurent Schwartz' creation of the theory of distributions.

Mathematics as one of the basic pillars of physical theory: historical and epistemological survey
Juraj Šebesta (*Department of Theoretical Physics and Physics Education, Comenius University Bratislava)*

Physical theory as an essence of physical knowledge. Four basic pillars of physical theory. Mathematics as tool of construction of physical theory - historical survey. Epistemological role of mathematics in physics up to J. C. Maxwell and in Maxwell´s theory. Intensive mathematisation of physical theories. Mathematics as source of physical knowledge. Axiomatisation - one stage in development of physical theory. "Erlangenisierung" of physical theories - past and present.

From q-numbers to Hilbert spaces: The interplay of mathematics and physics in the rise of quantum mechanics
Jan Lacki (*Histoire et Philosophie des Sciences, Université de Genève)*

In a remarkable paper from fall 1926, *Winkelvariable und kanonische Transformationen in der Undulationsmechanik*, the German physicist Fritz London achieves a novel and deeper understanding of the relationship and hence of the equivalence of matrix mechanics promoted by the Göttingen physicists and wave mechanics championed by Schrödinger. In an attempt to understand the meaning of the analogues of classical canonical transformations in the context of the new mechanics, London realizes, anticipating the transformation theory of Dirac and Jordan, the mathematical linear nature of the operations involved in the changes of variables required by solving quantum mechanical problems. In a note, he draws attention to the extant mathematical literature devoted to functional spaces, namely works the Italians Pincherle (1897, 1905) and Cazzaniga (1899) on “distributive functional operations” where a theory of formal operations on functional spaces is developed. Another reference given by London is French Paul Levy's "Lecons d'Analyse Fonctionnelle”, which deal with functional spaces emphasizing the “geometrical” (linear space) aspect of the situation. London's achievement and his lucid appraisal of the importance of his observations conveniently mark the opening of a fascinating chapter in the relationships between quantum mechanics, functional analysis and linear spaces which will shortly culminate in the work of von Neumann and his formulation of quantum mechanics as an operator calculus in Hilbert space. In my contribution, I shall review the situation of quantum theory as of 1926, the problems in its formulation and the contribution of London which deserves to be better known. I shall then examine some landmarks in the merging of the mathematical research trend starting with the Riesz-Fischer theorem, the growth of functional analysis, Hilbert's theory of integral equations and its avatars, with physical research on quantum theory, specifically transformation theory, von Neumann's Hilbert space formalism, and Weyl's own vision of quantum mechanics, with his deep results on the representation theory of the Heisenberg group.

Good reasons for and against a mathematization of physics: On forms of theoretical physics of Max Planck, Wolfgang Pauli and Max Born
Arne Schirrmacher *(Max-Planck-Institut für Wissenschaftsgeschichte, Berlin)*

Although Max Planck was the first theoretical physicist who did without facilities for experimentation, nonetheless his theoretical physics was often more influenced by experimental ways of thinking than by mathematical ones. He disapproved of rigorous axiomatic mathematization, e.g. of the theory of heat radiation, which David Hilbert suggested strongly. My contribution will start from the Planck-Hilbert dispute and then explores other examples in which not only good reasons in favor of a mathematization of physics were raised, as this would help physical theories to become more consistent and far-reaching, but also good reasons against it became apparent, since it could also distract from physical argument and insight. Wolfgang Pauli in his early work on atomic physics eventually became critical towards the attempt of a rigorous mathematization that should allow for calculating the stability of the hydrogen molecule ion by methods of celestial mechanics, while Werner Heisenberg found his way towards quantum mechanics rather by puzzling around with under-defined models and conceptions, and only slowly considered and accepted mathematical issues as did the other quantum physicists. Max Born in turn had used matrix calculus in physics not only in matrix mechanics, he rather had employed it before in several physics problems. Did he follow the gospel of mathematization? It was him, however, who spread the credo that in the end it is the correctly grasped mathematical formalism that carries more physical meaning than a direct physical interpretation would suggest at first.

Section 2: Local contexts

Wechselverhältnis von Mathematik und Physik an den Universitäten Leipzig, Halle und Jena - ein Vergleich
Karl-Heinz Schlote, Martina Schneider (*Sächsische Akademie der Wissenschaften zu Leipzig)*

The development of the interrelation between mathematics and physics at the universities of Leipzig, Halle and Jena shows various forms of how this intricate and complex relationship manifested itself in the 19th and 20th century. In our talk this diversity will be explored. We will touch upon connections with philosophy as well as with technology and experimentation (Jena, Halle). The case of advanced research in mathematical physics and, later, in theoretical physics at Leipzig will be covered and contrasted with that of 'normal' science at Halle. In Halle the research of theoretical physicists included extensive experimental investigations, and from the mid 1920s onwards the mathematicians no longer considered physical questions in research nor in teaching.

Die theoretische Physik an der Universität Hamburg in den Jahren 1921-1959
Karin Reich *(Hamburg/Berlin)*

Geometry as physics: Oswald Veblen and the Princeton School in the 1920s
Jim Ritter *(Mathématiques et Histoire des Sciences, Université Paris)*

In the years immediately following the First World War, the new theory of general relativity and its extensions inspired creative reactions in a large number of mathematicians around the world. One of the most structured and long term of these was that of a group of geometers at Princeton University, who, under the leadership of Oswald Veblen and Luther Pfahler Eisenhart, reoriented both the nature of the mathematics practiced at that University and the manner of doing it. On the one hand the “physicalization of geometry”, on the other the “industrialization of research”: the coordination and planning of the subjects to be investigated, the mobilization of resident mathematicians, and the use of a steady stream of invited scholars and of a newly-created group—post-doctoral fellows. Though the effort was only partially successful on both counts, the direction of modern differential gometry and the nature of the mathematical/physical interface today owe much to this program.

Mathematics meets physics in early twentieth century Paris: the case of Brownian motion
Charlotte Bigg (*Centre Alexandre Koyré, Centre de Recherche en Histoire des Sciences et des Techniques, Paris)*

“May an investigator succeed in deciding on the question raised here, which is of such importance for the theory of heat!“ Albert Einstein’s appeal, made at the very end of his first paper on Brownian motion (1905), was quickly answered. Most notably, the French physical chemist Jean Perrin and his students supplied in 1908-1909 what is considered as the decisive experimental verification of Einstein’s theory of Brownian motion. Yet Perrin did not subscribe to a division of work between theoretician and experimentalist, a division that was in fact quite recent at the time, and one still characteristic of the German universities where it had first been institutionalized.
Though in the French context no comparable field of theoretical physics emerged in the early twentieth century, new developments in physics, especially in statistical mechanics, put a new twist on the question of how chemistry, physics and mathematics could and should constitute ressources for each other. The investigations of Brownian motion by French scientists following Einstein’s publications, most notably the closely-related work of physicist Paul Langevin, physical chemist Jean Perrin, and mathematician Emile Borel was the occasion for a reflection on the practical and epistemological implications of interdisciplinary dialogue: how might experiments on Brownian motion be conceptualised and made fruitful for mathematical pursuits? how, conversely, were physicists and chemists to deploy mathematicians’ statistical theories in their study of physical and chemical processes?

Section 3: Scientists

Theoretical physics and relativity in Paris during the Belle Époque
Scott Walter *(Henri Poincaré Archives, Université 2, Nancy)*

French theoretical physics at the turn of the twentieth century can be characterized in large part by attending to the activity of Parisian physicists and mathematicians. The important contributions of the leading Parisian theorist of the time, Henri Poincaré, to celestial mechanics, relativity and quantum theory are well known, but by their very brilliance, they tend to mask the work of Poincaré's students and colleagues. Attention to the contributions of these lesser-known theorists helps us to understand the failure of Poincaré's program for relativity, and the related success of Einstein-Minkowski theory in Paris.

Poincaré and Saturn’s rings: equilibrium figures and the prestige of mathematics circa 1900
Tom Archibald *(Department of Mathematics, Simon Fraser University, Burnaby)*

Probability and Statistics as connecting links between mathematics and physics: the approach of Richard von Mises in the 1920s

Reinhard Siegmund-Schultze (*University of Agder, Kristiansand)*

One reason for Richard von Mises to create his new foundations for probability theory in 1919, based on the limit of relative frequencies, was inconsistencies in mathematical physics, in particular in the theory of Brownian motion. Another stimulus for von Mises' interest in probability theory was his effort to model indeterminism mathematically. In this respect he was not only influenced by his positivistic philosophical positions but also by contemporary developments in quantum theory, although he even anticipated some of these developments with his theory. Von Mises contributed also with details to statistical applications in physics, not last with his cyclical error theory, which was triggered by speculations about the integer-valuedness of atomic weights. In the following decade of the 1920s von Mises contributed to the theory of Markov chains, although based on his paradigm in probability and technically depending on matrix theory. This proved to be of at least indirect value both for ergodic theory and the theory of stochastic processes, which were later on developed with more mathematical depth at the hands of Birkhoff, Khinchin, P. Lévy and others.

Einsteins Verhältnis zur Mathematik

Tilman Sauer (*Einstein Papers Project, California Institute of Technology, Pasadena)*

More than once, Einstein profited from substantial help by professional mathematicians in his research. I will review the role that mathematics and mathematicians played in Einstein's work, specifically in what we have called a ``mathematical strategy'' in his search for a general theory of relativity. More generally, I will review the role that mathematics and mathematicians played in the emergence and early history of the general theory of relativity and of unified field theory. I will argue that in his reflections on his own work, Einstein after 1915 began to realize and acknowledge the productive heuristic and analytic power of mathematics for theoretical physics research, even though he himself never was nor became a mathematician proper.

Anmerkungen zu Hermann Weyls Auffassung der Beziehung zwischen Mathematik und Physik

Erhard Scholz (*Fachbereich C Mathematik und Naturwissenschaften, Bergische Universität Wuppertal)*

The two fields in which modern physics broke off most strikingly from the classical framework during the early 20th century were general relativity (including the ensuing attempts for unified field theories) and quantum mechanics (including the first steps toward more general quantum physics). For some actors, e.g., those sharing D. Hilbert's view of an axiomatization program for physics, the relationship between mathematics and physics seemed to be more or less the same in both fields. For others, among them H. Weyl, the peculiarities of the rising quantum mechanics brought sufficient surprise to rethink the relationship between mathematics and physics. The talk aims at illuminating the changing roles Weyl saw for mathematical theorizing in his contributions to the two physical fields, the common core of which became clearer during the years. In the words of our protagonist it was a "symbolical construction of the world" to which he saw mathematics and physics contributing in a common enterprise. In these formulations he also signalled a partial reconciliation with Hilbert's viewpoint.

Section 4: Development of concepts and theories

The unusual interaction of physics and mathematics in the formation of field theory
Friedrich Steinle (*Institut für Philosophie, Literatur-, Wissenschafts- und Technikgeschichte, TU Berlin)*

The formation of field theory in the second half of 19th century was a prominent case of mathematisation processes, but at the same time had characteristics that made it strikingly different from other those processes. Not only were Faraday's concepts not formulated in mathematical terms of his time, but there were no mathematical tools available that could easily be adjusted to those concepts. At the same time, however, both Thomson and Maxwell credited Faraday for having high mathematical qualities. In my talk, I shall analyze in detail what they had in mind, and how exactly the mathematical character of Faraday's concepts could be grasped that made an analytic approach possible in the end. The case sheds light, finally, on a rather uncommon type of relating experiment and mathematics.

The notion of "angular momentum" between mathematics and physics
Arianna Borrelli (*Interdisziplinäres Zentrum für Wissenschafts- und Technikforschung, Bergische Universität Wuppertal)*

The notion of angular momentum is a privileged starting point to explore the relationship between mathematics and physics: already in the context of classical physics there was a tension between the formal, mathematical definition of angular momentum and its use in studying rigid bodies or hydrodynamic systems. The relationship between mathematical and physical notions of angular momentum became even more complex with the advent of quantum theory. In the atomic model of Bohr and Sommefeld the term „angular momentum" took up a prominent position, yet Bohr himself cautioned against interpreting it as more than a symbol taken over from classical mechanics. Thanks to experiments involving magnetic moment, though, the classical and quantum notions of angular momentum were brought closer to each other. When the new quantum mechanics emerged in the 1920's, it brought with it a mathematical notion of angular momentum which implied the impossibility of ever measuring the physical quantity associated to it. At the same time, though, the new notion offered the chance of embedding the until then highly problematic concept of „spin", and this step would be of central significance for the future development of relativistic quantum field theory.

Mathematical and phenomenological rigor: distributions in quantum mechanics and quantum field theory
Klaus-Heinrich Peters (*Hamburg)*

Long before the theory of distributions was invented by Laurent Schwarts in the 1940s, physicists used them as "improper functions". Examining the attempts of mathematical oriented physicists to build a self-consistent formalism with the help of the mathematically correct theory, they seem to have one striking feature in common: The mathematically correct treatments also match the phenomenologically given facts of physics better than the usual "improper" way – provided that "facts" are understood in the sense of the Copenhagen interpretation of quantum mechanics. The aim of the talk is to demonstrate this parallelism between mathematical and phenomenological rigour along the lines of the historical development of quantum mechanics (Dirac and von Neumann) and quantum field theory (from Wightman to Epstein and Glaser).

The role of mathematics in the construction of the relativistic quantum mechanics
Helge Kragh *(Department of Science Studies, Aarhus University)*

Mathematical foundations and physical visions: Pascual Jordan and the quantum field theory program
Christoph Lehner (*Max-Planck-Institut für Wissenschaftsgeschichte, Berlin)*

> Pascual Jordan was more than any other leading quantum physicist a product of Göttingen mathematical physics. Within a few years of his immatriculation, he was Max Born's assistant and coworker on anything from lattice dynamics to quantum theory, had assisted Richard Courant on the Methods of Mathematical Physics, and James Franck on his Handbuch article "Anregungen von Quantensprüngen durch Stöße." His work in the following years shows a remarkable combination of mathematical sophistication and physical insights. But it also displays a puzzling tension between a radical positivism, expressed in his transformation theory, and a grand foundational vision, expressed in his program of quantum field theory. Despite his vocal support of Bohr's doctrine of complementarity, Jordan was never satisfied that the riddles of quantum physics had been resolved. I argue that this tension in his work is intimately related to Jordan's views about the relation between mathematical apparatus and the foundations of physics.

The conference ended with a panel discussion on continuities and discontinuities in the development of the interrelation between mathematics and physics presented by ERHARD SCHOLZ. The three members of the panel – the theoretical physicist BODO GEYER, the mathematician and founding co-director of the Max-Planck-Institute for Mathematics in the Sciences EBERHARD ZEIDLER (both Leipzig) and the historian of sciences JIM RITTER had very different opinions of the subject. Geyer tried to capture the interrelation on a general level and to look, among other things, for structural links between theoretical physics and mathematics. Zeidler basically presented the interrelation as a long history of successes in which delays and parallel developments usually were the result of a lack of communication between mathematicians and physicists. Yet, Ritter pointed out that the conference's talks showed that one could not talk of a continuous development in a dynamical sense. He rather compared this development to a nowhere continuous function. Therefore, a further historisation of the concepts of mathematics and physics was necessary. He put down the differing opinions of the members on the panel to their different interests, demands and wishes: active mathematicians and physicists are interested in things that have worked or might work in future times, whereas historians are also interested in theories which did not work, that is in science in the broad sense and in the dynamics of their development.