

## **Participationist discourse on mathematics learning**

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In the last decade or two, the claim that mathematics learning is a social process can be heard with such frequency that it became almost a cliché. And yet, those who declare their belief in the social nature of learning have an important statement to make: They signalize that in the ongoing debate between cognitivist and sociocultural research communities they side with the latter. This paper is devoted to explicating theoretical and practical consequences of this message.

These days, being explicit about what one means while claiming “the social nature of learning” seems a necessity. In spite of the omnipresence of the word “social” in the current literature – or perhaps just *because* of it! – there is much confusion about how this term should be understood when applied in conjunction with learning.<sup>1</sup> To avoid undesirable connotations, I use a different terminology. Due to the metaphor for learning underlying the particular family of sociocultural discourses to be presented on the following pages, I call these discourses *participationist*. To bring the special features of the participationism in fuller relief, I present it against the contrasting background of the more traditional *acquisitionist* approach. The origins of participationism can, indeed, be traced to acquisitionists’ unsuccessful attempts to deal with certain long-standing dilemmas about human thinking. After surveying some of these resilient puzzles and presenting basic participationist tenets, I show how the claim that participationism, if followed in a disciplined way, leads to the claim that human thinking originates in interpersonal communication. I finish with a few remarks on the consequences of the participationism for theory and practice of mathematics education and demonstrate how it helps in dealing with some of the questions that acquisitionism left unanswered.

## 1. Acquisitionism and its dilemmas

The roots of acquisitionist discourse on learning, which is usually seen as originating in the work of Piaget, go in fact much deeper. The underlying metaphor of learning as an act of increasing individual possession - as an *acquisition* of entities such as concepts, knowledge, skills, mental schemas – comes to this scholarly discourse directly from everyday expressions, such as *acquiring knowledge*, *forming concepts* or *constructing meaning*. To get a sense of the impact of the metaphor of acquisition on one's interpretation of human mathematical activities, let me take a look at the following episode, featuring young children talking with grownups about numbers. The brief scene is the beginning of a series of conversations about numbers between my colleague Irit Lavi and two young girls: 4 year old Roni, Irit's daughter, and 4 year 7 months old Eynat, Roni's friend. The event took place in Roni's house.<sup>2</sup>

### *Episode: Comparing boxes with marbles*

Speaker	What is said	What is done
1. Mother	<i>I brought you two boxes. Do you know what is there in the boxes?</i>	<i>Puts two identical closed opaque boxes, A and B, on the carpet, next to the girls.</i>
2. Roni	Yes, marbles.	
3a. Mother	Right, there are marbles in the boxes.	
3b. Mother	I want you to tell me in which box there are more marbles.	<i>While saying this, points to the box A close to Eynat, then to box B.</i>
3c. Eynat		Points to box A, which is closer to her.
3d. Roni		<i>Points to box A</i>
4. Mother	In this one? How do you know?	<i>Points to box A</i>
5. Roni	Because this is the biggest than this one. It is the most.	<i>While saying "than this one" points to box B, which is close to her</i>
6. Mother	Eynat, how do you know?	
7. Eynat	Because... cause it is more huge than that.	<i>Repeats Roni's pointing movement to box B when saying "than that"</i>
8. Mother	Yes? This is more huge than that? Roni, what do you say?	<i>Repeats Roni's pointing movement to box B when saying "than that"</i>
9. Roni	That this is also more huge than this.	<i>Repeats Roni's pointing movement to box B when saying "than that"</i>
.....	.....	.....

- 10a.** Mother     *Do you want to open and discover?  
Let's open and see what there is  
inside. Take a look now.*
- 10b.** Roni     *Abruptly grabs Box A, which is nearer to  
Eynat and which was previously chosen as  
the one with more marbles.*
- 11.** Roni     1.. 1.. 1.. 2, 3, 4, 5, 6, 7, 8.     *Opens box A and counts properly.*
- 12.** Eynat     1, 2, 3, 4, 5, 6.     *Opens box B and counts properly.*
- 13.** Mother     So, what do you say?
- 14.** Roni     6.
- 15.** Mother     Six what? You say 6 what? What  
does it mean "six"? Explain.
- 16.** Roni     That this is too many.
- 17.** Mother     That this is too much? Eynat, what  
do you say?
- 18.** Eynat     That this too is a little.
- 19.** Mother     That it seems to you a little? Where  
do you think there are more  
marbles?
- 20.** Roni     I think here.     *Points on the box , which is now close to her  
(and in which she found 8 marbles)*
- 21.** Mother     You think here? And what do you  
think, Eynat?
- 22.** Eynat     Also here.

The episode is likely to leave the acquisitionist researcher unimpressed. The girls' mastery of counting would just confirm what she knows only too well from previous studies: 4 and 5 year old children are usually advanced enough in their "acquisition of the concept of number" to be able to count properly (for a summary of the relevant research see e.g. Nunes & Bryant, 1996, Dehaene, 1997). Nor will the acquisitionist researcher be stricken by the fact that in spite of their well developed counting skills, the girls did not bother to count the marbles or even to open the boxes when asked to compare these boxes' invisible contents. Extensive acquisitionist research on early numerical thinking, in which young children have been observed implementing different versions of Piagetian conservation tasks, has shown that at this age, this behavior is quite normal: "Children

who know how to count may not use counting to compare sets with respect to number” (Nunes & Bryant, 1994, p. 35).

And yet, knowing what children usually *do not do* is not enough to account for what they *actually do*. An unprejudiced observer, whose analysis is not biased by the sole interest in the girls’ ability to “operate with numbers”, is likely to ask questions to which the acquisitionist researcher may have no answers. Thus, the young interviewees’ apparently arbitrary response to the question “Which box has more marbles?” cannot be accounted-for simply by the reference to ‘underdeveloped number schemes’. Similarly, the fact that the girls agreed in their surprising decisions does not seem to have much to do with insufficiency of their “conception of number”. Finally, one should rather not count on acquisitionist explanation while wondering what made the children “justify” their choice in a seemingly adequate way in spite of the fact that they had no grounds for the comparative claims, such as “this is the biggest than this one”, “It is the most” ([5]) and “it is more huge than that” ([7]). If there is little in the past research to help us account for this kind of phenomena, it is probably because the acquisitionists, while watching their interviewees, attended to nothing except for those actions which they classified in advance as relevant to their study. For them, the conversation that preceded opening of the boxes would be dismissed as a mere ‘noise’. The analysis of the remaining half of the event might even lead them to the claim that the girls had a satisfactory command over numerical comparisons, although this is not the vision that emerges when the second part of the episode is analyzed in the context of the first.

Probably the main reason for the shortcomings of acquisitionists’ accounts is these researchers’ belief in the invariability of learning processes across different contexts. In their research, these scholars are tuned to cross-situational commonalities rather than differences. For them, individual minds are the principal source of their own development, whereas the task of the researcher is to discover the universal blueprint of the process. In

result, acquisitionist discourse is ill equipped to deal not just with inter-personal and cross-situational differences, but also with those changes in human processes that transcend a single life span. Indeed, as long as human learning is seen as originating in the individual, and as long as this process is thought practically impermeable to other influences, notably those coming from interactions with other individuals, one has no means to account for the fact that human ways of doing, unlike those of other species, evolve over history. Within the confines of acquisitionist discourse, there is no cogent explanation for the fact that the outcomes of the ongoing transformations accumulate from generation to generation, constantly redefining the nature and extent of the individual growth.

## **2. Participationism and its solutions to acquisitionist dilemmas**

Although usually traced back to the work Vygotsky and other founders of Activity Theory,<sup>3</sup> participationism has, in fact, a more extensive genealogy. As a confluence of ideas coming from areas as diverse as philosophy, sociology, psychology, anthropology, linguistics, and more,<sup>4</sup> this relatively new school of thought is a *mélange* of approaches rather than a single research discourse. Some of these approaches depart from the acquisitionism only marginally, in that they merely add social considerations to the traditional individualist account. Lave (1993) speaks about ‘cognition plus’ whenever referring to the talk about the ‘social’ mounted on the top of an acquisitionist discourse. The basic claim that motivates the more radical form of participationism is that *patterned, collective forms of distinctly human forms of doing are developmentally prior to the activities of the individual*. Whereas acquisitionists view the individual development as proceeding from personal acquisitions to the participation in collective activities, strong participationists reverse the picture and claim that people go from the participation in collectively implemented activities to similar forms of doing, but which they are now able to perform single-handedly. According to this vision, learning to speak, to solve mathematical

problem or to cook means a gradual transition from being able to take a part in collective implementations of a given type of task to becoming capable of implementing such tasks in their entirety and on one's own accord. Eventually, a person can perform on her own and in her unique way entire sequences of steps which, so far, she would only execute with others. The tendency for individualization<sup>5</sup> – for turning patterned collective doings into activities for an individual – seems to be one of the hallmarks of humanness, and it is made possible by our capacity for overtaking roles of others.

The difference between the acquisitionist and the participationist versions of human development is thus not just a matter of "zoom of lens," as it is sometimes presented (Rogoff, 1995; Lerman, 1998). Above all, it manifests itself in how we understand the origins and the nature of human uniqueness. For acquisitionist, this uniqueness lies in the biological makeup of the individual. While participationism does not deny the need for special biological pre-requisites - such as, for example, the special voice cords and the ability to discern certain sounds, both of which are the basis for effective human communication - this approach views all the uniquely human capacities as resulting from the fundamental fact that humans are social beings, engaged in collective activities from the day they are born and throughout their lives. In other words, although human biological givens are what makes this collective form of life possible, it is the collective life that brings about all the other uniquely human characteristics, with the capacity for individualizing the collective – for individual reenactments of collective activities - being one of the most important. Human society emerges from the participationist account as a huge fractal-like entity, every part of which is a society in itself, indistinguishable in its inner structure from the whole.<sup>6</sup>

Another notable change that happens in the transition from acquisitionist to participationist discourse is in the unit of analysis. It is this new unit which I had in mind while speaking, somewhat ambiguously, about "patterned collective doings". Other eligible

candidates for the participationist unit of analysis are *form of life*, suggested by Wittgenstein (1953), and *activity*, the pivotal idea of the Activity Theory. The nowadays popular term *practice* is yet another viable option (see e.g. Lave & Wenger, 1991; Wenger 1998; Cobb 2002). Although all these terms are used in the current literature in numerous ways, with the differences between one use and another not always easy to tell, each of them is good enough for my present purpose. Indeed, all I want, for now, is to describe participationist innovation according to those central characteristics which remain basically the same across different renderings. Whatever name and definition are given to the participationist unit of analysis and whatever claims about humans are formulated with its help, the strength of this unit is in the fact that it has both collective and individual 'editions.'

Armed with this flexible analytic focus, participationists have a chance to address the question of change that exceeds the boundaries of individual life. While speaking about human development, participationists do not mean a transformation in people, but rather in forms of human doing. This non-trivial discursive shift is highly consequential, as it removes the sharp acquisitionist distinction between development of an individual and the development of collective. The developmental transformations are the result of two complementary processes, that of *individualization of the collective* and that of *collectivization of the individual*. These two processes are dialectically interrelated and, as a consequence, both individual and collective forms of doing are in a constant flux, resulting from inevitable modifications that happen in these bi-directional transitions.

So far, I have shown how participationism deals with the dilemma of the historical change in human forms of doing. In the rest of this paper I show how it deals with questions about mathematics learning that acquisitionism left unanswered.



### 3. Consequences of participationism for the discourse on mathematics thinking and learning

#### 3.1 *What is thinking?*

Although thinking appears to be an inherently individual activity, there is no reason to assume that its origins are any different from those of other uniquely human capacities: like all the others, this special form of human doing could only develop from a patterned collective activity. This claim is far from intuitively obvious. After all, whatever we call thinking is usually done by each one of us alone and is generally considered as inaccessible to others in the direct manner. It is thus not readily evident which ‘visible’ human activity might be the collective version of thinking. In fact, one has good reasons to doubt whether such collective edition exists at all. More than any other human activity, thinking appears biologically determined and growing ‘from inside’ the person. Still, participationist tenets speak forcefully against this deeply rooted conviction. The next thesis to explore is that *interpersonal communication* is the collective activity that morphs into thinking through the process of individualization.

A powerful, even if indirect, argument comes to mind immediately when one tries to substantiate this conjecture. The ability to think in the complex way people do is absent in other species – and so is the human highly developed ability to communicate. At a closer look, communication, like thinking, may be one of the most human of human activities. This is not to say that the ability to communicate is restricted to people. At least some animals do seem to engage in activities that one may wish to describe as communication. And yet, human communication is special, and not just because of its being mainly linguistic – the feature that, in animals, seems to be extremely rare, if not lacking altogether. It is the role communication plays in human life that seems unique. The ability

to coordinate our activities by means of interpersonal communication is the basis for our being social creatures. Our very survival, not to speak about our distinctive forms of living, depends on our being always a part of a group. And since communication is the glue that holds human collectives together, even our ability to stay alive is a function of our communicational capacity. We communicate in order to ascertain the kind of mutuality and collective doing that provide us with what we need and cannot be attained single-handedly. The list of human needs that would remain unsatisfied without interpersonal communication is long and multifarious, and it includes not just the most advanced and complex cultural needs, but also the most primitive biological ones, of the kind that most animals are able to provide by themselves, with only marginal collaboration of other individuals. In the view of all this, it is not surprising that Leont'ev (1930), one of the founding fathers of participationism, declared the capacity for communication as the hallmark of humanness: "[W]e do not meet in the animal world any special forms of action having as their sole and special end the mastery of the behavior of other individuals by attracting their attention" (p. 59).

All this, as important as it may sound, is not yet enough to substantiate the claim that thinking could be defined as *a form of communicating*. In fact, the current discourses go directly against this vision when they present thinking and communicating as separate, even if tightly connected. This, indeed, is how these two activities are pictured in colloquial forms of talk, through expressions such as 'communicating one's thoughts' or 'putting thoughts in words'. Our speaking about thoughts as being *conveyed* (or *expressed*) in the act of communication implies two distinct processes, that of thinking and that of communicating, with the former slightly preceding the latter and constantly feeding into it. According to this vision, the outcomes of thinking, pictured as entities in their own right, are supposed to preserve their identity while being "put in other words" or "expressed somehow differently".

Whereas acquisitionists have been working with this dualist vision of human cognition for centuries, participationists are likely to view the idea of ‘thought-conveyed-in-communication’ as but a direct result of an unhelpful objectification. With Wittgenstein (1953), they believe that "Thought is not an incorporeal process which lends life and sense to speaking, and which it would be possible to detach from speaking" (p. 108). Having accepted this claim, one can also see that it remains in force when the somewhat limiting word *speaking* is replaced with the more general term *communicating*. Consequently, thinking stops being a self-sustained process separate from and, in a sense, primary to any act of communication, and becomes an act of communication in itself, although not necessarily interpersonal. All this justifies the claim that *thinking may be usefully defined as the individualized form of the activity of communicating*, that is, one’s communication with oneself. Of course, this self-communication does not have to be in any way audible or visible, and does not have to be in words.<sup>7</sup> In the proposed discourse on thinking, cognitive processes and processes of inter-personal communicating are thus but different manifestations of basically the same phenomenon. To stress this fact, I propose to combine the terms *cognitive* and *communicational* into the new adjective *commognitive*.<sup>8</sup> The etymology of this last word will always remind us that whatever is said with its help refers to these phenomena which are traditionally included in the term cognition, as well as to those usually associated with interpersonal exchanges.<sup>9</sup>

To complete the task of defining thinking as an individualized form of communication, I need yet to explain how this latter term should be understood in the present context. Since the patterned nature of communication is due to the fact that different people act in similar ways, communication needs to be considered as a collective activity, and should thus be described in terms of its global patterns. Restricting the field of vision to a single node, or to single pair of ‘sender’ and ‘recipient’, as is done in the majority of known definitions, would be as unproductive as trying to understand the rules of

chess from the individual moves of one checker. The following formulation seems to fulfill this requirement: *Communication is a collectively-performed rules-driven activity that mediates and coordinates other activities of the collective.* More specifically, individuals who participate in the activity of communicating perform actions that are customarily followed by a certain type of re-action of other individuals. The re-actions may be either practical actions or other communicational moves. By *practical actions*, I mean actions resulting in a change in the physical environment. Opening a window or adding a brick to a wall while building a house are good examples of practical actions. *Communicational actions* are those that affect members of community and have no direct impact on the environment, although some of them may, in the end, lead to another person's practical (re)action. In human activities, communicational and practical actions are usually simultaneously present and inextricably interwoven. Clearly, communication is what enables inter-person coordination needed for the collective implementation of complex practically-oriented activities, from preparing foods and garments to building houses, publishing newspapers, producing films, transporting goods, etc. This said, let me add that it is also typical of humans to have long chains of purely communicational interactions, in which every re-action is, in itself, a communicational action bound to entail yet another communicational re-action. In this process, the participants alternate between the roles of actors and re-actors, often playing both these parts in one communicational move.

Let me finish this introduction to the participationist discourse on thinking with a number of remarks. First, the definition of communication speaks about *rules* that regulate communication (and thus the commognition in general). It is important to stress that these rules are to be understood as *observer's constructs*, and not as guiding principles, followed by individual actors in a conscious, deliberate way. Another fact to remember is that the rules of commognition, are not in any sense "natural" or necessary, as nothing "in the world" can possibly necessitate the given types of associations between actions and

re-action. The source of the patterns is in *historically established customs*. This contingent nature of communicational patterns is probably the reason why Wittgenstein (1953) decided to speak about communication as a kind of *game*.<sup>10</sup> Second, because of its being rules-driven, commognition has dynamics of its own, and it would not be possible without the natural human tendency for alignment. This said, it is equally important to note that in commognition, like in any other historically established activity, human players do have agency. Communicative action almost never determines a re-action. More often than not, both action and re-action are a matter of construction, to be performed according to rules that constrain but do not dictate. Third, whereas practical actions are direct actions on objects, commognitive actions are *about* objects, that is, they focus interlocutors' attention on an object. Fourth, commognitive actions are performed with the help of *mediators*, which can have auditory, visual or even tactile effects on individuals. In humans, language, which has both vocal and visual (e.g. written) editions, is the principal, although not the only, form of commognitive mediator.

Finally, just as there is a multitude of games, played with diverse tools and according to diverse rules, so there are many types of commognition, differing one from another in their patterns, objects, and the types of mediators used. Like in the case of games, individuals may be able to participate in certain types of communicational activity and be unable to take part in some others. The different types of communication that bring some people together while excluding some others will be called *discourses*. Given this definition, any human society may be divided into partially overlapping *communities of discourses*. To be members of the same discourse community, individuals do not have to face one another and do not need to actually communicate. The membership in the wider community of discourse is won through participation in communicational activities of any collective that practices this discourse, be this collective as small as it may.

### 3.2 *What is mathematics?*

Given participationist vision of thinking as a form of communication, mathematics can be seen as a special type of discourse, made distinct, among others, by its objects, mediators and rules.<sup>11</sup> Let me be more specific.

A discourse counts as *mathematical* if it features *mathematical words*, such as those related to quantities and shapes. The conversation between Roni, Eynat and Roni's mother, presented in the beginning of this article, is replete with such mathematical terms as number-words and comparison-words (e.g. *more*, *bigger*), and can thus count as a case of mathematical discourse. This, however, is just one out of several possible types of mathematical communication. While many number-related words may appear in non-specialized, colloquial discourses, mathematical discourses as practiced in schools or in the academia dictate their own, more disciplined uses of these words. As will be argued below, neither Roni nor Eynat is using any of the mathematical words the way they are used by mathematically versed interlocutors (and I do not mean just the grammatical imperfections of the girls' talk).

*Visual mediators* to be found in mathematical discourses tend to be quite unlike those used in many other types of discourses. While colloquial discourses are usually mediated by images of material things, that is, by concrete objects that are identified or pointed to with nouns or pronouns and that may be either actually seen or just imagined, mathematical discourses often involve symbolic artifacts, created specially for the sake of this particular form of communication. Such symbolic mediation, however, is still absent from the incipient numerical talk of our young interviewees. Quite understandably, the only form of visual mediation that can be found in our data is concrete rather than symbolic: The mathematical task performed by the girls is described in terms of sets of marbles provided by Roni's mother, and is visually (and tangibly) mediated by these sets.

*Endorsed narratives* are sets of propositions that are accepted and labeled as *true* by the given community. Mathematical narratives, to be endorsed, have to be constructed and substantiated according to a set of well-defined rules, specific to this discourse. In the case of scholarly mathematical discourse, these endorsed narratives are known as *mathematical theories*, and this includes such discursive constructs as definitions, proofs, and theorems.<sup>6</sup> In addition to the generally endorsed “abstract” narratives such as those listed above, one can speak about more specific narratives that pertain to concrete objects and may be endorsed in a given situation. The aim of Roni and Eynat’s activity, at least in the eyes of the grownups, is to create such locally endorsable narratives: The girls are supposed to explore the boxes with marbles and to come up with endorsable statements that answer the Mother’s question “Which of the boxes has more marbles”?

*Routines* are well-defined repetitive patterns characteristic of a given discourse. Specifically mathematical regularities can be noticed whether one is watching the use of mathematical words and mediators or follows the process of creating and substantiating narratives about number. In fact, such repetitive patterns can be seen in almost any aspect of mathematical discourses: in mathematical forms of categorizing, in mathematical modes of attending to the environment, in the ways of viewing situations as “the same” or different, which is crucial for the interlocutors’ ability to apply mathematical discourse whenever appropriate; and in production of narratives and their further substantiation. Routines may be algorithmic, and thus deterministic, or just constraining. The canonic routine of numerical comparison, which, in our example, the mother expects her daughter to perform, is an example of algorithmic routine.

### 3.3 *What is number?*

The transition from acquisitionist to participationist approach changes the way we talk even about the most familiar of phenomena. This special linguistic change has a far

reaching effect on the way we view relations between things and these things' nature and properties. In consequence, it also makes a big difference in ways we act upon them. All this is best illustrated by showing what happens in the transition from acquisitionist to participationist understanding of the notion of number. To have a better grasp of the conceptual upheaval that occurs on this occasion, let us give a thorough look at the two descriptions of the phenomenon known from literature as young children's unawareness of the Principle of Cardinality (see e.g. Nunes & Bryant, 1996). On the first sight, the two formulations, the acquisitionist and participationist, presented in Table 1 are hardly distinguishable.

Acquisitionist version	Participationist version
Children under the age of five often do not realize that it is the last word in the process of counting elements of a set that is <i>the number</i> of elements in the set	Children under the age of five often do not realize that repeated counting of the same set must end with the same <i>number word</i>

Table 1:

Two versions of the claim that young children are unaware of the principle of cardinality.

However, at a closer inspection, there are differences, some of which become obvious when you consider the words stressed in italics. The acquisitionist versions speaks about numbers as *self-sustained objects* of the existence of which the child is expected to be aware even if she is unable to tell their properties (e.g. the value of the number corresponding to a set of counters). In contrast, the participationist text does not mention these objects; instead, it speaks about number *words*, which acquisitionist thinker would consider as mere *names* of numbers. This difference seems quite important. To evaluate its significance, let us imagine that rather than speaking about abstract objects, such as



numbers, a person is talking about tangible creatures, such as animals. The difference between speaking about the object as such:

Children under the age of five often do not realize that it is the *whale* that is *the largest animal*

and talking about the name of the object instead:

Children under the age of five often do not realize that "whale" and "the largest animal" are synonyms and can be used interchangeably.

becomes clear (note that in the second case I used quotation marks to stress that we are talking about the *names* of objects and not about the objects themselves.) True, we are likely to say that both sentences express the same truth about the world (namely, that the whale is the biggest among all the animals). However, this effect is attained by different means, with the speaker's and listener's immediate attention turned to different types of "beings": to an animal in the first case, and to words in the second. Or, the two texts refer us, respectively, to the world's own creation (animal) and to human-made things (words).

I now wish to claim that the object/name-of-object duality, which is so clear in the case of concrete entities, is only an illusion in the case of abstract creatures such as number. More specifically, the fact that we think about number as a kind of self-sustained entity such as the whale is a byproduct of a "transplant" from the discourse about material objects to the one we call mathematical: we borrow syntactical templates characteristic of the talk about animals, stars and chemical substances whenever speaking about abstraction, such as number. This grammatical metaphor, however, carries with it some basic ontological and epistemological assumptions about the entities that are being talked about. Thus, for example, by saying that "Children under the age of five often do not realize that it is the *whale* that is *the largest animal*" we imply that being aware of the existence of the whale is independent of the question of how much one knows about its properties. The same assumption clearly transpires from the statement about the number

that constitutes the counterpart of this proposition, “Children under the age of five often do not realize that it is the last word in the process of counting elements of a set that is *the number* of elements in the set”. Indeed, by describing the phenomenon in this way we imply that children are well aware of the existence of objects called numbers before they are able to speak about them in a meaningful way.

In fact, however, numbers are not anything like concrete objects. Indeed, how can a child become aware of the existence of objects called numbers before she becomes a participant of numerical discourse? There is no separate perceptual channel nor a comparable tangible object by means of which the child could develop an independent sensory familiarity, either actual or just imagined, with “things” such as three or ten. The bottom line of all these deliberations is that numbers, as opposed to whales, stars and houses, are not separate objects that preexist the talk about them. Rather, *numbers are discursive constructs that emerge for a person through her using number words while telling stories about the world.*

This last statement signals the disappearance of the ontological gap between the world of abstract mathematical objects and the mathematical discourse as such. In the last section of this article, I will show that far from being just idle philosophical musings, these observations have important implication for our understanding of the phenomena we are witnessing while trying to engage young children in conversations on numbers. Hand in hand with this new understanding of learning come new ideas about teaching.

### *3.4 What is mathematics learning?*

Learning mathematics may now be defined as individualizing mathematical discourse, that is, as the process of becoming able to hold mathematical communication not only with others, but also with oneself. It is through the process of individualization of numerical

discourse as presented by others that the child will build new objects called numbers for herself.

At this point, somebody can protest, saying that we are talking here about mission impossible: to construct the object the child has to talk about it; but how can she talk about something that does not yet exist for her? Indeed, it is because of this difficult to overcome circularity of requirements that the children's initial execution of numerical tasks is so different from that of grownups. To explain this claim, let me go back to the *Comparing sets of marbles* episode and see whether this definition helps to make a better sense of children's actions. It is now natural to assume that the observed phenomena are related to the fact that the children have not yet individualized the numerical discourse. In particular, if the young participants are using a number word, it is not because it makes them think about some independent entity, but because this word is a part of the game that grownups are ready to play with them. Indeed, children have not yet turned numerical talk into a *discourse for themselves*.<sup>12</sup>

There are many signs showing that Roni and Eynat have not yet individualized numerical discourse and are probably at the very beginning of the process. The first evidence can be found in the fact that they do not use the compare-by-counting procedure on their own accord: The question "[I]n which of the boxes [are there] more marbles?" ([3b]) is clearly not enough to get them started, and nothing less than a clear hint by the mother ("Do you want to open and discover?", [10a]) would help. Further, the children need mother's scaffolding in order to perform the procedure in its entirety (note, for example, that they stop after having counted the marbles and they need to be prompted in order to draw the conclusion; see mother's question [15]). It is thus clear that if the girls participate in the numerical discourse, it is on other people's accord and is done by following other people's rules.

All this can be summarized in the following way: What for the grownups is the routine of *exploration*, geared toward enhancement of one's arsenal of "factual knowledge" (endorsed narratives), for the children is a *ritual* – a game played with others for the sake of the togetherness that game-playing affords. Note that touching the marbles one by one while also pronouncing subsequent number words is not unlike incantation of meaningless rhymes which is often a part of children's play. What is now but a ritual, will turn into exploration in the course of individualization.

The fact that the girls' participation in the numerical discourse is ritualized and undertaken for the sake of connecting with others becomes even more visible when children's actions in the second part of the episode are compared with what they do in the first. When the conversation begins, the girls spontaneously respond to the mother's query with pointing to one of the identical boxes. Evidently, the question "[I]n which of the boxes [are there] more marbles?" when first asked, is not received as a prompt for a conversation on numbers but rather as an invitation to what the children usually do on their own accord and willingly: to choosing one of the boxes for themselves. Making choices, unlike numerical comparisons, is the kind of activity which the girls have already individualized. It will yet take time until the two types of routines – those of choosing and those of comparing – combine one with the other into an individual activity of the child.

It is reasonable to assume that a certain proficiency in a discourse is a prerequisite for its individualization. Roni and Eynat do not yet exhibit sufficient fluency in numerical talk. For example, they have yet to change their use of number words. Right now, these words are for them but a part and parcel of counting. In the future, the words will be used in many different types of sentences and in multiple roles, as adjectives and as nouns, among others. Above all, the use of these words will become objectified: More often than not, expressions such as *one*, *two* or *two hundred* will be used as if they referred to self-sustained, extra-discursive entities. Similarly, the children's use of connectives such as

*because* will change dramatically. At the moment, this use is clearly ritualized: If the girls answer mother's *why* questions in a seemingly rational way (see Roni's utterance [5] and Eynat's utterance [7], which both begin with the word *because*), it is obviously due not to their awareness of the relations between boxes but to their familiarity with the *form of talk* which is expected by the grownups in response to this kind of question. At this point, the girls are already aware of *how* to talk when answering request for explanation, but are not yet fully aware of *when* – under which circumstances – it is appropriate to apply them. As for now, the mere appearance of the word *why* in the interlocutor's question may be enough to prompt an utterance that begins with *because* and then simply repeats, in a somewhat modified form, what the question was asking about. It seems reasonable to conjecture that in the process of individualization, the awareness of *how* discursive routine should be performed usually precedes the ability to tell *when* such performance would be appropriate. One may even hypothesize that it is the ability to make independent decisions about *when to apply* a given discursive procedure which is the ultimate sign of its individualization.

The manner in which all these changes in the girls' numerical discourse<sup>13</sup> are supposed to happen is implicated in the very claim that learning mathematics is the process of individualization of mathematical discourse: Discursive change can only originate in communicating with experienced interlocutors. This vision is quite different from the one professed by the acquisitionist who assumes, if often only tacitly, that learning results from the learner's attempts to adjust her understanding to the externally given, mind independent reality. Contradicting the participationist belief in the primacy of the collective, this latter version implies that learning, at least in theory, could take place without participation of other people.

Not every mathematical conversation is an opportunity for learning. For a discursive change to occur, there must be some discrepancy – a *communicational conflict*

– between interlocutors. Such conflict arises whenever different participants seem to be acting according to differing discursive rules. The difference may express itself in a disparity in the interlocutor's uses of words, in the manner they look at visual mediators or in the ways they match discursive procedures with problems and situations. More often than not, these differences find their explicit, most salient expression in the fact that the different participants endorse differing, possibly contradicting, narratives.<sup>14</sup> Dissimilarities between Roni and Eynat's numerical discourse and the numerical discourse of the grownups express themselves in different uses of words and disparate routines, and thus constitute a good example of communicational conflict, likely to result in a considerable learning.<sup>15</sup>

In order to fully individualize numerical discourse Roni and Eynat will have to overcome this conflict. This is not going to be easy. In addition to the circularity stemming from the need to speak about objects that do not yet exist, there is another pair of paradoxical requirements likely to hinder the process of bridging between conflicting discourses. If the child is to ever use the numerical discourse in solving her own problems, she must be aware of the advantages of the relevant discursive procedures. For example, she needs to realize that she may benefit from act of choosing performed according to numerical relation. And yet, in order to become aware of these advantages, she has to already use the numerical discourse. The next question to ask is what can possibly motivate the child to engage in the demanding task of overcoming all these circularities.

### 3.5 *Why do we learn mathematics?*

Why do people engage in mathematical discourse? Why has this discourse emerged over history? The answer seems straightforward and simple: mathematical explorations produce narratives that improve the way we do things. For instance, mathematical narratives about quantities and proportions improve the deed of preparing different kinds

of foods. Much more complex mathematical stories mediate the work of building computers and erecting skyscrapers. This, however cannot possibly be the reason that motivates very young children. As stated before, young participants are still entirely ignorant of the object called number and cannot possibly appreciate its deep relations with the tangible world, the relation that renders numerical narratives the power to shape human deeds. If they agree to participate in conversations on numbers, it must be for a reason other than the applicability of stories about numbers or of these stories' sheer beauty.

If the child participates in the numerical discourse before she is able to engage in genuine mathematical explorations it is because of the social gratification such participation may bring them. Roni and Eynat perform the ritual as an act of solidarity with the grownups and in the attempt to win their approval. Giving the answer that is expected by the interlocutor may be read as an act of pledging allegiance – and it does, usually, yield this kind of reward.

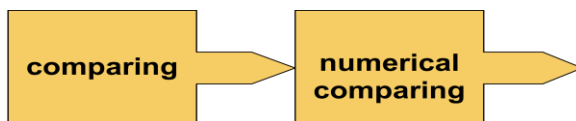
More generally, when the child first engages in mathematics learning, it is because of her overpowering need for communication, which grows out of the even more fundamental need for social acceptance. This social concern can clearly be seen all along the conversations with the girls. The way Roni monitors her mother's face, talks to her and follows her lead clearly indicates that getting the parent's attention and approval is the girl's main concern. This wish competes, and is successfully combined, with an equally strong need to belong with the peer. While making their choices, Roni and Eynat are careful to stress that their decisions are shared (in the further parts of our transcripts, this need for solidarity with the friend is further evidenced by Roni's repetitive use of the word *we*, through which she asserts the joint ownership of solutions.)

To sum up, the children have different goals than those envisioned by the grownups. While counting and comparing, the girls are in fact preoccupied with the

delicate social fabric of their little group, and the conversation on boxes with marbles is, for them, as good an occasion for inter-personal engineering as any other. While grownups count in order to get closer to the truth about the world, the children count to get closer to the grownups. The “exploratory” activities of the young participants are therefore a form of community-building ritual.

### *3.6 How does numerical discourse develop?*

In concert with basic assumptions of discursive approaches, having a good grasp of the historical development of mathematical discourse may help in understanding intricacies of the processes of teaching and learning of mathematics. When we try to figure out how and why numerical discourse emerged throughout history, the first question to ask is *What is the most elementary task that cannot be implemented without numbers?* Indeed, it is only with tasks for which numbers are indispensable that the emergence of numerical discourse could possibly begin. At a deeper thought, numbers have their roots in quantitative comparisons, and the only type of comparison in which they seem to be beyond competition involves things that are physically distant. More precisely, numbers enter the scene and become irreplaceable when the objects of comparison cannot be physically aligned, and there is also no possibility of using a physical “counterparts” to serve as “proxies” of the real things (as is the case, for instance, with a piece of string which is equal in its length to one wall and is being spread along another wall for the sake of comparison). My first claim about the history of numbers, therefore, is that their emergence has to do with turning “analogous” quantitative comparisons into ones based on counting:



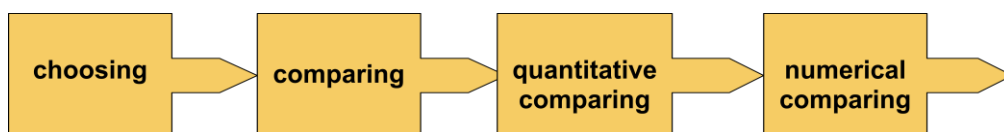


This story, however, leaves a “hole”. Quantitative comparisons, whether numerical or not, are explorations that produce narratives about relations between objects in the world.

People could not be interested in this kind of stories if these narratives had no practical value – if they did not mediate some kind of important tasks. The first thing that comes to mind in this context is the task of choosing. Indeed, it is when we are to choose one of several similar options that we say things such as “I opt for this one because it is bigger/wider/heavier”. My schematic model of historical development of numerical discourse can now be refined:



This model is grounded in the conjecture that the act of choosing for oneself may well be the tasks that human beings seem to be willing and able to perform almost from the day they are born. Of course, these first choices are not rational, and when they became reasoning-based, some of them turn out quantitative, that is, made according to the results of quantitative comparisons. Some, but not all. Indeed, the child may compare, and then choose, according to other features, such as color, shape, smell, and this is but the beginning of what seems like an infinite list of possible criteria. The fact that the primary discourse, that of choosing (“take one”, “choose”), gave rise to numerical discourse via quantitative discourse (one characterized by words such as “big”, “small”, “larger”, “greater than”, “smaller than”, “more”, “less”, “equal”, etc.) is encapsulated in a new refinement in my historical model:



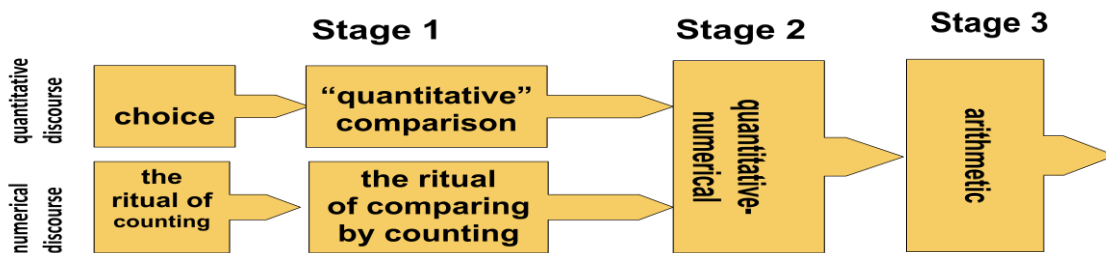
The developments summarized conclude in the primary numerical discourse in which numbers serve as ordered labels and can therefore be used in comparisons. It can be argued that the numerical talk of Oksapmin people who live in Papua New Guinea, as discovered and described in late 1970-ies by the American psychologist Geoffrey Saxe (Saxe, 2012), is a good example of numerical discourse the development of which has stopped at exactly this point. The next step in the growth of numerical discourse is the introduction of operations on numbers. This development comes hand in hand with the objectification of number, that is, with its turning from just a label into an object in its own right.

The basic assumption underlying the above historical model can now be summarized as follows:

- The activity of choosing from among several options is the primary source of quantitative discourses.
- Quantitative non-numerical discourse, one that evolves around the activity of comparing sizes and is thus distinguishable by the use of words of comparison such as “larger”, “greater than”, “smaller than”, “more”, “less”, “equal”, and words for sizes, such as “big”, “small”, etc., develops to support this task.
- Numerical discourse develops from the quantitative discourse as a result of the attempt to make quantitative comparisons also between objects that are too distant in space or time to be compared either through their physically aligned or with the help of physical “proxies”.

These tenets constitute inevitable entailments of the assumption that human discourses develop to serve human deeds, and that the act of choosing is the most basic among those deeds. These claims impose the special developmental order suggested by the above historical model. The question which has to be answered in the transition to the

ontogenetic growth is whether the development of the child's numerical discourse should follow a similar trajectory. There is a reason, it seems, to expect some differences. Unlike in the case of historical development, the numerical discourse is already around when the child is born. In our culture, words of quantitative comparison and elements of numerical discourse are introduced to the child well before she has developed quantitative discourse. In consequence, there is a difference between historical and ontogenetic development: in the life of a child, quantitative and numerical discourses develop in parallel and in both cases they begin as a bunch of rituals to follow. The quantitative discourse enters the scene as the discourse of making seemingly quantitative choices, and the numerical discourse is initially but a collection of brief litanies to be recited in certain well defined situation. This is why the diagram below pictures the first parts of these two discourses as parallel, thus unconnected.



To summarize, up to a certain moment, the child's talk about quantities does not involve numbers and quantitative statements are often not translatable into numerical ones; and her talk about numbers is disconnected from the quantitative. At a certain point, these two parallel discourses will coalesce and become one. The pace at which this unification will happen depends, undoubtedly, on the intensity of the child's discursive experience. One of the most important lessons that can be learned from these analyses is that the development of numerical discourse cannot be understood without considering the development of quantitative discourse.<sup>16</sup>

#### **4. Consequences of participationism for the practice of mathematics teaching and learning**

Our ability to make sense of what we see depends on our uses of words. As illustrated above, the interpretation of the notion “social” that gave rise to the commognitive framework made a significant difference in our vision of learning and in this vision’s theoretical entailments. In particular, it allowed to account for phenomena that escaped acquisitionist’ explanations and it offered alternative explanations for some others. Thus, for example, what acquisitionists interpreted as showing children’s unawareness of the “conservation of number” became, in our interpretation, the result of the simple fact that in the situation of choice, young learners had no reason to privilege the ritual of counting over other routines that they had already at their disposal.

Perhaps the most dramatic difference between the acquisitionists’ and participationists’ visions of mathematical thinking is in their respective messages about the origins of mathematical learning. Whereas acquisitionists views learning as resulting from the learners’ direct efforts to arrive at a coherent vision of the world, participationists sees learning as arising mainly from one’s attempt to make sense of other people’s vision of this world. The former perspective implies that learning, at least in theory, could take place without participation of other people. In contrast, the idea of mathematics as a form of discourse entails that individual learning originates in communication with others and is driven by the need to adjust one’s discursive ways to those of other people.

Participationism also provokes second thoughts about some common pedagogical beliefs. For instance, it casts doubt on the current call for “learning with understanding,” at least insofar as this call is interpreted as the exhortation to never let the student practice routines which she cannot properly substantiate. According to the present analysis, students’ persistent participation in mathematical talk when this kind of communication is

for them but a *discourse-for-others* seems to be an inevitable stage in learning mathematics. If learning is to succeed, all the interlocutors must agree to live with the fact that the new discourse will initially be seen by the newcomers as a game to be played with others, and that it will be practiced only because of its being a discourse that others use and appreciate. It is thus now time to rehabilitate the learning that is based on ritualized action and on thoughtful imitation of the grownups' ways with words. Trying to figure out and then to meet the expert participants' expectations is sometimes the only way to initiate the long process of individualization of discourses. Making sense of other person's thinking is not any less demanding (or respectable!) than the direct attempts to understand reality. Indeed, entering "foreign" forms of talk (and thus of thought) requires a genuine interest and a measure of creativity. To turn the discourse-for-others into a discourse-for-oneself, the student must explore other people's reasons for engaging in this discourse.

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<sup>1</sup> To illustrate, let me just mention two differing interpretations of the word ‘social’ to be found in the context of the famous dichotomy *individual vs. social*, that lies at the very heart of the current controversies on human development. At a closer look, those who contrast “the social” with “the individual” may have two different distinctions in mind. In one of these dichotomies, the term *social* means that whatever is described with this adjective has been done or attained *by an individual* through interaction with others. In this case, the *social* could probably be replaced with *interactional*. The other dichotomy that hides behind the opposition *social versus individual* regards not so much the ‘technicalities’ of individual learning as the nature and origins of what is being learned. This time, the issue at stake is that of the ontological-epistemological status of knowledge, with the word *individual* functioning as almost synonymous with *natural* or *genetically necessitated*, whereas the *social* is tantamount to *human-made*. It is this latter, strong interpretation of the “social” that seems to have spurred Vygotsky’s famous criticism of the Piagetian doctrine (the fact of which Piaget was likely to be aware only partially, if at all).

<sup>2</sup> The study from which the vignette is taken has been reported in Sfard & Lavie (2005). The conversation was held in Hebrew. While translating to English, I made an effort to preserve the idiosyncrasies of the children’s word use.

<sup>3</sup> For Activity Theory see, e.g. Leontiev (1947/1981), Nardi (1996), Engeström (1987).

<sup>4</sup> In this context, one should mention the significant influence of Wittgenstein, as well as that of two inter-related, but still distinct schools in sociology: The *symbolic interactionism* usually associated with Mead (1934), Goffman (1958), and Blumer (1969); and the *ethnomethodological* approach initiated by Garfinkel (1967). Of relevance in this context is also the *sociological phenomenology* that originated in the philosophical thought of Husserl’s and was founded in the first half of 20<sup>th</sup> century by Schutz (1967). The direct influence of this latter school of thought on psychology and education can be seen in the work of German and American researchers – see e.g. work by Bauersfeld (1995), Voigt (1985), Krumheuer (1995), and Cobb and his colleagues (Cobb et al., 1993; Cobb & Bauersfeld, 1995). All these schools, be them diverse as they are, share a number of basic assumptions, which can also be found in most of the current versions of participationism. They all take the inherently social nature of humans as their point of departure and agree that actions of the individual cannot be understood unless treated as part and parcel of collective doings. The patterned collective activities, in turn, are objects of their participants’ sense-making efforts. The different schools begin to diverge only when it comes to their respective responses to the question of where the regularities come from and whether the observed patterns are in any real sense ‘real,’ as opposed their lying exclusively in the eyes of sense-making insiders.

<sup>5</sup> The terms *individualization* and *collectivization* may be viewed as strong participationist versions of what Vygotsky and Activity Theorists call *internalization* and *externalization*. The important advantage of the present terminology is that it is free of acquisitionist undertones of the traditional vocabulary. In result, the proposed version of strong participationism does not imply that thinking and behavior are two ontologically different types of processes but rather promotes the idea that they are two forms of basically the same phenomenon, which may be termed simply as ‘individual human doing.’ These two forms differ only in the degree of their visibility to others.

<sup>6</sup> One should not, of course, take this metaphor too far. Not every collective activity can be fully individualized (reenacted by a single person). Suffices to think about building bridges or performing complex surgeries. And yet, whatever distinctly human activity has been mastered by a person, the source of this ability is in this person’s earlier participation in its collective implementations.

<sup>7</sup> This definition resonates well with the conversation metaphor of mind to be found in Ernest (1993, 1994), Mead (1934), Bakhtin (1981), Holquist (1990) and Marková (2003). See also the idea of discursive psychology in Harré & Gillett (1995), Edwards (1997).

<sup>8</sup> The act of coining my own neologism is certainly rather daring, and I feel I owe an explanation. While trying to give a name to the just defined discourse on thinking I could, of course, follow the usual practice of employing a word that already exists in the English language. In fact, after having said that thinking is an individualized form of communication, I could use the word communication to encompass both categories – that of thinking, and that of inter—personal communication. Indeed, many other human activities that begin as collective and are liable to individualization do not change their names as a result of individualization: the

individually performed mathematical problem solving is still called *problem solving* and the task of complex data processing is called *data processing* whether it is implemented by a single individual or by a group.. However, calling thinking (individualized form of) *communication* would require the users to overcome our deeply entrenched habit of using the words *thinking* and *communicating* as denoting different, non-overlapping types of activities. In introducing the new name I was motivated by the conviction that our view of communicating as being collective *by definition* may be too strong to be removed by a mere act of redefining.

<sup>9</sup> At this point, a skeptic can bring yet another argument against the idea of thinking as individualized form of interpersonal communication. The dilemma of relations between thinking and speech has been stirring one of the most persistent and encompassing debates in the history of human thought. Considering the fact that no solution, not even those offered by the most revered of thinkers, managed to bring about a durable consensus, it may be difficult to understand why the simple statement “thinking is (can be usefully defined as) a form of communication” should now be accepted as an answer. In response, let me stress two differences between my present attempt and most of those undertaken in the past. First, what I did has been framed as an act of *defining*, not as an attempt to find out what thinking “really is.” Thus, the agreement may be possible provided I manage to convince others about the usefulness of the proposed *thinking = self-communicating* equation. The second difference stems from the fact that the time-honored dilemma which, for centuries, has been boggling philosophical minds is that of the relations between thinking and *language* (or *speech*), whereas the proposed definition links thinking with *communication*. The relation between thought and speech has been, indeed, a leitmotif of philosopher’s musings about thinking. This is easily explicable, considering the centrality of verbal communication in specifically human forms of life and the resulting tendency to equate human communicating with talking. Speech and communication, however, although related, are not the same: The former is but a special case of the latter. There are numerous non-verbal forms of communication, and all of them must be considered. Thus, the descriptions of thinking as “talking to oneself” or as “inner speech” are more restrictive than the communicational definition proposed above and as such, they do not make full justice to the phenomenon we wish to fathom. If the attempts to capture the gist of human thinking have been invariably deemed futile, it was probably because of the fact that the problem has been restricted to the issue of relations between thinking and language.

<sup>10</sup> More precisely, Wittgenstein (1953) spoke about *language games*. The metaphor of game, however, is clearly applicable also to non-verbal forms of communication.

<sup>11</sup> Equating mathematics to discourse should not be confused with the time-honored, and often contested, claim that mathematics is a language. The word *language* is usually understood as referring to a tool for representing objects, with this objects being external to, and independent from, the language itself. Therefore, the statement “mathematics is a language,” unlike its discursive counterpart, could imply that the objects of mathematics are not a part of mathematics itself. Second, discourses involve many mediators, not just language.

<sup>12</sup> The term *discourse-for-oneself* is close to Vygotsky’s idea of *speech-for-oneself*, introduced to denote a stage in the development of children’s language (see e.g. Vygotsky 1987, p.71). Our terms also brings to mind the Bakhtinian distinction between *authoritative discourse*, a discourse that “binds us, quite independently of any power it might have to persuade us internally”; and *internally persuasive discourse*, one that is “tightly woven with ‘one’s own world.’” (Bakhtin, 1981, pp. 110-111.)

<sup>13</sup> Since the only way to actually observe such changes is by watching the child in mathematical conversation with others rather than with herself, we will need to remember that whatever is found has been informed by the other participants as well. Still, with an appropriate analyses and the sufficient amount of observations, we may be able to make conjectures about some general properties of the child’s participation, as well as of the individualized form of this child’s discourse, if any.

<sup>14</sup> Since discursive conflict arises in face of differences in meta-discursive rules, a mere difference in narratives cannot count as a sufficient evidence for such conflict; for example, if one objects to the claim that “The weather is beautiful today”, it is indicative of the conflict of opinion, not of discourses

<sup>15</sup> The notion of communicational conflict, although reminiscent of the acquisitionist idea of *cognitive conflict*, is in fact a different type of theoretical construct: Communicational conflict results from a disparity

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between student's and teacher's discourses rather than from a clash between the learner's vision of the world and the real state of affairs; it is indispensable for learning rather than optional; and it is resolved through students' acceptance and rationalization of the discursive ways of an expert interlocutor and not via their direct, independent reasoning about the world.

<sup>16</sup> This fact was probably guiding the work of the Vygotskian psychologist and educator Vasili Davydov when he proposed to foster children's non-numerical quantitative discourse (Davydov, 1975).