COMMOGNITION

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Definition

Commognition, the portmanteau of *communication* and *cognition*, is the focal notion of the approach to learning grounded in the assumption that thinking can be usefully conceptualized as one's communication with oneself. This foundational tenet goes against the famous Cartesian split between the bodily and the mental. According to the resulting non-dualist vision of human cognition, *mathematics* is a historically-established discourse and *learning mathematics* means becoming a participant in this special form of communication. The basic assumption about thinking as communicating has multiple entailments that combine into a comprehensive non-dualist theory of learning.

Origins

The idea of commognition emerged within the context of mathematics education in response to certain weaknesses of traditional visions of human development. Whereas learning has always been seen as a process of change, proponents of the various conceptualizations that emerged in the 20th century differed in their answers to the question of what it was that changed when learning took place. According to behaviorists, learning was a change in the learner's *behavior*, whereas cognitivist thinkers proposed to conceptualize learning as a process of acquiring – receiving or constructing – mental entities called *concepts*, *knowledge* or *mental schemes*. One common weakness of suh 'acquisitionist' approaches was that being focused exclusively on the individual, they fell short of fathoming the mechanisms of the historical change in human ways of acting.

In the second half of the 20th century, the acquisitionist stance was countered by the claim that in those processes of learning that are unique to humans, the learner becomes a participant of well-defined historically established forms of activity (Vygotsky, 1987; Cole, 1996). This 'participationist' thinking on learning was taken one step further when different domains of human knowing, with mathematics among them, have been recognized as *discursive activities*. This latter idea, which constitutes the foundation of commognitive vision of learning, arrived almost simultaneously from two directions. On the one hand, it was an inevitable conclusion from the work of psychologists and philosophers who claimed the untenability of any attempt to separate thought from its expression (Vygotsky, 1987; Wittgenstein, 1953).

On the other hand, the statement about the discursive nature of human knowing has been made explicitly by postmodern philosophers interested in societal-historical rather than individual-ontogenetic change of the activity known as *science, research* or *knowledge-building* (Lyotard, 1979; Foucault, 1972; Rorty, 1979). With its double focus on individual and collective discursive processes, which are now seen as different aspects of the same phenomenon, the commognitive approach made it possible to account for historical transformation of human activities (Sfard, 2008).

Although discursive activities constitute the main source of data in almost all types of learning sciences, the commognitive approach may be the only one that rests on the explicit claim on the unity of thinking and communication. Tacitly, this tenet seems also to be present in the branch of psychology known as *discursive* (Harrè & Gillett, 1995; Lerman, 2001).

Foundations

According to the basic commognitive assumption, *thinking mathematically* means participating in a historically developed discourse known as *mathematical*. Here, the term *discourse* applies to a form of communication made distinct by a number of interrelated characteristics: its special *keywords* (for instance, 'three', 'triangle', 'set' or 'function' in mathematics); its unique *visual mediators* (e.g. numerals, algebraic symbols, and graphs); its distinctive *routines*, that is, patterned ways in which its characteristic tasks (e.g. defining or proving) are being performed; and its generally *endorsed narratives*, (in mathematics, theorems, definitions and computational rules, among others). The descriptor "generally endorsed", used in this last sentence, is to be understood as referring to endorsement by the community of the discourse, with this latter term signifying all those who are recognized as able to participate in that discourse.

In tune with this conceptualization, *learning of mathematics* becomes the process of individualizing mathematical discourse. Here, the term *individualizing* refers to the process as a result of which learners gradually become capable of employing the discourse agentively, in response to their own needs.

People develop specialized discourses, such as mathematical of scientific, so as to be able to generate potentially useful stories on chosen aspects of the world around them and of their own experiences. Just as biologists narrate the worlds of living things and physicists tell stories about unanimated objects, so do participants of mathematical discourse tell stories about the universe of *mathematical objects*. Unlike the majority of other discourses, however, mathematics is a genuinely *autopoietic* system: it creates all those entities its participants talk about. In this special discourse, introduction of new nouns or symbols, rather than being an act of signifying existing mathematical entities, is the initiation of the process of *objectification*, in which new objects are constructed. At least one of the following discursive devices is used in this latter process:

- *saming*, that is, giving a common name to things that, although seemingly unrelated, can be seen in certain contexts as equivalent (this is what happens, for instance when the term *the basic quadratic function* is introduced to refer simultaneously to things as different as the expression x^2, a certain curve called *parabola*, the set of numbers paired with their squares, etc.);
- *encapsulating*, that is, replacing the talk about separate objects with the talk about a single entity (this takes place, when several objects are referred to collectively as a single *set*; for instance, when numerous ordered pairs of elements are claimed to constitute *a function*);

- *reifying*, that is, turning talk about a mathematical process with talk about an object (this is the case, for example, when we replace “When I add 5 to 7, I get 12” with “the sum of 5 and 7 is 12”).

Once a new noun is introduced in one or more of these ways, the *alienation* of the new object gradual occurs: the noun will eventually be used in impersonal narratives, implying that its referent exists independently of the discourse. Thus created discursive construct becomes an object of mathematical explorations, as a result of which new mathematical narratives will eventually emerge.

Our actions with mathematical objects at large, and our mathematical storytelling in particular, are governed by discourse-specific routines. These relatively stable patterns of action reflect our human tendency for repetition: While in a situation in which we feel a need to act (task-situation, for short), we usually recapitulate what was usefully done in those past situations that we deem similar enough to the present one to justify such repetition. Thus, the routine performed by a person P in task-situation TS may thus be seen as a pair of elements: (1) the *task*, which is P’s vision of all those elements of the precedent events that must be repeated in TS, and (2) *procedure*, which is the prescription for action that aptly describes both the present and precedent performances. The same procedure may become a basis for different types of routines, depending on the performer’s vision of the task.

Expert participants of mathematical discourse interpret most task-situations as requiring a (re)formulation and endorsement of a particular type of mathematical narrative. Such outcome-oriented routines can be called *explorations.* In contrast, if these are the actions of the previous performers, not just their outcome, that the person considers as requiring exact recapitulation, it is justified to describe her process-oriented routine as *ritual*. Since the ritual performance does not count in the eyes of the performer as an act of production, it can only be motivated by this person’s expectation of social rewards. Of course, most routines people actually perform are neither pure rituals nor perfect explorations, and between these two extremes there is a wide spectrum of possibilities.

Method

Mathematical discourses are the principal object of commognitive research and the development of these discourses is its main theme. In contrast to psychological studies that tend to analyze learning as the process of change *in the learner*, commognitive investigations seek transformations in mathematical *discourse*. As a form of communicational activity, learning is now conceived as inherently collective, or social, rather than individual phenomenon (and it is so even if it is practiced in solitude).

Detailed records of multimodal interactions and their meticulously prepared transcriptions constitute the main type of data in commognitive research on learning. Among the rules that govern data analysis there is the *principle of wholeness*, according to which the discourse as a whole, rather than its particular objects (or concepts), constitutes the unit of analysis; the *principle of operationality,* which requires defining the keywords with the help of perceptually accessible properties of the discourse; and the *principle of alternating perspectives,* which states that analysts have to constantly alternate between the perspectives of insiders and of outsiders to their own discourse. Although each study requires its own analytic scheme, effective heuristics are available for constructing such scheme. Finally, when reporting their findings, commognitive writers favor direct quotations from data over reported speech, and they are always wary of “ontological collapse”, which is the case whenever the participant’s vision of reality is offered as the researcher’s own narrative on that reality.

Commognitive theory of the development of mathematical discourses

One of the main strands in commognitive research is the study of the development of mathematical discourses, with the word *development* pertaining to both ontogenetic and historical growth of this special form of communication. Although these two types of development are quite distinct – the former is mainly productive (creative) and the other mainly reproductive – there are reasons to believe that they share some basic mechanisms and are subject to a number of comparable constraints.

Objectification, the first common feature to mention, is widely practiced across mathematics as a means of compressing the discourse, and thus of making it possible to say more with less. The periodic compression allows for practically unbounded growth of mathematical discourse. This growth happens in cycles of objectifying and formalizing of the current meta-discourse and then annexing it as a new layer of the full-fledged mathematical discourse. Elementary algebra, which constitutes a formalized meta-discourse of arithmetic (Caspi & Sfard, 2012) may be seen as a prototypical product of this process.

Another common feature of historical and ontogenetic developments of mathematical discourse is that they involve changes on both object- and meta-level. *Object-level developments* result in extending the existing sets of endorsed narratives about already constructed mathematical objects. This type of growth is mainly accumulative. *Meta-level developments* are those that involve changes in meta-rules of the discourse. This type of transformation is not a matter of a simple accretion: it usually results in a discourse incommensurable with its predecessor. This means that within the new discourse, some of the endorsed narratives of the old one will be considered as "misconceptions". Incommensurable discourses, therefore, rather than being mutually exclusive, complement each other in their applicability. In encounters between incommensurable discourses, such as those occasioned, for instance, by successive extensions of the number system, the old discourse (e.g. that of integers) may become subsumed within the new one (that of rational numbers). This, of course, will happen at the price of losing some of the old endorsed narratives (for instance, it will no longer count as true that "multiplications makes bigger") and of modified word uses.

Historical development. To get a sense of their historical development, it is necessary to consider discursive activities within the context of other ones, especially of those that result in changes, re-organization or re-positioning of objects, and can thus be called *practical*. One of the main commognitive assumptions is that practical and discursive activities have always been spurring each other's development. Thus, for instance, it is reasonable to hypothesize that the emergence of numerical discourse was prompted by our ancestors' wish to extend the practical activity of making quantitative choices. This task was initially performed by putting small finite sets in one-to-one correspondence. Once numbers were introduced, it became possible to compare also sets that were too large or too distant in space or time to be physically mapped one into another. The invention of counting opened opportunities for new types of practical activities, which, in turn, gave rise to further discursive extensions. More generally, practical and discursive activities co-evolved in cycles, functioning like two legs that by a constant attempt to get ahead of the other one keep moving the whole system toward an ever greater complexity.

This vison of the co-evolution of practical and discursive activities has been recently corroborated by findings of a cross-cultural research on the learning of mathematics in the Polynesian state of Tonga (Morris, 2017). The study has shown that discourses developed in one culture to support practical activities specific to this culture may not be easily transferrable to a culture, in which these special activities are absent. Commognitive approach has also been found useful in mapping shorter term historical changes, such as those that happened over the period of a few decades in the discourse of school mathematics in England (Morgan & Sfard 2016).

Ontogenetic development. Although it is reasonable to expect some parallels between the historical and ontogenetic developments, it is just as justified to expect differences. Rather than being brought into being by some practical, genuinely felt need, new discourses may appear in the life of a learner as ready-made patterns of communicating, widely practiced in the community. For instance, in today's societies, children are taught to count prior to being properly exposed to the quantitative discourse, recognizable by descriptive keywords such as *more*, *less*, *greater, large*, etc., and long before they are aware of how the resulting numerical discourse may be applied in any activity (Lavie & Sfard, 2016). Similarly, the development of the discourse on rational numbers begins with an introduction of the calculus of fractions. In both these cases, the new discourse, if successfully developed, will be incommensurable with its predecessor, and this means that there is a need for a meta-level learning.

In contrast to object-level learning that, theoretically, can happen without the teacher's deliberate intervention, meta-level learning requires interacting with a person who is already adept in the new discourse. This type of learning cannot be motivated or guided by the learner's own genuine interest in the outcome. For the student, the only way to enter the discourse is to imitate teacher's expert performances. At this point, the routines she performs cannot yet constitute true mathematical explorations, because the learner, not being acquainted with the focal objects, cannot judge the success of her performance by the endorsability of the mathematical narrative produced in the process. Meta-level learning is thus bound to begin with rituals.

The rituals, which are arguably inevitable at the earliest stages of meta-level learning, may later morph into explorations. For this to happen, the leaner must keep participating in the new discourse, while also making persistent efforts to figure out its usefulness. In the progress of de-ritualization, the performer's attention gradually shifts from the performance as such to its outcome. This shift may manifest itself, among others, in the strengthening of such characteristics of routines as *flexibility* or *applicability*. With time, the routine will become *vertically bonded*: every step in its procedure will build on the outcome of the previous ones. It will also be *horizontally bonded* with other routines: its procedure will branch into a number of alternative paths as a result of realization that other routines perform the same task. As found in research, the process of de-ritualization may be gradual and slow (Sfard & Lavie, 2005; Lavie & Sfard, 2016), and only too often is not be completed in school. The question of what it is that fuels or obstructs processes of de-ritualization is being addressed in numerous commognitive studies.

Commognitive theory of factors that shape the learning of mathematics

Conditions for learning. Commognitive approach offers its own vision of circumstances under which learning of mathematics becomes possible. Object-level learning requires no more than the ability to deduce new narratives from those already endorsed, and thus can, in principle, be attained by learners on their own, without help from a more experienced participant. For meta-level mathematics learning to occur, however, some special conditions are necessary. The opportunity for meta-level learning offers itself when the learners encounter a discourse incommensurable with their own. Three conditions must be fulfilled to turn such *commognitive conflict* into a genuine opportunity for learning: (1) all the participants have to agree on the question of which discourse should be the leading one, that is, common to all the particiants; (2) the experienced participants of the leading discourse must accept their role as leaders (teachers), whereas other ones must be willing to act as followers (learners); (3) the participants need to have shared expectations with regard to the possible form and pace of the learning process. Together, these three conditions constitute a *learning-teaching agreement*. Commognitive theory offers a vision of factors likely to support or counter this kind of agreement, thereby shaping the learning of mathematics.

Culture. Any mathematical discourse, when taught in different institutional or cultural settings, may give rise to different learning processes. That this is the case has been corroborated in a study that compared mathematics learning of native Israelis to that of immigrants from the former Soviet Union (Sfard & Prusak, 2005), in the commognitive research on the learning about infinity and limits by Korean-speaking students and by English-speakers from United States (Kim, Ferrini-Mundy, & Sfard, 2012), and in a study on the learning of fractions and probability in Tonga (Morris 2017).

Identity. While *mathematizing*, that is, participating in a discourse on mathematical objects, we tend to be simultaneously involved in the discourse od *subjectifying*, that is, in an overt or covert talk about participants. Clearly, the activity of subjectifying, unless tightly related to the performance of mathematical tasks, may reduce the participants' engagement in mathematical discourse, thereby undermining the effectiveness of their mathematics learning. Particularly strong may be effects of subjectifying that takes the form of *identification*, that is, of telling stories on the properties of the learner rather than of her actions. Identity-constituting narratives, offered directly or indirectly by their protagonists, the learners, and by people around them, tend to function as self-fulfilling prophecies and may thus have a long-term effect on learning: the student identified as "weak" will now be more likely to fail, and the one labeled as "strong" will be more determined to achieve success. The result will reinforce the previously constructed identities, reducing the chances for a change in a reverse direction (Ben Yehuda et al. 2005; Sfard & Prusak, 2005; Heyd-Metzuyanim, 2015).

Teaching. In our society, young people enter the world of formalized mathematics mainly through opportunities for learning created for them by mathematics teachers. The teacher models the discourse for the learners and issues invitations for their active co-participation. One of the main question to ask while trying to figure out possible outcomes of the teacher's efforts is whether the students are offered an access to explorative mathematics or are rather encouraged to satisfy themselves with ritualized discourse (Adler & Sfard, 2017).

Contributions of commognitive research – past and future

The commognitive approach may be claimed to have a number of strengths. First, research methods grounded in its underlying non-dualist onto-epistemology make it possible to investigate learning on both individual and collective level and lead to a high-resolution picture of the relevant processes. Commognitive analyzes reveal the highly consequential nature of even the tiniest of the teachers' moves. Second, the constantly expanding commognitive theory brings its own insights about mathematics learning and informs the teaching of mathematics in ways that often go against widely endorsed pedagogical principles. Last but not least, the disappearance of the though-communication dichotomy dissolves some of the time-honored dilemmas that proved untreatable within the confines of the traditional dualist approaches. The non-duality implies that both types of phenomena can be researched, at least in principle, with the same set of conceptual tools, even if not in the same ways and not with an equal ease. One time-honored quandary that becomes treatable with these unified tools is the question of our uniquely human capacity for changing our ways of doing things from one generation to another (for *societal learning)*. Unaccounted for by the traditional theorizations of learning, this special capacity for accumulating the complexity of our actions can now be explained by taking a close look at processes of development, in which discourses remain in a co-constitutive interaction with physical tools. With the tools together, they function as practically unbounded compressors, repositories, and disseminators of complexity. Since societal learning is the signature feature of the human species, commognition may be said to have made a tentative contribution to solving the puzzle of human uniqueness.

Whereas some of the old quandaries may now be regarded as dissolved, some other ones invite further commognitive study. In spite of the progress already made, figuring out the mechanisms of discourse development, whether ontogenetic or historical, is nowhere close to disappearing from the researcher's to-do list. The same may be said about the task of mapping the co-constitutive relations between our discursive and practical activities, or about project of fathoming mutual influences of mathematics and other discourses practiced in different societies. If successful in tackling these and similar issues, commognitive researchers may produce insights, the relevance and impact of which are likely to go beyond the practice of learning and teaching mathematics.

Cross-References/See also

Discursive Approaches to Learning, Discourse Analytic Approaches in Mathematics Education, Mathematizing as Social Process, Theories of Learning Mathematics

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