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# ON THE NEED FOR THEORY OF MATHEMATICS LEARNING AND THE PROMISE OF "COMMOGNITION"

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*In this paper, research in mathematics education is defined as a special type of discourse, in which potentially useful stories about learning and teaching mathematics are being told. Stories coming from a given discourse and considered as useful constitute* a theory. *A* commognitive *theory of mathematics learning, made distinct by its foundational assumption about the unity of thinking and communicating, is presented in accord with this discursive definition.*

For several decades now, the idea of theory, arguably one of the most impressive inventions of the human intellect, has been falling out of favor, at least in human sciences. The fierce manifesto "Against theory", issued more than three decades ago by literary theorists Knapp and Michaels (1982) has been echoing ever since in other human sciences. Today, with the advent of computerized pattern detection, we seem to have yet another reason to be nonchalant about theories. Titles such as "Big data and the death of the theorist" (Steadman, 2013) herald the possibility of theory-less research. More often than not, the harbingers sound relieved, if not outright triumphant.

No, theories do not seem to be well, at least not in human sciences. Being very much theory-minded, I regret this state of affairs. I am also worried by what is happening around. Most PhD students I know are quite desperate. They are in a constant quest after the holy grail of theory, but, depending on how they manage to look, all they can see is either a dazzling abundance of candidates or an almost total absence thereof. Either way, they feel helpless. Those of them who experience *embarras de richesse*, enquire about the possibility of theoretical "networking"; those who complain about the paucity of supply, start asking why they need theory at all.

I wish to claim that these doubts and predicaments are the result of certain misunderstandings about what theory is, what it is good for, and what can or cannot be done when several theories compete for everybody's attention. In this paper, after stating my position on these matters, I outline and justify the theoretical perspective that guides my research at this point in time.

## WHAT IS THEORY AND WHY DO WE NEED IT?

As a researcher, I feel strongly about theoretical thinking. For me, theory is the ground to stand on while trying to move and the signpost to follow while looking for a direction. Without it, I feel like walking on a thin ice in the middle of night. My aim in this section is to explain why I feel this way. For this, I will now try to clarify what theory is for me, what turns it into an indispensable part of my research, and what it is that makes me favor one theory over another.

Because theories arise in *research*, it is natural to begin with a definition of this latter concept. Probably the simplest way to describe research is to say that it is the activity of telling stories about some aspects of reality. Thus, research in history produces stories about past events, research in physics tells us about material objects (think, for instance, about the formula $S = ½gt^2$, which can be

read as a story of the free fall), and mathematics education researchers narrate the processes of learning and teaching mathematics. The resulting tales are not just-so stories. Most of them are meant to mediate ad improve those human activities around which they evolve (and let me leave for later the question of what counts as improvement and who is to tell). Thus, for instance, the researcher in the field of mathematics education strives to tell teachers and parents things about teaching and learning mathematics that may be useful for them to know, but across which they would probably not come themselves simply because of their being too preoccupied with precisely those activities, around which the researcher's stories evolve. What makes the researcher's narration different from other types of storytelling is the systematic use of discursive tools with which she can reach beyond the obvious and then communicate her insights in an unambiguous, precise way.

Different types of research are definable by how their stories are told and by the stories themselves. In other words, their first distinct characteristic is the *discourse* that the researchers employ in order to tell their stories. The other feature – the set of stories produced within a given discourse and endorsed by the researchers because of the general agreement among them that the stories present reality in an accurate and useful way – is what we call *theory*.

Except for the *theories* they produce, research discourses differ from each other in their storytelling tools: their *keywords* and these words' uses, the *visual mediators* with which the storytellers make clear what they are talking about, and the *routine* actions of their participants. Routine is a discursive pattern that may be presented as a set of rules followed, mostly implicitly, by competent storytellers. Perhaps the most conspicuous among the special features of research discourses are the relative strictness of their routines and the explicitness of many of the routine-defining rules. Those of these meta-rules that regulate the use of words are called definitions. Ideally, the stringent rules guard the narratives against ambiguity and make it possible for the storyteller to defend them against doubts and criticism. The researcher, therefore, may be expected to be much more accountable for what she says than is usually possible in informal colloquial discourses.

The distinctive feature of the collection of stories created with the help of a given set of tools and deserving the name "theory" is the tight interrelatedness of its components. Thus, for instance, together with any sub-set of stories already endorsed, the theory includes, at least potentially, all those narratives that can be derived from this sub-set with the help of rules known as laws of logic. Theories begin their life as small sets of stories specifying some basic properties of a given discourse's focal objects. These are known as theoretical assumptions. Once established with the help of these assumptions, theories start growing by absorbing new narratives constructed either by logical derivation from previously endorsed narratives or on the basis of observation and in concert with the rest of the theory. The required property of theories is their overall internal consistency: there must be no pair of narratives that would mutually exclude each other's endorsability. Of course, this is a highly idealized picture of theories, and it was presented here as but a signpost one must follow in spite of the unattainability of the ultimate destination.

Having distinguished between discourses and theories, I should now correct myself and recast my own declared need for a theory as the need for a properly constructed research discourse. Indeed, the difference between those among us who are "theoretically-minded" and those who doubt the indispensability of theory is not in our answers to the question "To theorize or not to theorize?" After all, according to the definitions just given, all researchers are storytellers, and thus theory-builders.

The relevant difference is in the strength of their commitment to unambiguous communication. I now have no difficulty explaining my own deeply sensed need for rigorous discourse. Indeed, only the form of talk defined as precisely and explicitly as possible lets me arrive the level of accountability and communicability which I feel obliged to sustain as researcher. At the risk of sounding a bit dramatic, I would thus say that for me, working toward a well delineated research discourse is a matter of the researcher's professional ethics. For the same reason, I believe in the necessity of making my basic assumptions explicit. In research in mathematics education, these initial stories only too often remain tacit, whereas the researchers themselves claim to be starting their study "with an open mind" and "with no theory". This, however, cannot be true. To say the least, the very choice of things to investigate implies the assumption about these things' possible significance for educational practice, and this indicates, recursively, the presence of tacit assumptions about some underlying causal relations! To sum up, I feel obliged to spell out explicitly both the definitions that guide my use of words and those primary stories that, for one reason or another, I endorse as the point of departure to my research. This is exactly what I will try to do in the rest of this talk, when I present my current research discourse.

First, however, let me say a few words about the epistemological status of our theories, as implicated in the just outlined idea of research as a discursive activity. This issue needs to be addressed, because even today, more than fifty years since this idea's inception (see e.g. Foucault, 1972), followers of the still powerful positivist approach to scientific research view some entailments of the discursive vision as unacceptable. Below, after presenting their arguments, I will show that these considerations are grounded in a faulty logic.

The discursive approach implies that there is no such thing the "ultimate" theory and that the theoretical plurality is the inherent feature of research. Different discourses may be incommensurable, that is, differ in their use of words (which in traditional terms may mean differing ontological foundations) and in the rules according to which their component stories are being endorsed (which some would say is the issue of diverse epistemologies). Theories coming from such discourses, even if they appear as contradicting each other, are not necessarily mutually exclusive. Indeed, assuming that one of them must be "untrue" would be as wrong as claiming that only one of the Euclidean and non-Euclidean geometries can be true. In fact, the adjective "true" does not really apply to theories, and if we wish to choose between them, we should rather ask about their *usefulness*. As to the adjective *useful*, it is clearly a matter of what those in the position to decide consider as such rather than of any universal, rigidly defined criteria. Indeed, one's vision of usefulness depends on this person's values and on her conception of the needs of those for whom research stories are told. All this implies the inherent impossibility of ordering all the theories along one line. We thus seem to have no choice, but to let thousand theories bloom.

On the other hand, it would be a mistake to deduce from here, as some people do, that "any theory goes". Claiming this would be as weird as asserting that the proposition "No single pair of trousers can be regarded as absolutely and forever the best one for Mr. X" entails that for Mr. X, any pair of trousers is as good as any other. Yes, some theories may appear to us better than some others, and some may be even seen as superior to all the alternatives we know. One must, however, view this superiority as inherently provisional and treat any current "winner" as a permanent candidate for dismissal. This is exactly how I see the *commognitive* approach, which I am about to outline now (the

sources of the name will be explained later): although this discourse answers the current needs of my research better than any other I am aware of, I do not delude myself about its ability to retain this status for much longer or, for that matter, about its being as attractive to others as it is to me.

The rest of this paper is devoted to the introduction to commognitive discourse. This is done according to the discursive definitions of research and theory: in the next section, I take care of commognitive keywords; I then proceed to perceptual mediators, equivalent in this case to what is known in research as data; I follow with a section devoted to routines, also known as methods of analysis; I conclude with a brief survey of stories generated so far by the commognitive research, thus outlining the commognitive theory of learning and teaching mathematics in its present version.

## THE COMMOGNITIVE KEYWORDS AND THEIR USE (BASIC CONCEPTS)

Because the research in question deals with mathematics learning, the main keywords that have to be defined here are *mathematics* and *learning*. The first of these terms has already been implicitly defined in the former section: being a type of research or a "domain of knowledge", mathematics is a discourse. Of course, school mathematics is not the same as mathematics of research mathematicians. These two types of discourse differ in all four characteristics, but above all, in the nature and strictness of their meta-rules. But although in this respect school mathematics may seem closer to colloquial discourses, it is still tightly related to the discourse of mathematicians. The talk and stories about shapes, numbers, sets and functions that the members of contemporary societies meet in school and are obliged to master can be considered as a product of "customization" of the mathematicians' mathematics to the needs and capacities of the young learners, as these needs and capacities are understood by whoever is responsible for the customization.

From here it follows that *learning* mathematics is the process in which students extend their discursive repertoire by individualizing the historically established discourse called mathematics. To *individualize* a discourse means to become able to communicate according to its rules, and to do so not only in conversations with other people and possibly with their help, but also while "talking" to oneself and solving one's own problems. Thus, to say that a person individualized mathematical discourse means that this discourse became a discourse of her thinking.

This last sentence obliges me to explain the commognitive interpretation of the word *thinking* as well. The way this term has just been used makes it clear that thinking is considered as tantamount to communicating with oneself, and not necessarily in words. This explains the source of the neologism *commognition*, coined from *communication* and *cognition* so as to serve as a constant reminder that communicating with others and thinking "in one's head" belong to one ontological category. All this implies that in spite of differences in these two activities' visibility, we can use a single set of tools to investigate them both.

Uniting the thinking-communicating divide, the idea in which the commognitive discourse takes its roots, has been inspired, among others, by philosophical writings of Ludwig Wittgenstein and by psychological musings of Lev Vygotsky. Both these thinkers repeatedly stressed the inseparability of thought and its expression. Wittgenstein debunked the view of thinking as ‘incorporeal process which lends life and sense to speaking, and which it would be possible to detach from speaking’ (§339, Wittgenstein, 1953/2003, p. 109). Vygotsky's (1978) insistence that any uniquely human competency originates in a historically established, collectively executable activity implied that thinking, arguably

the most unique of human activities, must also had a developmental predecessor in the form of some historically established, collectively implementable activity. Since communi- cation is the most obvious candidate, one cannot but conclude that to think means to communicate with oneself. As will be explained in the next section, the feature of non-duality has multiple consequences for how researchers identify cases of mathematical learning and how they subsequently interpret these events.

## COMMOGNITIVE MEDIATORS (DATA)

Mediators are the generators of perceptual experiences that help us in getting to know the objects around which our stories evolve. In research, this includes all those things, known as *data*, which we are looking at or listening to while crafting our narratives. Because the object of commognitive research is discourse, its mediators are mainly, although not exclusively, recordings and transcripts of learning-teaching events, such as classroom interactions or research interviews.

This type of data is not unique to commognitive research. Its less common feature is the re-searcher's insistence on recording events with the help of video and on the uncompromising rigorousness of the subsequent transcribing. Indeed, the researcher tries to document anything that can impact communication. Of course, since nothing short of the event itself can provide the observer with information about all potentially relevant features of an interaction, the transcriber will have to choose which aspects to follow. This must be done according to the needs of the upcoming analyses. Two features, though, should never be compromised. One is the compre-hensiveness of the transcript: Because the commognitive researcher studies discursive processes rather than mental structures in the heads of the learners, she needs to view the exchange in its entirety, with no participant's part – not even that of an "unobtrusive" observer – being dismissed as of lesser importance. The other necessary feature of transcripts is their verbal fidelity. Since the non-dualistically minded researcher rejects the word-meaning and form-content dichotomies, she needs verbatim records of things said, with no word changed and no pause omitted. Because of the nature of the task, she must live with the fact that her work as transcriber is never done.

For all the emphasis on full records of communicational events, commognitive research does not dismiss written questionnaires that can be submitted to quantitative analyses. These are welcome, provided they come as an integral part of a study in which records of interactions constitute the primary corpus of data. Each type of data has its own unique role: only videos and transcripts can give rise to conjectures about hitherto unknown phenomena; the quantitative follow-up helps to decide whether what was observed can really count as *phenomenon*, a pattern frequent enough to be considered as a possible explanation for any comparable occurrence.

## COMMOGNITIVE ROUTINES (METHOD OF ANALYSIS)

Commognitive methods for crafting and endorsing new narratives, also known as methods of analysis, benefit from the operationality of the commognitive vocabulary. The fact that the re-searcher is studying discourse, the activity that may be public (as is the case in classroom discussions) or private (as in the case of thinking), but is always describable with the help of the same, operationally defined set of characteristics, lets the researcher be fully accountable for what she claims on the basis of her analyses. True, some important parts of the discourse under study may be inaccessible to direct inspection. And yet, a researcher who makes conjectures about the learner's

inner dialogues may be compared to an archeologist who reconstructs an ancient vessel by complementing the excavated pieces with her own additions. In both cases, the available and the added pieces are of the same kind, with the latter ones informed by the researcher's understanding of the relevant context and by her awareness of what would count as reasonable. In this respect, commognitive research differs from cognitivist studies, in which thinking is treated as a different, and more basic kind of phenomenon than interpersonal communication. Out of several further differences between commognitive analyses and more traditional methods let me mention just two.

First, for the commognitive researcher, the object of study is the participants' discourse as such, and not anything that can be seen "through" it. Thus, for instance, if she is interested in the participant's learning, she makes it clear that her findings are about *the learner's stories* of mathematical objects. This, as opposed to talking about entities in the learners' heads, supposedly detectable through their stories. Similarly, the interviewer who inquires students about their experience of mathematics presents her findings as a story *about her interviewees' stories* rather than her own direct testimony about the interviewee's experience. To those who wonder about the significance of stories about stories, as opposed to stories about the reality itself, let me remind that we live by the stories we tell, that is, by our *perception* of reality, rather than by the unmediated reality as such.

Second, the commognitive analyst must constantly alternate between the *insider's* and *outsider's perspectives*: to be able to do the work of "archeologist", she needs to act as an insider and use her own interpretations of words to make sense of what other participants are saying; in parallel, however, she must be able to act as an outsider to her own discourse and, by suspending her understanding of words, allow herself to think about these words' alternative uses. Thanks to the insider's perspective, her stories take care of those aspects of learning processes that are, as a rule, left out from behaviorist accounts. Thanks to the outsider's perspective, her narratives are about what and why the learners *do* rather than about the tasks they are "still" unable to perform, and this feature makes commognitive narratives quite different from typical cognitivist stories of learning, and also more useful than most of them.

## COMMOGNITIVE STORIES OF MATHEMATICS LEARNING (THEORY)

The commognitive theory has been evolving for 25 years now, and the stories on the development of mathematical discourses told on the way cannot be summarized on the remaining pages. Below, I signal in italics some of the best developed commognitive storylines, adding a few words about the ongoing process of fleshing them out with additional detail, new insights and further evidence.

But first, let me remark that *all the objects around which commognitive narratives revolve must be understood as* metaphors *originating in discourses about material objects.* This means that stories of numbers or functions cannot be forged in the same way in which scientists construct an account of an exotic plant, by simply exploring these objects. Instead, the mathematical storyteller must "communicate these objects into being" as she goes. All the storylines listed below attend closely to different aspects of this unique process of objectification, that is, of turning stories about processes into narratives about as-if independently existing objects.

*Storyline 1: Historically, mathematical discourses evolved hand in hand with practical activities, which these discourses helped to expand.* Each such expansion, in turn, called for an additional growth of the discourses, and this led to yet another development of practical activities, and so on, up

to a point where the further development of mathematical discourse began coming from inside this discourse rather than from the reality around. The initial co-dependence of cultural practices and discourses has been corroborated in the study recently conducted in the Polynesian country called Tonga, in which the particular difficulty with fractions and probability experienced by otherwise successful mathematics learners has been explained by the fact that no native Tongan activity could be found that would benefit from any of these discourses (Morris, 2014).

*Storyline 2: Mathematical thinking is developmentally secondary to interpersonal communication about mathematical objects.* Unlike objects studied in, say, biology, at least some of which can be experienced by the child before she is able to speak, mathematical objects make their first appearance in one's life as words or symbols used by other people. Only the persistent use of these signifiers in mathematical conversations with others can eventually turn them into a signifier-signified pair. In our empirical studies, we had ample opportunities to see children grappling with the inherent dilemmas of the task. In result, we managed to present in some detail early versions of discourses of numbers (Sfard and Lavie, 2005; this study is ongoing), of algebra (Caspi and Sfard, 2012), and of functions (Nachlieli & Tabach, 2012). We believe that in this way, we have outlined trajectories travelled by many, if not by the majority, of mathematics learners.

*Storyline 3: Routines are the basic building blocks of discourses, and they make their first appearance in the life of a child as rituals.* This storyline comprises narratives on individual routines that develop as precursors to discourses of which they will eventually become a part. Before the child created the objects that such routine is supposed to involve, she can perform this routine only in a ritualized way. As a result of objectification, such separate rituals will eventually coalesce into a full-fledged discourse and will lose their ritualized character. This process, and especially those occurrences that interfere with its completion, have been investigated in much detail in the studies mentioned above.

*Storyline 4: Objectification, the key occurrence in the process of turning rituals into explorations, happens in leaps rather than through gradual change.* This claim was spelled out already in the earliest version of commognitive theory, and was based mainly on mathematicians' testimonies about their *aha* experiences. I was subsequently convinced by some critics that there was practically no chance for catching such events on camera. I am thus happy to report that Shai Caspi, in his study completed in 2015 and yet to be published, has been able to actually record such occurrence more than once, and with unexpected clarity.

*Storyline 5: Learning-teaching interactions are a special variety of discourse, with its own routines that need to be learned.* Among the rules that govern the "cocoon" interactions within which children's first mathematical routines incubate let me count *learning-teaching agreement* that requires all the participants to play their respective roles as teachers and learners. We became aware of the need for this agreement in studies in which it was violated (Sfard, 2007). In our current research we keep studying the evolution of learning-teaching routines, asking what can help the child start acting independently and agentively out of the cocoon interaction.

*Storyline 6: Mathematical success and failure of an individual are collective achievements.* In her recent studies, Einat Heyd-Metzuyanim (2013) was able to describe in finest detail how students' families, teachers, friends and, in a sense, the entire society, collaborate in creating and sustaining

these students' identities as either successful or failing learner of mathematics. The inherently discursive processes of identity building has been shown to result from common, but usually ignored classroom occurrences: from the interplay between *mathematising* – talking about mathematical objects, and *subjectifying* – telling stories about participants of the process.

By way of conclusion, let me express the hope that some of these still-evolving commognitive stories will count as original contributions to our understanding of learning mathematics. To those who view some of these narratives as an attempt to sell old wine in new bottles, let me say that the similarity of the italicized bottom lines to some widely endorsed claims about mathematics learning conceals an important difference in the stories themselves. The added value of commognitive narratives is that except for stating *that* something is the case, they bring detailed insights into the questions of *how* and *why* things happen. This makes commognitive stories particularly well suited for their role of practice changers. The detailed narratives point their audience to specific occurrences that result in specific outcomes, and as such, yield advice specified at the level of the participants' elementary moves. This seems to be the kind of comprehensive, operational and reliable guidance that we need if we are to change learning-teaching processes effectively, responsibly, and in concert with our intentions.

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