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FUNDAMENTAL CONCEPTS IN DIDACTICS: PERSPECTIVES PROVIDED BY AN ANTHROPOLOGICAL APPROACH

Yves Chevallard [1]

ABSTRACT

This lecture is intended to provide a first introduction to the approach hereafter styled anthropological in the analysis of the didactic.
Some preliminary observations help make clear what the "rules of the game" are, and two introductory examples illustrate the spirit of the thing. Then comes a presentation of the theory and of its main concepts, as they have grown out of earlier formulations relating to the study of the didactic transposition process.
The development out of the theory stresses the need for the didactics of mathematics to position itself, within the framework of a didactic anthropology of knowledge, in relation to a didactics of knowledge to which it has hitherto contributed more than any other didactics.

In the university system, it is the general custom to speak in terms of half-yearly or yearly lectures, courses, etc. But what meaning can be given to a "course" which takes up at most one and a half hours of our time?

In light of this, I shall consider my presentation today as a simple *lesson*, which ideally takes its place within a course, which I hasten to add remains for me largely a whim of the imagination.

For things to have been different, a prior transpositional operation would have had to have been completed, which would have "created" precisely such a course. However, I have never, up to now, been given the opportunity to produce — in the sense in which one creates something, but also in the sense in which one produces a document — a real course on "the anthropological approach in the didactics of mathematics". This is generally, it is true, the fate of researchers in relation to their own work: at best it falls to him to inaugurate its didactic transposition and to oversee "teething troubles", so to speak.

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Nevertheless, I will emphasise here in what way our spontaneous expectations — yours and mine — may be unrealistic, and how the time we shall spend together may prove frustrating, if we are mistaken on this point: the "lesson" I have prepared takes its place, hypothetically, at the start of a course *which I can only imagine*. I am therefore incapable of giving it a semblance of an *inaugural* lesson which would allow us "to make a guided tour", calmly, to get the perspective of a subject to be taught which enjoys only potential existence.

More modestly, this is simply an *introductory* lesson to which you are invited. I am, of course, delighted that the organisers of this summer school have forced my hand, and given me the possibility of achieving this aim, however limited it may be.

1. Metatheoretical observations

1.1. Critical hyperrealism and representationalist illusion

The notions of *theory* and *model* — which I shall barely distinguish in what follows — are fairly incomprehensible if we start from current culture which is imbued with what I shall call the *representationalist illusion*.

This, briefly, encourages us to consider theories or models of a given system as *representations*, or *images*, of the system which we then claim to theorise or modelise.

The cultural metaphor of the image represents a real epistemological obstacle, the clearest symptom of which is the common practice of *hyperrealistic criticism* regarding theories and models. This "critical" point of view has the merit, it is true, of being almost always available: indeed, there almost always exists at least one culturally authorised point of view which allows a given theorisation to appear "incomplete" and "unfaithful" to the original.

I would like to suggest that such criteria of judgement, — "completeness" and "faithfulness" to the reality studied — are illegitimate not only in practice but also in principle.

1.2. Metaphors as tools of thought

Contrary to what is often thought, I would like to state here that any scientific activity (including mathematics) is made up (through its language) and described (through its metalanguage) by means of *metaphors*. Thought develops basing itself on metaphors; more generally, "rhetoric" is a component of scientific activity, as of any noetic economy.

It is not, therefore, in any way illegitimate to think of theories and models in terms of images and representations. However, the greatest problem here lies in the choice of "good" metaphors, of genuinely fecund metaphors which will not get out of hand. The Greek mathematicians, thus, were unable to consider the ratio of integers *as numbers*. They lacked the audacity to adopt this metaphor, and this excessive respect for cultural convention raised an obstacle to the development of the science they had created.

If things had been different, moreover, no doubt some philistine would have raised the following objection: "If I multiply an integer by an integer, I find a *greater* integer; however, that could be false if the multiplyer is one of your famous "numbers", a ratio of integers which is not a whole number! It is therefore *extremely dangerous* to call anything a number when it clearly is not."

Under this misleading criticism, which stems from a poorly controlled use of the number metaphor, we encounter an epistemological obstacle with which we are quite familiar. It appears, in today's pupil, to be a "problem of comprehension" rather than a critical argument addressed to the number metaphor (as a pupil does not, generally, claim the necessary cultural and epistemological authority to allow conversion from one type of symptom to another), and which I will call, to qualify it more precisely, *the metaphor obstacle*.

Metaphors are, however, indispensable in scientific work.

Hence modern functional analysis is based around a cluster of geometrical metaphors, considering a function as a "point" in a certain "space" or — contrary to what we are used to in ordinary geometrical space — considering that two "points" (two functions) "are similar" the closer they are to each other. Metaphors of geometric origin, let us note in passing, are here

impregnated by another metaphor — that of similarity — which has a greater cultural surface area.[2]

However, let us return to the theme of theorisation. In order to consider the activity of theorisation (or modelisation), I shall now introduce *another metaphor,* just as legitimate in principle as that of conceptualisation, but, I believe, giving much more freedom.

A theory or a model of a given system is a *machine,* which, when started up, allows *knowledge to be produced* relative to the modelised (or theorised) system. (Here I use the word knowledge in its common meaning, thereby using a metalanguage accessible through common culture, as opposed to the theoretical language which I shall present later.)

First, we see that the metaphor of the knowledge-producing machine allows one to discard the false obsession with the "similarity" between theory and the reality theorised. A machine for the production of a certain component has no reason to resemble the component to be made, nor to resemble the object in which the component will be placed. In what follows, I shall give two simple examples which should help to understand the use I make of a metaphor which is culturally uncommon: the first stemming from mathematics, the second from the life sciences.

1.3. Two "systems" and their model

Let us consider the number $a = \sqrt{3}$. This will be our system. (I purposely leave to one side what you may regard as a difficult point: regarding a number as a "system", which must be modelised.)

I shall start from a model of the number which is derived from the definition of our system. This model takes the form of a set of algebraic expressions: $a^2 = 3$, $a > 0$. While "working on" this model, I will now produce other, mathematically equivalent models. Consider, successively, the following:

$$a^2 - 1 = 2,\ a>0;$$
$$(a-1)(a+1) = 2,\ a>0;$$
$$a - 1 = 2/(a+1),\ a>0;$$
$$a = 1 + 2/(a+1),\ a>0.$$

2. On this point, see Chevallard and Johsua 1982.

I shall stop there. The last model obtained now allows me to write the following, *by means of which I produce knowledge relative to the system studied:*

$a = 1 + 2/(a+1), < 1+2/(0+1) = 3$
$\Rightarrow\ a = 1 + a/(a+1) > 1+2/3+1) = 1.5$
$\Rightarrow\ a = 1 + 2/(a+1) < 1+ 2/(1.5+1) = 1.8.$

I thereby obtain the inequalities $1.5 < a < 1.8$. If I continued operating my "little machine" — the model $a = 1 + a/(a+1)$, $a > 0$ —, I could, of course, further improve the state of my knowledge of the number a.

Of course, the model which I have "put to work" does not resemble the system it modelises *in any culturally familiar way.* However, it definitely appears as *a machine for producing knowledge* relative to this system. Again of course, it does not produce with equal facility — with equal yield — just any *type* of knowledge concerning it. *Other models,* equally "dissimilar", should no doubt be constructed and perfected in order to produce *other types of knowledge* relative to the *same* system.

The type of system I will now study is... a dog. I construct a first — geometric — model by simply considering a cylinder of length L and radius R.

Here we get a better picture of how the representational-pictural metaphor slipped into scientific culture. Of this cylinder, which can be imagined in "horizontal position" (axis parallel to the ground), it could be said that it is a *stylised,* simplified, dog, several elements of which have been "left out" (the head, the paws, the tail, etc.) or that it "represents" the dog and more subtly, that its surface area represents the surface area of the dog, whereas its volume represents the volume of the dog.

The representational metaphor — in principle not illegitimate in the least, remember — here finds a *local* use, allowing us, without much cost, to think out and guide the job of modelisation to be carried out.

Let us continue the job of modelisation, moving from the preceding *geometric* model to an *algebraic* model — which the geometric model *will only have served to produce.* We thus reach the following final model:

$V \propto R^2L,\ S \propto RL + R^2,$

where the symbol $\propto$ designates the relationship of (direct) proportionality.

What can such a model be used for? To produce (rudimentary) knowledge concerning an essential problem in the life sciences: that of the metabolism of the animal. First let us recall the following, which constitutes prior knowledge required to put the model to work: the metabolism of an organism produces a quantity of energy proportional to the *volume V* of the organism; loss of energy, however, is proportional to the *surface area* of the organism.

Let us now consider a dog of a given size (fixed R and L) whose metabolism ensures a thermal balance; and let us consider a hypothetical dog of half the dimensions of the first dog considered (R' = R/2, L' = L/2). The former model allows us to conclude that the volume V' of the hypothetical dog is *8 times smaller* than the volume V of the first dog considered, whereas (with analogous notations) the surface area S' is only *4 times smaller* than S. One can conclude in the language of images, that, if the metabolism of our hypothetical dog remained unchanged, the animal would end up dying from cold: small dogs are warmer than large animals.

1.4. Neglect what is negligible

From a personal point of view, which remains totally distinct from the theorisation which I will present hereafter, I would not see any problem in replacing most dogs and cats by simple metallic cylinders, or even by the geometric representation of such cylinders, or even by the equations written above.

However it must be recognised that it might be difficult to persuade animal lovers of the "similarity" between the "system" dear to them (and whose litter must be changed at least every other day) and the model which we could propose as a replacement — which does not require food and has only to be dusted once a week.

I shall not linger any longer over the question of similarity. However, the previous example will now provide the chance to formulate an essential commentary. When one theorises or modelises, to use a formula provided by G. Bachelard, *everything that can be neglected must be neglected.*

This formulation must however be corrected up and completed. First completed: what can be neglected can never

be neglected *absolutely,* and is only neglected, at most, in relation to the project of producing *a certain type* of knowledge relative to the system studied. Next corrected: what can be neglected can only, at the start of the job of modelisation, be *supposed* to be negligible. For there is no absolute guarantee concerning negligibility, and any theorisation will thus be a risky undertaking. This is why, some, as they watch the caravan go past, through lack of scientific education, accumulate pessimistic forecasts on the outcome of the adventure — simply because there are risks involved. One can dream of sure knowledge; there is, however, no sure road to knowledge.

2. The style of the anthropological approach

2.1. The problem of zero

I shall give two examples here, one microdidactic, the other macrodidactic, in the aim of illustrating a priori the peculiar atmosphere of analyses of anthropological style in didactics. The first — microdidactic — example will also serve to suggest that anthropological sensitivity is, as opposed to what one might think, constantly present in the work of G. Brousseau, and has been from the start — even if it is often situated slightly in the background compared to explicitly formulated analyses.

In what is now older work — around 1980 — dealing with the question of elementary algebra, we encountered what is called *the problem of zero*[3].

The number zero, indeed, gives rise to errors which seem to be specific and which, beyond their obvious erratic nature, show undoubted recurrence in behaviour: in primary school, on occasion, pupils might write for example 0x4 = 4, etc.

From this arose, in the noosphere and beyond, a set of considerations on the supposed difficulty of the "concept" of zero.

The theorisation developed from this time by G. Brousseau taught us that the significance of a type of behaviour must be referred to the overall frame in which this behaviour arises: that of the *didactic contract.* We analysed to a certain extent the

3. See Pascal 1980.

didactic contract relating to calculations — numerical and algebraic — and we drew up a model, rudimentary but adequate, to confront the problem of the significance of errors linked to the presence of zero.

This model brings a certain *metaphor* into play, which must be "put to work" rather than camouflaged. It is this: giving a calculation to a pupil means entrusting him with an expression (numerical or numero-literal) which he will have to *manage* correctly, which will guarantee the validity of the expression to be "delivered" at the end of the operation.

One of the characteristics of this correct management is linked to the fact that the calculatory transformations operated by the pupil produce, at every step of the operation, *small* variations in the *ostensive complexity* of the expression with which he has been entrusted. In other words, acceptable calculatory management, consistent with the didactic contract, should not include any "catastrophic" operation, *which would brutally modifiy the ostensive structure of the expression whose successive transformations must be managed by the pupil.*

One can immediately see that in this regard zero poses a problem, in that it appears inevitably *as a factor of calculatory catastrophes,* which make "correct" management of such expressions impossible. Let us refine our description a little.

The transformations carried out must, more precisely, satisfy a rule, which could be called *a postulate of the conservation of ostensive differences,* and which can be stated thus: a change in one of the parametres defining the ostensive structure of the written expression entrusted to the pupil at the start must be translated *by a correlative change in the expression "delivered" at the end.* Here is an example.

Take the equation $3x = 12$. The pupil must provide the statement $x = 4$. If, changing the coefficient of the unknown, we now give him the equation $4x = 12$, he must provide $x = 3$. In the same way, for $2x = 12$, he will provide $x = 6$, etc.

Hence, to each visible change in the original statement corresponds a visible change in the end statement. This is a rule which at first would seem to admit of no empirical exception in the pupil's practice. At a more elementary level, likewise, he will provide 8 when entrusted with the statement 2x4, 12 for 2x6, etc.

Let us now introduce a zero: instead of equations such as $3x = 12$, $4x = 12$, $2x = 12$, etc., let us get him to "manage" equations such as $3x = 0$, $4x = 0$, $2x = 0$, etc. The correct answer, here, is uniformly, $x = 0$. Hence, to the visible variation in the statements entrusted to the pupil there is now *no corresponding variation in the expression to be delivered by the pupil.*[4]

If this purely numerical phenomenon is taken into account on one hand, and the contract relating to calculations on the other, there is no need to expound on the "difficulty" of the "concept" of zero in order to interpret the classic errors, seemingly erratic, relating to zero. These errors in fact arise from what is, from the pupil's point of view, *a strategy of "healthy" calculatory management.* This strategy allows the pupil to respect, *against the objective laws of algebra,* the didactic contract he may believe himself involved in — the "correct" answer proposed by the teacher ($x = 0$) may even appear to him to cause a *unilateral and incomprehensible breach of the contract.*

What will a pupil supposedly caught in the trap of the contract do? Take for example the equation $2x = 0$. In order to respect the contract, the end statement would have to feature an ostensive trace, characteristic of the original statement, so that *something* remains of the coefficient 2 which featured in the equation given.

For this reason one can observe, among the answers actually provided at a certain point in the learning process (and therefore in the setting up of the didactic contract governing situations of calculation), statements such as $x = 2$, $x = 1/2$, $x = -1/2$, etc, *which all meet the required condition.* Unless — and this is another way of not breaching what is thought to be the contract — the pupil simply refuses to reply!

The type of analysis which illustrates the above developments is marked, in my opinion, by the fact that the analytic frameworks proposed by G. Brousseau are taken seriously, those of *didactic contract* and *situation.*

The pupil is considered, not from the point of view of Sirius, but in the real situation in which he finds himself during

4. Similarly, in elementary addition, one moves from 2x4 = 8 and 2x6 = 12 to 0x4 = 0x6 = 0.

the first few years of studying mathematics in school. To use a formula which has now gone out of fashion, "a concrete analysis of a concrete situation" is made.

For example, it is only much later in the course of the study of mathematics that zero, or more precisely "nil quantities", appear to the pupil in a regular manner as *fortunate* factors of calculatory catastrophes.

One might think, for example, of a pupil's encounter with a classic situation of barycentric algebra. Point G being the barycentre of points A and B bearing respectively coefficients 3 and 5, the pupil will learn to recognise the happy effects of the nullity of the vector 3GA + 5GB when entrusted with the management of the expression $3MA^2 + 5MB^2$ (where M is any point in the plane):

$$3MA^2 + 5MB^2 = 3(MG+GA)^2 + 5(MG+GB)^2$$
$$= 8MG^2 + 2MG.(3MA+5MB) + 3GA^2 = 5GB^2$$
$$= 8MG^2 = 3GA^2 + 5GB^2.$$

Once again I shall set the type of analysis presented here against what, in my private language — which you are at liberty to adopt or not! — I call *cultural delirium.* This procedes almost systematically by reification of idealities — here, the "concept" of zero — considered abstractly, *without relating it to the concrete conditions of the pupil's activity.*

2.2. The child and playing

The second example I shall give is of quite another magnitude. We are familiar with the role assigned to playing (and to games) in the child's early school and preschool learning. Playing is taken as the fulcrum of various didactic strategies: this is the observable fact. However, once again, around this fact cultural delirium has developed: the child and playing are believed to be of one substance; the child is taken to be *homo ludens* in his original perfection, etc.

The anthropological analysis I have proposed is quite different[5]. Let us start from the following problem: on entering into an institution, one must, in order to become a "viable" subject of the institution, become subject to the corresponding institutional contract. However, how can the entrance into the

5. See Chevallard 1988.

school's institutional contract, of a child who has not yet received any schooling, take place, concretely? How can the first didactic contract come to life between "subjects" who are a priori foreign to any school didactic order?

Let us look further into the problem. The institutional (didactic) contract supposes that the child in the position of pupil will participate as an actor in the activity sequences which I will call, without nuances, *long* (relatively) and *connected.* However, much of the child's life outside the school institution sees him involved in activity sequences which are both *short* and *disconnected.* Entrance into school, therefore, from this point of view alone, marks a fundamental change.

On what can the school institution base itself in order to promote this change? In what continuity with the child's life prior to starting school can this "new state" establish itself?

Fortunately, through his experience of institutions prior to school, the child who has not yet received schooling is not totally unfamiliar with long and connected activity sequences. More precisely, he is familiar with two categories of activity, which have previously drawn him into this sort of situation.

The first category is, precisely, that of *play* activities, which can involve fairly long sequences which are not necessarily independent of each other (a game may be taken up again after having been abandoned for a few minutes, a few hours or a day).

The second category of activity is *story-telling,* in which the child participates as *listener,* while the narrator is an adult: this is a long sequence, which the child would often like to see continued, and which can be taken up evening after evening at bedtime for example.

This second type of activity is the prototype of what will become, in school, the "lesson" or "class". The first category — playing — is the prototype of what could be called, generically, "practical work" — even if this terminology is not used in primary school.

Hence, the emergence of the first didactic contract bases itself on contracts already familiar to the pupil through his experience of the institutions which govern preschool childhood. The didactic use of playing, early in schooling, is one particular way of making an almost universal institutional strategy concrete: a new institution emerges, for its subjects,

against the background of institutions which already exist for them, and it is based on this "genus" even in order to make them understand, a little later, its "differentia".

3. Cognitive anthropology

3.1. Objects, persons, institutions, relations

The theorisation I shall now deal with was present in seed form in the first formulations relating to the theory of didactic transposition. It must be seen as using and articulating notions whose elaboration is aimed at allowing unified reflection on a large number of didactic phenomena encountered through various analyses.

I would just like to add that my presentation will become somewhat formal, almost axiomatic, even though I shall not enumerate explicitly the "axioms" on which the theory is based.

Three primitive terms are necessary to begin with (others will be added later): *objects* O, *persons* X, *institutions* I.

Objects, however, here hold a privileged position: they are the "basic material" for the theoretical construction envisaged. Just as, in the contemporary mathematical universe, based on set theory, everything is a set (integers themselves are sets), in the universe I would like to consider, *everything is an object.* Persons X and institutions I, as well as other elements which will be introduced, *are objects of a particular type.*

For this reason, I shall first look at the generic notion of object, which the theory thus places at the source of its development.

From the point of view of the "semantics" of the theory, anything can be an object. An object exists as soon as a person X or an institution I recognises this object as *existing* (for it). More precisely, it can be said that the object O *exists for X* (respectively, *for I*) if an object exists, which I shall note R(X,O) (resp. $R_I(O)$), and which I shall call the *personal relation of X to O* (resp. *institutional relation of I to O*). In other words, the object O exists if it exists for at least one person X or one institution I, that is if at least one person or one institution *relates to this object.*

In practice, the first analyses proposed in *La transposition didactique (Didactic Transposition)*[6] were confined to distinguishing "mathematical", "paramathematical" and "protomathematical" objects. The widening of the framework required by the needs of analysis led me to propose a theorisation in which *any* object could appear: the logarithmic function is, of course, a (mathematical) object, but there is also the "school" object, the "teacher" object, the "learn" object, the "knowledge" object, the "toothache" object, the "toilet" object, etc.

Hence one passes from a "machine" for reflection on a *limited* didactic universe to machinery of wider influence, in theory suited to allowing us to situate *didactics within anthropology.*

I add here another notion: *acquaintance*[7]. *To be acquainted with an object O,* in the sense of the theory presented (and not in the sense of the various institutions it should allow us to study), is, for a person as for an institution — *to relate to O.* Person X (or institution I) is acquainted with O if R(X,O) (respectively, $R_I(O)$) exists. It can then be said that an object exists if it is known by at least one person or one institution (it may, moreover, exist — a limited case — only for this person or this institution). An object exists only because it is *an object of acquaintance.*

The conceptual framework which I have just sketched is that of what I call *anthropology of acquaintance,* or *cognitive anthropology.* The notion of object and the associated notions which I have just introduced may, a priori, appear rather rough; however, I would like to suggest, using a single example, that they allow detailed reflexion on anthropological reality, that is on the stuff of acquaintance itself.

Take the following experiment (which I cannot yet give a name to, we shall soon see why): when going downstairs, I reach the last step thinking that there is still one step to go; my foot hits the ground abruptly, just when I expected to go down again. The object I want to talk about arises from this type of

6. See Chevallard 1991, the first edition of which dates from 1985.
7. In agreement with the author, the translator of this text has rendered the French distinction between "connaissance" and "savoir" (both normally translated by "knowledge") thus: connaissance = acquaintance; savoir = knowledge.

situation, which I shall now call: "the sensation felt on reaching the bottom of the stairs when one thinks that there is still one step to go".

I think I can confirm that this object *exists for each of us*. At the same time, it does not exist, as far as I know, *for any institution* that I know of. That is why, to name it, there is *no recognised denomination*. One could however imagine that, for example, if the type of incident in which this object came to life for each of us gave rise regularly to traumatic consequences, requiring specific treatment, if it entailed a certain type of sprain for example, this object would exist for one or more institutions: for the medical institution, or for the institutions of daily life — such as is the case for "whiplash".

It is now time to say a few words about this central character, *institutions*. Once again, an institution can be just about anything. In reality, because of the current meaning of the word, some of you may be surprised to see on which objects I may be led to stick this label. A school is an institution, as is a class; however, there is also the institution of "practical work", the institution of "lectures", the institution of "family". Daily life is an institution (in a given social setting), and so is the state of being in love (in a given culture), etc.

I would like to point out in passing that, as concerns the notion of object, I know of few elaborations which could have inspired the theorisation presented here, apart perhaps from the phenomenology of Husser and some of his descendants, and apart from some developments of Soviet psychology; the same cannot be said concerning the notion of institution; in this respect one may usefully consult Mary Douglas' work, *How Institutions Think*.

Having noted this, let us see how objects and institutions are linked. With each institution I is associated a set of objects, O_I, called the set of *institutional* objects (for I), which is the set of objects O known by I, that is to say for which there exists an institutional relation $R_I(O)$. An object O is institutional for I, in other words it exists for I, if I has defined an institutional relation with O. I add that certain analyses call on an extra notion: that of an object which is *institutionally visible from I*, and yet which is not an institutional object for I. I shall not spend any more time on this point in this presentation.

However, it is necessary to introduce a new primitive term, through which the reader of *La transposition didactique (Didactic Transposition)* will recognise an extension of the notion of didactic time. For every institution I, there exists what I will call *an institutional time* t_I.

Two essential notions may now be *defined* (these are no longer primitive terms). They derive, of course, from the corresponding notions — of *didactic contract* and *milieu*, respectively — introduced by Guy Brousseau in the theory of didactic situations.

We designate by $C_I(t)$, and name *institutional contract* relative to I in time t, the set of pairs $(O, R_I(O,t))$, where O is an element of $O_I(t)$.

We name *institutional milieu* relative to I at time t, and we note $M_I(t)$, the sub-set of $C_I(t)$ formed of the ordered pairs $(O, R_I(O,t))$ "stable" at time t.

The formalization is incomplete here, as we have to use a notion of stability which has not been defined. However, it can be clarified here, anticipating the rest of the presentation: the elements $(O, R_I(O,t))$ which make up the milieu — the "stable" elements — are those which, subjectively, that is for the subjects of the institution I (the notion of subject will be introduced shortly), appear *self-evident, transparent, non-problematic*.

3.2. Subjects, positions, learning

Hitherto I have spoken about *persons*, X. I shall now introduce the primitive notion of *subject*. Take an institution I. A person X becomes a *subject of I* when it becomes "subject" to I. Metaphorically, one could say that X becomes a subject of I "on entering I".

Let us suppose therefore that the person X enters the institution I, and let O be an institutional object for I. The object O will "come to life" for X *under the constraint of the institutional relation* $R_I(O)$. In other words, a personal relation R(X,O) will be constructed, or will change, under the constraint of $R_I(O)$ — and in a wider sense, under the constraint of the institutional contract C_I.

The object O may or may not have existed for X before his entrance into I. In any case, R(X,O) (which we may suppose

equal to the empty set when it does not exist) will change. I would say that there is then *learning* (relative to O).

There is thus learning (for the person X, relative to the object O) when R(X,O) *changes*. Of course, R(X,O) may not change: it will then be said that X "has learnt nothing". It should be noted here that we are in the purely *cognitive:* there is not yet anything *didactic*. The institution I is not supposed to manifest an intention to bring about a change in the personal relation of the persons X to certain objects, that is so that X changes cognitively. It is the fact that X becomes a subject of I which can bring about cognitive changes — which may make X a "good subject" of I.

Let us clarify a little this notion of "good subject" (and its corollary, the notion of "bad subject") of an institution I. A person X becomes a good subject of I relative to the institutional object O when his personal relation R(X,O) is judged to be *consistent* with the institutional relation $R_I(O)$. This person may also prove to be a bad subject, incapable of entering into the institutional contract C_I, and may, in the end, be expelled from I. Here is where a development relating to *intra-institutional evaluation* comes into play, relating to the mechanisms according to which I is led to pronounce, through some of its agents, a verdict of conformity (or non-conformity) of R(X,O) to $R_I(O)$. I shall say no more on that point.

To make headway, we must make the above scenario a little more complex. Up to now I have implicitly presented institution I as a "homogenous" space. In reality, among the objects of I, there exists a particular category of objects which I call *positions within I,* the set of which I note as P_I. Given an institutional object O, there exists — contrary to what I have appeared to say up to now — not a single institutional relation $R_I(O)$, but, for each position p within I, an institutional relation to O *for the subjects of I in position p*. I note this relation as $R_I(p,O)$.

These notions now allow me to introduce some particular institutions, that is, *didactic* institutions. Take an institution I and let p = pu a position designated by convention as *pupil position* (within I). It can be said that *I is didactic relative to the position pu* if there is a non-empty set $S_I(e)$ included in O_I, whose elements are called *didactic stakes for subjects in position pu,* in so far as I shows the intention of rendering

R(X,O) consistent with $R_I(pu,O)$ for every X in position pu and for every O in $S_I(e)$.

It should be added here that the set $S_I(e)$, is always strictly contained in O_I and that, if p is different from pu and if O belongs to $S_I(e)$, one can have *empty* $R_I(p,O)$: the subjects of I in position p "do not have to be acquainted with" O.

Given an institution I, didactic or not, one could name *institutional education* (provided by I) the set of changes brought about in the personal relations R(X,O), where O is an institutional object of I, when X becomes a subject of I. It can be seen, once again, that one must speak of institutional education *in position p* within I. If I is a didactic institution and pu a position of a pupil within I, we will then call *institutional instruction* (for the subject in position e) the part of the institutional education relating to the objects which are didactic stakes for the subjects in position pu.

A person X is subject to numerous institutions. I shall pose here the axiom that a person is only the *emergent of a complex web of institutional subjections.* What we call the "liberty" of the person thus appears as the effect obtained by playing off *one or more institutional subjections against each other.*

In this respect, institutions are continually "swindled" by their subjects. Where they expect to find *pure subjects,* which they imagine to have been entirely fashioned by them, they encounter *persons,* who always appear to them, more or less, as *bad subjects*. In particular, the institutional relation $R_I(p,O)$ *is nobody's personal relation,* as a subject of I in position p: conformity is not identity.

Moreover, the personal relation R(X,O) presents itself to I as *split*. It includes a *public component* (relative to I), which is shown in I and on examination of which may be based the verdict of conformity of R(X,O) to $R_I(p,O)$; and, institutionally invisible, (from I), a *private component,* which escapes evaluation by I. It must be noted here that this split is *in no way absolute:* it is relative to the institution I, and that part of the personal relation which escapes one institution may appear in full light in another.

It is the denial of the public/private split which maintains in I the illusion of the pure subject. However, paradoxically, it is

this split which allows I to attract loyal subjects — who, because they are persons, will never, however, be pure subjects.

Indeed, one might think that it is through the private component of his personal relations, which escapes I, that the person, having become a subject of I, might turn out to be a bad subject of I: since it is within this person that the "alien" subjections (produced by other institutions) work on his personal relations, subversively, one might say (from the point of view of I), and lend their own "colourings". However, on the other hand, it is thanks to the existence of this private component, inaccessible to I, that the person will experience his adhesion to I *as a voluntary adhesion, the fruit of his free will and personal convictions.* This mechanism of voluntary servitude produces, moreover, in some subjects of the institution, real *institutional passions,* which are the clearest symptoms of the institution's desire for a pure subject.

4. Didactic anthropology of acquaintance

4.1. The theorisation of the didactic

The above, I feel, gives meaning to the idea of *anthropology of acquaintance* or *cognitive anthropology.* The notion of *didactic* institution however forces us to go further. It leads us to distinguish, within cognitive anthropology, a *didactic* anthropology of acquaintance, which I will also call *didactics of acquaintance* or *cognitive didactics.*

The starting point, in this respect, is the following: *any institution is to some extent a didactic institution.* Or, to put it another way: any institutional *education* includes a degree of institutional *instruction.* In what I shall later call, generically, *schools,* instruction is the essential part of education.

How can one think out, or modelise, the way in which the didactic intention present in a given institution I is carried out? Concretely, this didactic intention takes shape through the formation of institutions which I shall call, generically, *didactic systems.* A didactic system (DS) includes one or more subjects of I who come and fill a *position of teacher* T, one or more

subjects of I who fill a *position of pupil* pu, and finally an object O belonging to $S_I(e)$, which is the set of didactic stakes for I.

This is the "basic structure" of a didactic system. You may note that I have not yet mentioned milieu; but I shall come to that. I shall first give an example. Let I be a *family.* At certain moments of the family time, one can see a didactic system building up around a child taking on the position of pupil, the teacher in this didactic system being either the father, the mother, or an older sibling, for example (to simplify, I consider here the nuclear family). Such a didactic system can have a fleeting existence: it is formed then dissolves after a few minutes, for example. In contrast, *school* didactic systems — which form in the particular didactic institutions of schools — have a longer life-span: hence, in the case of French secondary teaching, they appear at the end of the summer or the beginning of the autumn, and only disappear at the end of the next spring or early summer.

In my family education — that of a boy living in a small Mediterranean port — a didactic system formed, at a certain point in the child's life, and in this particular case, between this small boy and his grandfather, around the following didactic stake (a stake for which the "motivation", as educationalists call it, or the cultural relevance, was great): to be able to attach a hook to the end of a fishing line. If you have never received this type of family instruction, you can always try it: you will see that it is not that simple! But I shall leave that example there, to take it up later, after having examined a fundamental theoretical question — that of milieu.

4.2. Conditions for the functioning of didactic systems

When a didactic system comes into existence, it has been said, it supposes at least three "terms": teacher, pupil, and one or more didactic stakes. However, several "ecological" conditions must be satisfied in order for a DS not only to exist but also to continue to exist: so that it may *function.* There is one which I shall leave aside entirely here: a didactic system *never exists alone.* Ecologically, its existence generally requires that of *other types of didactic systems,* which function *together,*

bringing together, for primary school, the same pupil and the same knowledge around other "teachers": the child with his mother, or his father, etc.; and above all, the child with *himself* — the case of *autodidactic* systems, in which the same *person* fills the position of teacher and learner, and which research in didactics has up to now neglected, investing its energy in the study only of the school didactic systems of the official teaching system.

I shall spend more time on another, much more subtle condition of the functioning of didactic systems. In order for a didactic system to function, a didactic contract must be created. However, for such a contract to be created, one needs a sure starting point. Remember at this point the role of playing — as I proposed to analyse it above — at the beginning of the schooling process. This example now appears as a particular case of a more general phenomenon which can be described thus: in order for a DS to function, at any moment — in relation to the specific time of the DS as an institution — a set of institutional objects must exist which, for the subjects of the DS, *are self-evident.* Objects O, therefore, such that the institutional relations $R_I(p,O)$ (where p = pu, S) are *locally stable.* In other words, *a milieu must exist.*

I shall now return to the example given above. If one day I wish to teach my grandson, having relearnt myself for the occasion, to attach a hook to the end of a fishing line, the objects "hook", "fishing line", "attaching a hook to the end of a fishing line" must *exist for him.* If this were not the case, I would have to manage to *bring these objects into existence for him,* so as to be able to teach him how to attach a hook onto the end of a fishing line.

You will have recognised here a classic problem in any teaching. In the daily life of the little boy I remember being, such objects existed vividly for me. In other words, they made up what was for me "a milieu". Learning could begin. The didactic system I formed with my grandfather (and with the didactic stake around which it built up), could function.

Of course, the functioning of a didactic system "shakes up" the milieu: it could be said that it is precisely for this reason that didactic systems exist! Some of the elements of the milieu will be destabilised and will momentarily cease to belong to the

milieu, before subsequently becoming stable within it, in an economically and ecologically different organisation.

However, I shall now return to my childhood memory. The object "hook attached to the end of a fishing line" is self-evident for me. Hitherto I know only of *already attached* hooks (and — but I don't know it yet — *well* attached); and also *hooks which have not yet been attached,* waiting quietly in their plastic bag. But I now learn to attach a hook. Then I attach my first hook. And now things get complicated! The hook I thought I had attached, I discover, does not stay on. Miserable failure of the learner, who discovers a reality he did not suspect, and which he thought transparent! At the end of my learning — because there is an end, that is a durable success in this noble exercise of attaching hooks — a new object comes to life for me — the object "attach a hook..." — and another object takes on a *different* life in my eyes, because my personal relation to this object has changed — that is the object "hook well attached."

At each moment, the milieu appears subjectively as given; but it is in fact a *permanent building-up.* In order to use a well-known image, one could compare this process of "mesogenesis" (genesis of milieu) with the process by which a building is built by reconstructing — or "rehabilitating" — the lower floors each time a new floor is added to it. Without a milieu — without the lower floors — a didactic system cannot function: it is impossible to add a floor when there is nothing underneath, not even any foundations. However, conversely, the functioning of a didactic system changes the milieu.

I would add here that the transformations of the milieu required by learning cannot in general be withstood indefinitely by the subjects in the position of pupil (or even by the subjects in the position of teacher). The evolution of the milieu means that subjectively, the transparent, self-evident, stabilised universe — that is the milieu — in which the subject is used to living, suddenly grows dark, becomes confusing, filled with uncertainty — before finally regaining its original subjective transparence, the criterion of completed learning.

Not everyone is indefinitely capable of withstanding such a repeated traumatism. Consequently, *he ceases to be capable of validly occupying the position of pupil in a didactic system.* His learning through instruction is therefore terminated. Later, he

will perhaps claim, with a backwards glance over his past, that he has since learnt a lot, learnt "from life" that is — and one might add: *learnt willy-nilly.* The examination of the facts on which this type of declaration is based generally shows that such supposed learning, real in the sense of the theory presented here (his personal relation to numerous objects will really have changed, new objects will have come to life for him), will hardly have operated as the effects of a shared didactic intention taken on within a didactic system.

The state of the person is therefore characterised by a structure which I call *adultism,* which can be described as the hardening of *an institutional passion for the milieu-as-it-is.* Paradoxically, the milieu, which is the *sine qua non* condition of learning, *then plays the role of an insurmountable cognitive obstacle.*

I shall now introduce a second condition vital to the functioning of didactic systems. Like the previous ones, it is a priori so transparent for us that we could be tempted to ignore it, relegating it without comment to the realm of the didactically contingent. It is a contingent which was necessarily present at the first theorisation elaborated around the phenomenon of didactic transposition. I shall formulate it thus: no didactic system can exist *in vacuo,* in an institutional vacuum. *Behind any didactic system you will find another system,* which appears generically as one of its conditions of possibility, which I call *teaching system.*

Let us return once more to the example of the hook. The institution I here is the family; and we saw what was the didactic system. Where is the teaching system postulated? One could, as a first approximation, identify it with the institution I itself, and this modelization may in many cases be enough (it is a "machine" capable enough of producing a certain type of knowledge). Let us be more precise however. Inside I appears a sort of more specialised sub-institution which will function as a teaching system, or rather as *one* teaching system, within I. Let us have a closer look.

"Where is he?" asks the mother, worrying about her little boy. Here is a first possible reply: "Where do you think he is? In school, of course!". "That's true" replies the mother, "I thought it was Thursday (formerly a day off in France)!... And

its already Friday!". The question — and the worry — give way here before the invocation of a legitimising institution of the "elsewhere" of this little boy. But here is another, equally possible answer, and equally legitimising — on certain days and certain times of day: "I think he's with his grandfather!". The sub-institution functioning as a teaching system in which the "didactic system of the hook" can come to life, is here: it is the institution "being with grandfather", quite simply. The little boy with his grandfather, that is a type of institution which has become institutionalised within the family institution, to the point of appearing to be an institution itself. This is what, concretely, will allow the type of didactic system mentioned above to come to life.

The important thing is to see that the formation of a didactic system — whatever it may be — presupposes a systemic environment, whose role is essentially to create a set of conditions necessary for the existence of the didactic system.

At this point I must introduce the notion of *situation.* Given an institution I, I call *institutional situation* any *state* of this system which is the institution. Take as I a didactic system DS(X,Y,O), in which X is the pupil, Y the teacher, and O the didactic stake. Like any institution, such a system cannot exist under certain environmental conditions. This affirmation may seem rather banal: it is, anthropologically, fundamental. Let us take an example, which generalises the one we have just left.

An adult man and a little boy are together, alone, in a room of the man's appartment. If I do not say any more — at risk of shocking virtuous souls — I have every chance of describing an illegitimate situation in the society in which it takes place. It can be seen straight away that, for this situation to appear normal — for it to match a behavioural or ethical norm — it simply has to take place inside an institution which legitimizes it. One can easily imagine, in our society, what such an institution could be. One could have, for example I = a family (the man and the child are simply *father* and *son,* and they are in the same room of the family appartment). Or, I is the institution "private classes" (the little boy is the man's *pupil,* the man is his *teacher,* and the boy has come to his house for his private class). Or else the man is a hairdresser (respectively, a dentist), and the little boy is having his hair cut (respectively,

having his teeth fixed), etc. Anyway, in our society, an adult cannot, *without an institutional justification,* be alone with a child in a closed room. I have heard tell that in the past, in our teaching establishments, a teacher could not teach fewer than three pupils at a time. If the number fell to two, the door of the classroom had to remain open; if there was only one pupil left, the lesson was cancelled. I don't know if it is true; but I shall stop before becoming improper.

In the above I have frequently called upon what I call *ecological analysis* in the approach to anthropological (and, in particular, didactic) phenomena. The assumption of the existence of a teaching system behind every didactic system proceeds from this type of analysis. However, the ecological core question has also been presented in relation to another theme: that of "good" and "bad" subjects of an institution.

I could have presented this point thus: in order for an institution to come to life, it must have good subjects. As a matter of fact, the institutions take a lot of trouble to create good subjects. This is obvious when I is a *profession.* In the modern evolution of our society, every profession tends to set up a teaching system — which I will call generically the *school* associated with the profession, *professional school* — whose aim is to create future good subjects of the institution, good "professionals". I shall come back to this later on. However, there is a case which you would perhaps not dream of linking to the one above, and which the tools of thought provided by the theorisation presented here allow us nevertheless to classify in the same category (that is *the same type of phenomenon*). Once again, I shall start from an example.

Let I be a terminal C class in one of our lycées (final year of secondary school, maths majors). Not everyone can be a good subject of this institution. In order to create suitable subjects of I, there is a whole teaching system which takes the future subjects in their tender years — let us say, at the start of primary school — and trains them by eliminating successively all those for whom the institutionally — given prognosis is pessimistic — and this happens over eleven consecutive years! It is said that it takes seven years to train a doctor. One forgets that it takes eleven years to hope to train a pupil for terminal C.

Here the teaching system is that which corresponds to all the classes from the start of primary school to the year before terminal C: it is part and parcel of the "official" teaching system.

If I insist on this point, it is to remind you that, as concerns didactic systems, as for other types of institutions no doubt, we are frequently tempted to forget this conditioning of the very existence of a didactic system by its environment. Some current ideologies of the noosphere, which would like to see a car engine run on water, lead one to think that any person should be able to feature as a "good subject" of any didactic system. (This is obvious for example concerning in-service teacher training, access to which is usually officially open to all).

I would be tempted to clarify my argument by adding this restriction: anybody, say these ideologies, should be able to feature as a good subject *in the position of pupil.* For, one will of course object, as concerns the position of *teacher, things are different,* and on the contrary there is currently great emphasis on the necessity of training (the creation of university training colleges in France (IUFM) is a precise example). In my opinion, however, this insistence is misleading. At the risk of shocking, and simply to clarify my point of view here, I would like to state this: even the most fervent advocates of the development of teacher training surreptitiously believe that *anybody, in fact, can do the job.* And personally I will only take their allegations seriously concerning teacher training when they behave in the same way in regard to the training of pupils, *that is training for the job of pupil.*

4.3. The link with the theory of didactic situations

The modelisation put forward up to now allows us to explain a number of phenomena. It only retains, concerning a didactic system, the existence of this other institution, i.e. the teaching system within which it exists, and beyond that, the "social" environment of the teaching system, i.e. society. Nevertheless, from the first formulations relating to the theory of didactic transposition, I felt it necessary to distinguish a fourth institution, the *noosphere* of the teaching system.

In fact, a schema such as this can be generalised. I propose, therefore, that to a greater or lesser extent, *every institution I has its noosphere*. I leave you to imagine what the noosphere of the institution made up by a particular family could be, or that of the institution "family", or, again, that of the institution "private classes" for example. Without noospheres, society could not function.

That being said, I will add nothing further here on this theme and I will continue to present a schema in which — for economy's sake — all mention of the fact of noosphere is excluded. However, I would like to point out the following, without going into a closer analysis. In general, the possible states of an institution, that is the types of institutional situations possible, *depend heavily on the environment of the institution*. In restricting this proposition to the case of didactic systems, I say, therefore, that the functioning of a didactic system, or what can possibly be produced within it, depends heavily on its environment, and in particular on the teaching system of which it is part.

This assertion may appear rather weak. I would like to show, through one simple example, what can be got out of it. I shall refer here to a didactic system in which we participated recently, as pupils: I mean the class given by Guy Brousseau here[8]. I imagine a person who would like to study the theory of didactic situations. In order to do this, he must find a didactic system DS(X,Y,O), in which O = the theory of didactic situations, and in which he could take the place of X. This person therefore seeks a teaching system in which such a didactic system exists. He thinks he has found it: the 1991 summer school! The didactic system sought is Guy Brousseau's class: can one hope for a better Y? The person is then accepted as a participant: at least that is what I shall suppose.

But there is a hitch: this teaching system is such that it allows the didactic system in question to exist for only *one and a half hours*. Our candidate can follow one and a half hours of class in 1991 and, with a bit of luck, another hour and a half in 1993, etc. At this rate, he will have received three hours of class in three years! I shall leave you to draw your own

8. Under the title *Fundamental concepts in the didactics of mathematics and the theory of didactic situations*.

conclusions about the efficiency of such a teaching system, and about the fate of the "pupils" and "teachers" subject to it.

I stated above that my insistance on the existence, behind any didactic system, of a teaching system was not innocent. We tend to forget — to reject, one could say — the teaching system and the determinations it imposes on the didactic systems which exist within it. Hence, following a cultural slope down which we are only too ready to rush, some will comment on Guy Brousseau's class — or any other! — as if it were a reality which can be grasped in itself and for itself, outside any institutional context, forgetting the environmental constraints which made it what it is. This forgetfulness, which already prevails among us, is not absent from several didactic analyses produced currently in the didactics of mathematics.

The above example will serve as a transition towards the theme I would now like to discuss: that of the link between the theorisation presented here and the theory of didactic situations. Concretely, this link is an operation which, up to now, has not been carried out in a satisfactory way, by either Guy Brousseau or myself. However, we can clarify some of the data of the problem.

First, it must be clear that the theorisation presented here *calls on a theory of didactic situations*, i.e. of the possible states of didactic systems, and more generally, a *theory of institutional situations*. Given an institution I, how can a certain institutional situation, i.e. a certain state of I, be located, described, characterised? The paradigm of an answer provided by systemic analysis leads one to attempt to describe a state by a set of institutional variables (didactic variables in the case of didactic systems). From this it can be imagined that one can elaborate, or at least sketch, a theory of institutional situations relating to the type of institution to which I belongs.

It can be seen, if there was any doubt, that there is no contradiction between the two theorisations which have been presented to you one after the other at this summer school. A priori, the theory of didactic situations does appear to *complete* the theorisation presented, since it occupies, in the cartography I am forcing myself to sketch, a place which was hitherto blank. On the other hand, the theory I present allows the theory of didactic situations to be situated in a wider theoretical field.

The second point I would like to highlight in relation to this link is a partial answer to a question which may have been suggested by what I have just developed above. It is possible that the two theorisations complete each other, and even that they need each other in order to make up together a more powerful tool for the production of knowledge; but can these two theorisations be linked in a coherent way? Will "grafting" one on to the other not produce rejection?

I shall answer these questions thus: what I call the style of anthropological analysis and the swarm of objects whose necessary appearance one can anticipate in a didactic analysis of anthropological style, seem to me to be present, in the modalities I shall clarify, if not in the theory of didactic situations itself, then at least in the work by which it has been produced and continues to be produced, as well as — and I insist on this point — in the way in which this theory has been put to work in the concrete analysis of concrete didactic situations.

It is for this last reason, it seems to me, that Guy Brousseau's lesson seems to be so complex, full of objects and relations, which cannot be explained simply by the "official" or explicit theory of didactic situations alone. In order to explain them, one must recognise that the system which generates the didactic analyses from which it proceeds, takes into account much more widely, although incompletely developed or theorised, the anthropological dimension of our object when fully grasped.

From this point of view, the present state of the "official" theory of didactic situations can be described, very roughly no doubt, in the following terms. This theorisation tends to favour the point of view of the *economy* and leaves aside somewhat the point of view of the *ecology* of didactic systems. Or, to put it more concretely, it tends to focus on the *functioning* of the machine, leaving to one side the study of *the conditions of possibility* of this functioning. It lists the sets of conditions under which a certain type of learning, characterised by certain personal relations which are "targets" of the didactic process, becomes possible, but it tends to leave unanalysed the general conditions which make these conditions themselves possible. Or again, in order to put forward another formula, more personalised but also just as approximative as the former: Guy

Brousseau seems to me to be "obsessed" by the conditions for the *correct* functioning of didactic systems; as far as I am concerned, I am more fascinated by the study of the conditions which make their functioning possible, be it correct or less correct.

There is no doubt here, to use an analysis dear to Gerald Holton, a difference between the *themata* of the two researchers which it may be difficult to surmount but which does not render them in anyway incompatible.

5. Didactics and didactic disciplines

5.1. The cognitive and the didactic

Hitherto I have spoken first of *cognitive anthropology*, and then of *cognitive didactics*. The second appears as a specialisation of the first. If you want to study *cognitive didactics*, you will frequently have to *move out* of your field in order to confront problems of cognitive *anthropology* in which the didactic dimension will either appear to be absent or negligible: one cannot study cognitive didactics while ignoring cognitive anthropology. However, I shall now turn my interest towards the *converse* of this last assertion: for one cannot go any further in cognitive anthropology without almost constantly being confronted with *problems of cognitive didactics*.

Indeed, one can hardly take an interest in the personal relation R(X,O), or in the institutional relation $R_I(O)$, in the *cognitive* that is, without also taking an interest in the *formation and transformations of these relations*. At this stage, it must be said, we are still in the "purely" cognitive. But an undeniable reality must be recognised in our society: *the purely cognitive does not exist, or hardly exists*. Changes in personal relations are frequently linked to *an institutional intention that these relations change;* they are institutionally, that is anthropologically, correlated with the appearance of *didactic intentions*.

It is then that "the anthropology of acquaintance" *encounters the didactic* and must, in order to explain the anthropological reality which is being studied, become *a didactician of acquaintance*. A fully developed cognitive

anthropology presupposes the development of a didactics of acquaintance. For, in our modern society, *the didactic is to a certain extent "dense" in the cognitive.*

Although anticipating the rest of my discussion, I would say, for this reason, that the historic appearance of *various* didactic disciplines (of mathematics, physics, French etc.) *was a true revolution in the anthropology of acquaintance,* and therefore *in anthroplogy in general.*

In order to clarify the position of the cognitive and of the didactic in the perspective of the anthropology of acquaintance, I shall briefly examine two objects which are fundamental in this respect: the object "learn" and the object "teach".

These objects no doubt exist in most societies: the Indo-European root from which the word *didactic* derives (through Greek) goes back to the idea of "acquiring — or helping to acquire — knowledge"; hence it testifies to the presence of our objects ("learn", "teach") in a large range of societies.

A few observations must however be added to these minimal remarks. In a given society, or in a given institution within a given society, the institutional relation to an object O can be such that it *excludes the idea that one can "learn O".* The relation that one person may have to O is regarded to some extent as a "natural" relation, which cannot strictly speaking be the result of learning.

It is for example this type of relation that, even in our societies which have a long tradition of schooling, many people still have concerning the object "mother tongue". Hence one will readily say of a small child *that he cannot speak yet*, or *that he is just beginning to speak*, not that he has not yet *learnt* to speak, or that he is *learning* to speak. The following story — which is probably fictitious — is worth telling in this respect. The director of a language teaching institute for adults one day sees a recently retired couple come in, with a low level of education, who express the wish to learn Chinese. Realising how delicate the situation could be, and believing that behind their wish lies the intention of keeping themselves busy with a "cultural" activity, the director tries to direct them towards another language, closer to French, another Romance language, for example. These people then point out to him that their intention of learning Chinese is purely utilitarian. For, they say,

"we are going to adopt a Chinese baby". And they add, as if to justify their wish: "If we can't speak Chinese, how will we understand him when he starts talking?"...

The relation we maintain — and moreover the relation we can maintain — to an object depends on our relation to the object "learning". In a given culture, one finds not only objects which are supposed to be "unlearnable" (the relation to these objects is considered to be a completely "natural" gift), but we also encounter objects which it would be considered scandalous to regard as being "learnable". Moreover, the "learnability" of a given object can vary according to the position of the subject in the institution. Once again, we encounter here the particular case of the problem of adultism: for an adult, the number of objects labelled "unlearnable" increases with the passing of time. A French child can learn English, or German, or Russian or skiing; a French adult would tend to present himself saying that he *knows* — or *does not know* — English, German or Russian; that he *knows,* or *does not know,* how to ski; etc. The time for learning, for him, is largely over. It has been replaced by the time for submission to the world as it is, and which cannot be changed: a naturalised world, in which the person will develop from then on in a supposedly unchangeable milieu.

That being noted, a movement in the opposite direction must be highlighted: in contemporary society, more and more objects enter into the field of "learning". Hence, today one can learn how to breathe, smile, walk, etc. Hence, we learn, at least since the beginning of compulsory schooling, our "mother tongue"... Of course, anything which it is not supposed to be possible to learn, cannot, *a fortiori, be taught.* However, that an object is recognised as being possible to learn does not imply that it is also recognised as being possible *to teach.* In general, in a given institution, that would be the status of most of the objects which are considered as entering into the field of institutional *education* without however being counted among the didactic stakes of institutional *instruction.* In order to function as a "good" subject, for example, the pupil must learn many things which will never be taught to him formally, and some of which will, a priori, be regarded as unteachable in the culture of the didactic institution.

5.2. Knowledge and didactics

I shall leave that question there — although it is a "sensitive" question in any institution. I now come to the introduction of the essential protagonist of our business. Up to now I have spoken about *acquaintance*. I shall now talk about *knowledge*.

You will have observed that in the theory I use words taken from everyday language. I do not, however, take them at their face value. I shall in fact introduce two objects. First of all, the object knowing in the sense of *to know something*.

I have at times noted this object ironically \$. I would like to suggest here that this object is peculiarly "sensitive". To this end, I shall simply note that the expression *relation to knowledge* (used and studied in numerous works which Jacky Beillerot has analysed brilliantly[9], and which I helped, somewhat clumsily, to spread among us as the emblem of the theorisation presented here) simply designates in my opinion, what I shall note as R(X,\$), in which X is a person. "A person's relation to knowledge", the person being X, is therefore simply *the relation of X to the object "knowing"*, that is to the object \$.

The consideration of this object appears vital in some analyses, both micro- and macrodidactic. But I shall not go into further detail here.

I shall now introduce as a primitive term the object "knowledge" (to be distinguished, therefore, from the object \$), in the sense of *a body of knowledge;* in the sense that mathematics or geometry, or the theory of geometric transformations of a plane etc, are *bodies* of knowledge.

I note a body of knowledge generically by S. Knowledge forms a particular category of objects which can rapidly be characterised thus. First, these objects can be *learnt*, and they can be *taught;* moreover, one cannot become acquainted with them without having *learnt* them. What is more, they can be *utilised*, and in order to exist, they must be *produced*[10].

9. Beillerot, 1987.

10. I distinguish here, partly arbitrarily, between utilisation and putting to use. In contrast to knowledge, the *institutional systems of acquaintance*, which are more or less the "practical knowledge" of which Bourdieu speaks, are not always supposed to be learnable, and in any case are not taught (they would indeed have to receive the status of knowledge in order to become teachable). They are *put to use*, and not utilised. They evolve, enrich themselves, etc., but are not really *produced*.

Indeed, with any knowledge S is associated an institution, which I shall note as $P(S)$, which is the institution of the *production of S*, and which in our society, tends to function as the emblem and metonymy of knowledge. I say therefore that it is in the nature of knowledge to be produced. Of course, the class of bodies of knowledge is vast: I do not designate under this name only the knowledge which, culturally, we designate as *scientific* knowledge[11].

That being noted, I now come to an essential point: to the class of bodies of knowledge there corresponds a specific sub-domain of the anthropology of acquaintance: a sub-domain which I call *the anthropology of knowledge* and which is simply what we usually call *epistemology* — but an epistemology renewed by the anthropology of knowledge. For epistemologists interest themselves, traditionally, only in the *production* of knowledge; much less in its *utilisation;* and banish from their field the question of its *teaching* and its *learning*.

As above, one can therefore trace, in the field of the anthropology of knowledge, the domain of a *didactic* anthropology of knowledge, which I shall also name, simply *didactics of knowledge*.

I am now able to formulate a fundamental epistemological thesis. I state indeed that this object which cannot be found, erratically present in our discourse and in our way of thinking the didactic, this controversial and sometimes taboo object, which we call *didactics*, whose notions of didactic contract or didactic transposition appear to us *a priori* as potential concepts (among others), *is simply the didactics of knowledge*.

In other words, I state here that "didactics" lives a fully legitimate existence; it has an object — bodies of knowledge, and the didactic phenomena they give rise to; and that didacticians of mathematics, in their effort to develop their own discipline, have continually encountered it, and on these occasions have made the greatest contributions to its constitution as a science.

11. The very notion of science does not, at this stage, enter into the theory I present: in the analysis of a great number of phenomena studied by the didactician, it appears as irrelevant — although it is extremely pertinent in the analysis and monitoring of the conditions of production of these analyses.

I can now complete the picture sketched in the preceding developments. Let S be a piece of knowledge. There exists — or can exist — a sub-domain in didactics (of knowledge) which takes as subject of study the didactic phenomena relative to S and which we shall call the *didactics of S*. We thus have *the* didactics of mathematics, *the* didactics of physics, etc. According to a general schema already seen, one cannot be a *didactician of S* without, at certain times, becoming not only an *epistemologist of S,* of course, but also an *anthropologist of knowledge* and in particular, a *didactician of knowledge,* that is simply, a *didactician.*

Conversely, just as *linguistics* cannot be constructed from the linguistics of particular languages alone (linguistics of French, English, etc.), which dialectically suppose it (without necessarily presupposing it), so *didactics* not only develops based on *particular* didactics, from which, today, it appears to emerge, and in which up to now, it found its real producers, although they did not know it, — among which, as I have already said, the didacticians of mathematics have played a vital role.

5.3. Return to didactic transposition

At this point I would like to show how, in the framework developed up to now, the theory of didactic transposition can be situated and generalised. Let I be an institution. You might think first of these particular institutions which are *professions* — that of doctor, or car mechanic, etc. The historical evolution of the societies in which we live today has led to the elaboration of a very general schema of social organisation, which I shall consider here.

In order to become a good subject of I, a person X must learn certain knowledge S, in particular because some of this knowledge exists in I, and in order to become a good subject of I in the position p which he will occupy, X must have a relation R(X,S), corresponding to a certain relation R_I(p,S).

Do not forget at this point that a body of knowledge S, from the point of view of its social genesis, exists first in the institution P(S), which is its original habitat: that at least is the hypothesis which I shall examine on its own here. Its presence

in I supposes therefore that there is, or has been, "transport" of P(S) to I — or what I call *institutional transposition of P(S) into I.*

The majority schema today is the following. Throughout the history of the institution I, one sees a system of teaching emerge associated with I, which I call generically *the school associated with I* and which I note Sc_I[12].

The institutional transposition of P(S) to I, linked initially to somewhat erratic, asystematic mediations, therefore tends to become normal in the following manner: its main flows now take the channel of a process of didactic transposition, which goes from P(S) to Sc_I — and not directly from P(S) to I.

The existence and the "quality" of this transposition are now in the field of *the noosphere of* Sc_I, in which one finds, in particular, the actors of I playing at part-time noospherians[13], and whose specific work in this respect, which I will call generically *transpositive work,* hence appears *essential in the functioning and the survival of institutions.*

When this schema is implanted in the social organisation, the future subjects of I will not encounter S when they enter into I, but before that, in the vestibule constituted by Sc_I. One cannot over-emphasise at this point the extent to which the didacticians' object of study appears to be linked to social organisation and its evolution, which tends to *increase the social mass of the didactic,* and notably *school* didactic — which appears within schools (in the generic sense given this word here).

From this schema, one can develop a generalised theory of didactic-institutional transpostition: a fascinating theme for a researcher to study, and crucial for society, but which I will consider in the following, and to conclude, only in a more particular form with which we are more familiar and to which we are no doubt collectively much more attached.

The schema presented above is, a priori, valid for any institution I. You could apply it yourselves, given what I said above, in the case in which I is a terminal C class: Sc_I is therefore the system of classes going from CP (or from nursery

12. As has already been noted, when I is a profession, one speaks of *professional school.*

13. Ecologically, noospheres are *ecotones.*

school) up to première S (penultimate year of secondary school). However, here I will take as an institution I = *S*, designating by *S* the particular institution which is society.

What, therefore, is Sc_S? It is the school society provides itself with in order to "create" good subjects; empirically, *it is today's compulsory school.* Let us note in passing — it could also have been seen in the preceding example, at another level of social organisation — that another essential problem arises for society, on which recent developments in the theory of didactic transposition can throw some light: that of the *identification and choice* of the bodies of knowledge S which society would like to see taught in its school.

This is a particularly sensitive problem because "society's school" is considerably more *exposed* to the public eye than, for example, the school of a given profession. However, from the point of view of the theory presented here, this problem is one particular case, among others, of a general problem: every institution is confronted with the problem of the "choice" of the knowledge it considers as relevant to the training of its subjects.

In compulsory teaching, as in fact, *in any institution,* the answers provided to the problem of the choice of knowledge to be taught are necessarily polemical, historically ephemeral, and take shape as a "menu" of knowledge taught which evolves by its very nature.

The outline of a study of this problem would suppose at this point the introduction of a new primitive term, that of *social practice,* which would allow us to go more deeply into the very notion of knowledge, and which, in fact, leads to a remarkable extension of the validity of the theory of didactic transposition. But this is where I shall end.

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