THE LIGHT GAME: AN ACTIVITY OF LINEAR SYSTEMS

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Short description of the workshop: aims and underlying ideas

This booklet contains four activities in which it is proposed that students imagine two lamps (for the first activity) or three lamps (for the third and fourth activities), each one containing one switch. Concepts of congruence modulo 2 are developed on the first activity in this booklet as follows: as the switch under the first lamp is activated, the state (luminosity) of both lamps is altered. However, as the switch under the second lamp is activated, only the state of the second lamp is altered. On the second activity, congruence modulo 2 is developed more directly, when the student is asked to define, by means of the activities, modular addition and subtraction. On the third activity, where congruence modulo 2 is also developed, one more lamp is added, amounting to three lamps to consider. When the first switch is activated, it alters the state of the state of the first and second lamps. When the second switch is activated, it alters the state of the second and third lamps. When the third switch is activated, it alters the state of the first and third lamps. On the fourth activity, when congruence modulo 3 is introduced, lamps can assume three different colors. In this activity, the first switch alters the color of the three lamps; the second switch alters the first and second lamps' color; and the third switch alters the color of the second and third lamps.

Planned structure:

Insert the planned structure of the Workshop here after leaving ONE empty line below the abstract. Please use this style for the timetable and insert necessary rows. Due to technical reasons the timetable shall not exceed 10 rows.

Planned timeline	Topic	Material / Working format / presenter
10 minutes	*Activity 1*	Activity sheet and electrical circuits
15 minutes	*Activity 2*	Activity sheet and electrical circuits
25 minutes	*Activity 3*	Activity sheet and electrical circuits
35 minutes	*Activity 4*	Activity sheet and electrical circuits

Activity 1

Consider two lamps which can either be turned on or off. The lamps are placed side by side so that under each lamp is a switch that changes the state of the lamps according to the following scheme: Switch X, under lamp A, changes the state of both lamps - what was on is turned off and what was off is turned on. Switch Y, under lamp B, only changes the state of lamp B. Considering the two lamps initially are turned off, the following table portrays the changes in state of the lamps when the Switches are activated.

	Switch X	Switch Y
Lamp A	1	0
Lamp B	1	1

Table 1

Answer the following questions, with final answers as 0 if the lamp is turned off and 1 if the lamp is turned on.

1) If Switch X is activated once and then Switch Y is activated once, what is the final state of each lamp if they both were initially turned off?

2) If Lamp A is off and lamp B is on, and Switch X is activated twice, and Switch Y, once, what is the final state of each lamp?

3) If both lamps are initially on, and Switch X is activated once, and Switch Y once, what is the final state of each lamp?

4) Complete the following table. It shows the number of times each Switch was activated, as well as the final state of each lamp. For each line, consider the initial situation with both lamps turned off. Follow the two examples:

Switch X	Switch Y	Lamp A	Lamp B
0	1	0	1
1	0	1	1
1	2		
1	3		
3	2		
5	1		
2	1		
2	2		
2	0		
3	1		

Table 2

5) Regarding the table above, what can you conclude about what happens when you press a single switch 2 times in a row?

6) What is the minimum amount of times you have to activate each switch in order for the two lamps that were turned off to reach the state in which lamp A is on and lamp B is off?

Activity 2

On the previous activity, the state of the lamps was either 0 or 1. We can establish addition and subtraction operations in the set {0,1}, that represents the set of possible rests after a division by 2, so that the result of these operations still belong to this set.

1) Complete the following table:

Addition	Result in {0,1}
1 + 1	
1 + 2	
2 + 0	
3 + 1	
2 + 3	
4 + 6	
11 + 6	
10 + 7	
7 + 6	
12 + 9	

Table 3

2) What can we conclude from the results of the previous table?

3) The same relation stated on the previous question, valid for addition in the set {0,1}, is also valid for subtraction in {0,1}. Follow the examples and complete the table for subtraction in {0,1} that follows:

Subtraction	Result in {0,1}
2 – 0	
3 – 2	
2 – 3	
4 – 2	
2 – 4	
6 – 3	

3 – 6	
1 – 4	
4 – 1	
11 – 16	

Table 4

4) Using the previous activities, build a definition for addition and subtraction in the set {0,1}.

Activity 3

Consider now three lamps, that can either be on or off. There is one Switch under each lamp, so that each Switch changes the state of the lamps as follows: Switch X, under lamp A, changes the state of lamps A and B; Switch Y, under lamp B, changes the state of lamps B and C; and Switch Z, under lamp C, changes the state of lamps A and C. Considering all lamps were initially turned off, the following table show the changes in state of the lamps when one of the Switches is activated.

	Switch X	Switch Y	Switch Z
Lamp A	1	0	1
Lamp B	1	1	0
Lamp C	0	1	1

Table 5

Answer the following questions, with final answers as 0 if the lamp is turned off and 1 if the lamp is turned on.

1) If Switch X is activated once, then Switch Y is activated once, and finally Switch Z is activated once, what is the final state of each lamp if they both were initially turned off?

2) If Switch X is activated twice and then Switch Y is activated once, and next, Switch Z is activated twice, what is the final state of each lamp if they lamp A was initially off and lamp B and C were initially on?

3) Now complete the following table. It shows the number of times each Switch was activated, as well as the final state of each lamp. For each line, consider the initial situation with all lamps turned off. Follow the examples:

Switch X	Switch Y	Switch Z	Lamp A	Lamp B	Lamp C
0	1	0	0	1	1
1	1	1	0	0	0
1	1	0			
0	1	1			
1	1	3			

5	1	0			
2	1	2			
1	2	1			
2	0	1			
3	1	2			

Table 6

4) It is possible to determine the minimum number of times you should activate each switch in order to end up at a given final state for each lamp? To answer, create three equations, one for each lamp, matching the equations to the final state of each lamp and considering X, Y, and Z the number of times Switch X, Switch Y and Switch Z were activated, respectively.

5) The three equations from the previous question constitute a linear system of equations, which's solution enables us to determine the minimum number of times each Switch had to be activated for a given final state of each lamp. This way, how many times should each switch be activatedso that Lamp A and Lamp C end up turned on and Lamp B ends up turned off? Consider all lamps are initially turned off.

6) What is the minimum number of times each Switch must be activated so that Lamp A ends up turned off, and Lamp B and Lamp C end up turned on? Consider all lamps are initially turned off.

7) Is there a solution, that is, is there a minimum number of times that each Switch must be activated so that all lamps end up turned on? And so that Lamp A ends up turned off, lamp B ends up turned on and Lamp C ends up turned off?

Activity 4

Consider now three lamps that can emit white, blue or red light. Under each lamp is a Switch that changes the color of the light emitted by the lamps as follows: Switch X, under lamp A, changes the color of all three lamps; Switch Y, under lamp B, changes the color of lamps A and B; and Switch Z, under lamp C, changes the color of lamps B and C. Use 0 to represent white, 1 to represent blue and 2 to represent red. Consider all changes in color are made following the order: white, blue and red.

1) Activating one time Switch X, one time Switch Y and one time Switch Z, what will be the color of each lamp, if A and C were initially white, and B was initially red?

2) What will be the final color of the light of each lamp it Switch X is activated 3 times, Switch Y, 4 times and Switch Z, once? Consider the initial colors as red for lamp A, white for lamp B and blue for lamp C.

3) What happens when you activate the same Switch three times in a row? And what happens when you activate it four times in a row?

4) Complete the following table. On the first two lines, there is the amount of times Switch X, Switch Y and Switch Z were activated and also the final state of each lamp. Consider that all lamps were initially white in all cases.

Switch X	Switch Y	Switch Z	Lamp A	Lamp B	Lamp C
1	1	1	2	0	2
2	2	2	1	0	1
2	1	2			
3	1	1			
2	1	3			
2	3	2			
3	3	3			
5	3	4			
2	3	0			
5	4	7			

Table 7

5) What is the minimum number of times you should activate each switch so that lamp A is blue and lamps B and C are white? Consider lamp A was initially red, lamp B was initially white and lamp C was initially blue.

6) Using the same idea of Activity 2, complete the following table concerning subtraction in {0,1,2}.

Subtraction	Result in {0,1,2}
7 – 3	
8 – 5	
10 – 5	
3 – 0	
4 – 3	
5 – 3	
5 – 7	
3 – 4	
3 – 5	
2 – 5	

Table 8

7) Is it possible to create one equation for each lamp using the variables X, Y, and Z, matching these equations to the desired final color for each lamp? If possible, what equations are these?

8) Using the preceding equations, what is the minimum amount of times each switch should be activated to obtain the following colors for the lamps: red for lamp A, blue for lamp B and white for lamp C? Consider all lamps were initially white.

9) What is the minimum number of times you should activate each switch each switch should be activated so that lamp A ends up blue, lamp B red and lamp C white? Consider all lamps were initially white.

10) What is the minimum number of times each switch should be activated so that lamp A ends up blue, lamp B red and lamp C blue? Consider all lamps were initially white.

11) What is the minimum number of times each switch should be activated so that lamp A ends up blue, lamp B blue and lamp C red? Consider all lamps were initially white.

12) Is there a solution, that is, is there a minimum number of times each switch should be activated so that the lamps have the following colors: lamp A white, lamp B blue, lamp C red, considering they were all initially white?

References

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