Workshop: Designing and evaluating mathematical learning by a framework of activities from history of mathematics

1. About the organizers

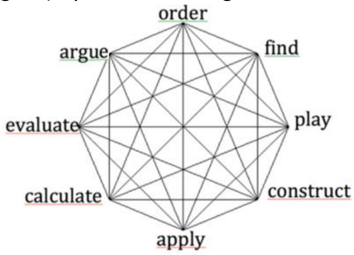
Prof. Lenni Haapasalo: PhD in Mathematics, additional qualification in education, Professor of Education including mathematics at the University of Eastern Finland, more than 35 years of experience in teacher training, specialized on modern learning theories, esp. procedural and conceptual learning, mathematical problem solving and use of IT in mathematics education, some 10 books published or coauthored, some 200 additional publications on these issues.

Prof. Bernd Zimmermann: Master (Diploma) in mathematics, PhD on mathematical problem-solving, habilitation on mathematical beliefs and history of mathematical problem solving and heuristics, retired professor of mathematics and computer science education at the University of Jena, some 10 years of experience as a teacher at German Gymnasium and in a project for mathematically talented students (Hamburg Model, guided by Prof. Kießwetter), some forty years of experience in training of teachers of all grades, specialized on mathematical problem solving and its history, creativity in elementary mathematics, mathematical beliefs, mathematically gifted students, edited 6 books and seven textbooks books, published some 130 papers on his research interests.

2. Theme and aims

In this workshop our aim is to offer a new framework from history of mathematics for designing and evaluating of mathematical learning environments.

On the basis of comprehensive studies of the history of mathematics (encompassing some 1000 text-resources, cf. Zimmermann 1991, 2003, esp. Haapasalo/Zimmermann 2015, pp 199) the following eight main sustainable motives and activities were revealed, which proved to lead frequently to new mathematical results at different times and in different cultures for more than 5000 years. These different activities are connected and interrelated in many ways, which are represented in the following figure (“Zoctagon”) by the connecting lines.



Z-activities which proved to be successful in producing new mathematics
(Zimmermann 2003, p. 42).

To get at least some impression about the meaning of the words in the figure above, we give a short summary:

At the very beginning of nearly all large cultures the documentation of quantities and their manipulation was of major interest (from the “Ishango-bone” up to scientific computing of today). This leads to the first important mathematical activity: *calculate*. Problems, e. g., from astronomy and agriculture, are until our days - from every-day problems up to space-industry and ecology - very important domains to *apply* mathematics and to develop new mathematical models, respectively (cf. the dominating “philosophy” of PISA). *Construct* is the most important activity, not only in (classical) geometry but also in architecture – which was taken as a part of mathematics for a long time.

These three activities are to some extent the oldest and most fundamental ones and therefore they form the basis of the figure.

Methods to *invent* (or *find*) something like heuristics (e.g., working backwards, analogizing, successive approximation, change of representation etc.) were applied at least implicitly since the ancient times of, e. g., the Babylonians and Chinese. To *argue*, esp. proving is at the core of modern mathematics and belongs to the more challenging mathematical activities. The tension to bring new knowledge, a set of new theorems or clusters of solved problems in a systematic *order,* including approaches to axiomatization (Euclid), led very often to a deeper understanding and to more insight into theoretical interrelations.

These activities are more challenging and sophisticated than those at the bottom of the figure. Therefore, we put them on the top.

Finally, there are two activities, which seem to be underestimated to some extent, but which proved to be major stimulators for new mathematics and mathematical insights over and over again, too.

Striving for religious cognition as well as for esthetical (geometric or proof-) configurations and related systems of values generated also new problems and their solutions and produced in this way also new mathematical knowledge during history of mathematics (e.g., Vedic geometry, combinatory, tessellation and ornaments in Islamic Mathematics). So the underlying activity of *evaluate* proved to be of importance, too.

The same holds for an approach to mathematics by *play*ing and the development of recreational mathematics. In this way very oft new branches of mathematics were created like stochastic and game-theory.

We skip a more detailed discussion on the development of the Z-activities because it can be found in Zimmermann 1999, 2003. Some first results coming from this approach were already presented at a MAVI-meeting in Duisburg (Zimmermann 1998).

LH will describe his idea to use this octagon as an instrument to measure how the eight activities are supported within school mathematics, university mathematics, and the usage of ICT in everyday life, respectively. The results suggest that the support gained from all those areas is modest, and amazingly the support gained from the usage of ICT seems to have even a descending trend. LH will represent the design of ICT-based learning environments (actually investigation spaces) orchestrated within the so-called pit-stop philosophy, promoting a promising support for the Z-activities. This would mean a thorough shift in curriculum design, including dynamic assessment.

There will be a third invited lecture by Professor Dr. Harry Silverberg from the University of Turku (he is an expert not only for questions of assessment in mathematics but also in science education, see e. g. http://www.utu.fi/en/units/edu/research/phd-studies/supervisors/harry-silfverberg/Pages/home.aspx). The title of his lecture will be: "Possibilities of using the "Zoctagon"-construct as a frame of evaluating the curricula".

3. Key questions.

I) How are these eight activities related to the fundamental ideas, established by Bruner 1979/77 and the six activities mentioned by Bishop 1988? Similarities, differences, consequences? All colleagues, who are interested in this workshop, should have read the corresponding parts of these books.

II) To what extent the octagon can be used as a framework for designing and evaluating mathematical learning environments, including modern ICT?

4. Structure.

The structure will be as follows:

- Three consecutive presentations giving some impulse for the following working groups (3x15 min + 5 min discussion each).
- Parallel working groups corresponding to the two the questions
    1. Possible improvement of the instrument?
    2. Possible testing and use of the improved instruments in different countries, possible common projects?

  Formation of the groups by random, if appropriate (20 min).
- Presentation of the results of the group-work corresponding to the questions in b) (10 min).

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If you want to join the workshop, please send name + e-mail-address until 7/15 to lenni.haapasalo@uef.fi + pertti1@gmx.de