Thematic Afternoon 27/Jul/2016

European Didactic Traditions
– the case of the Netherlands

ICME13
Hamburg 2016

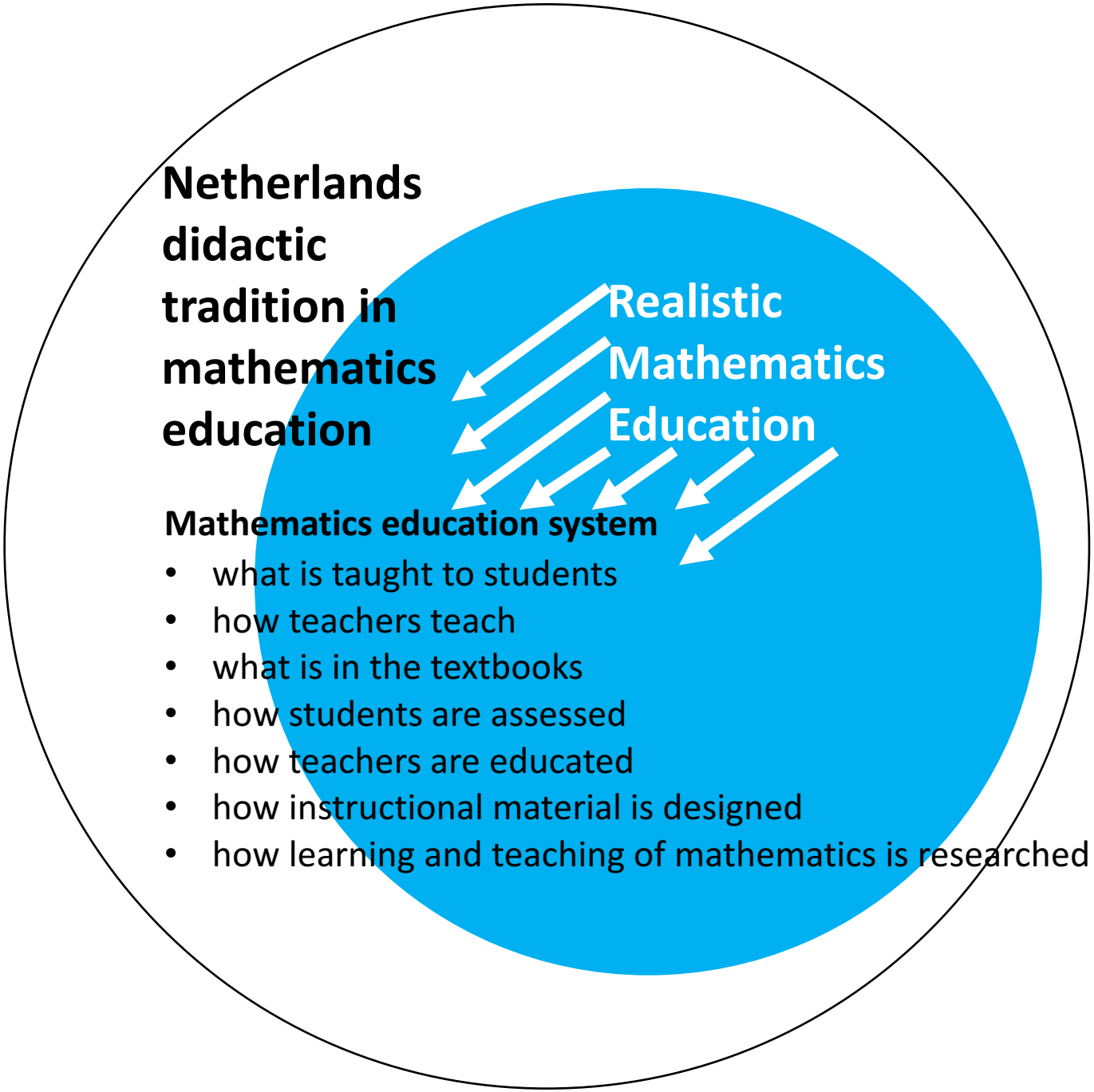
RME within the Dutch tradition

Marja van den Heuvel-Panhuizen

Freudenthal Group, Faculty of Social and Behavioural Sciences

Universiteit Utrecht

Freudenthal Institute, Faculty of Science

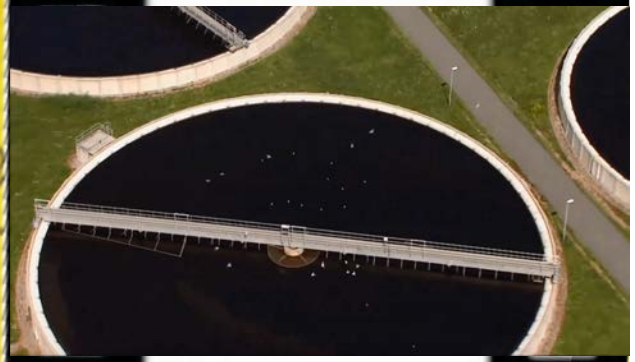

Netherlands didactic tradition in mathematics education
Realistic Mathematics Education
Mathematics education system
• what is taught to students
• how teachers teach
• what is in the textbooks
• how students are assessed
• how teachers are educated
• how instructional material is designed
• how learning and teaching of mathematics is researched

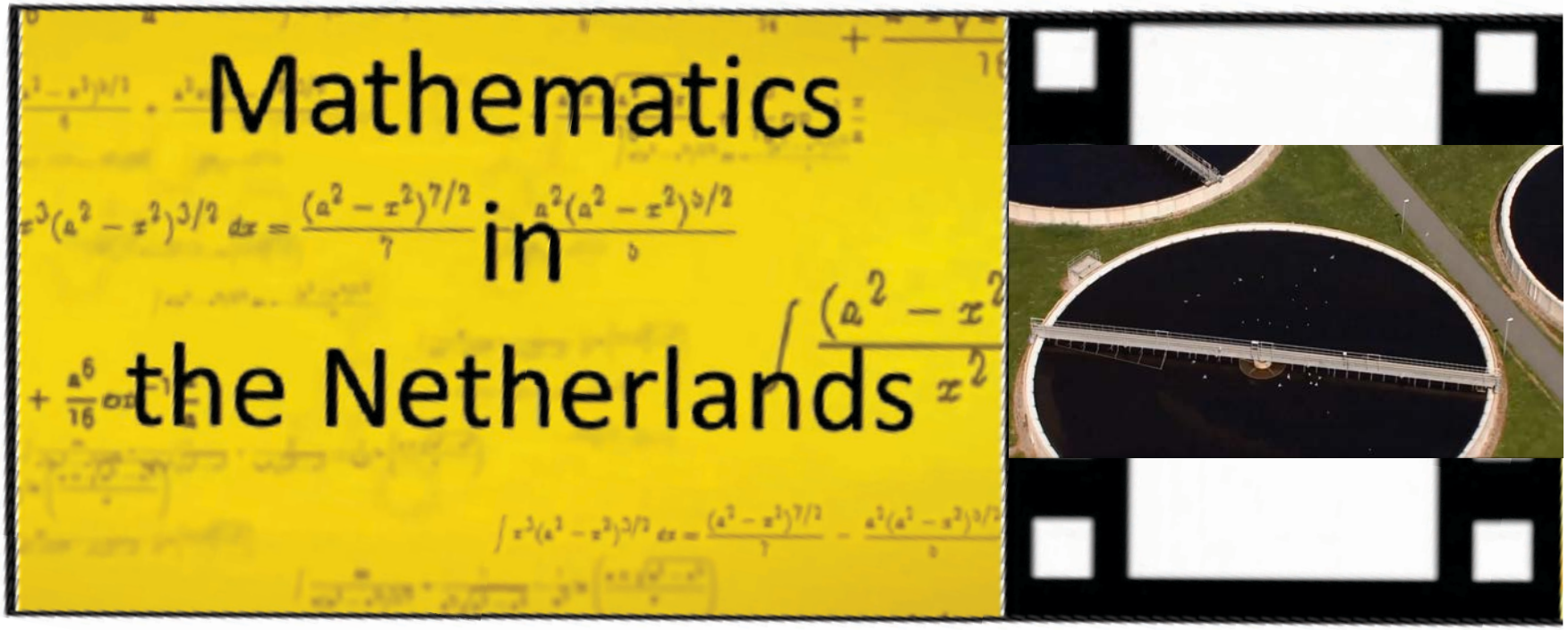
NL is not a
"one state – one didactics" country

Furthermore

- RME is not a fixed and unified theory
  - different accentuations and focus on different groups
  - different conceptualizations of RME
- Large variety in how RME is implemented in textbooks and classrooms

Mathematics in the Netherlands





Reality

- as a ***source context*** for developing mathematical understanding
- as a ***target context*** where mathematical understanding can be applied

Central role of reality is characteristic for RME

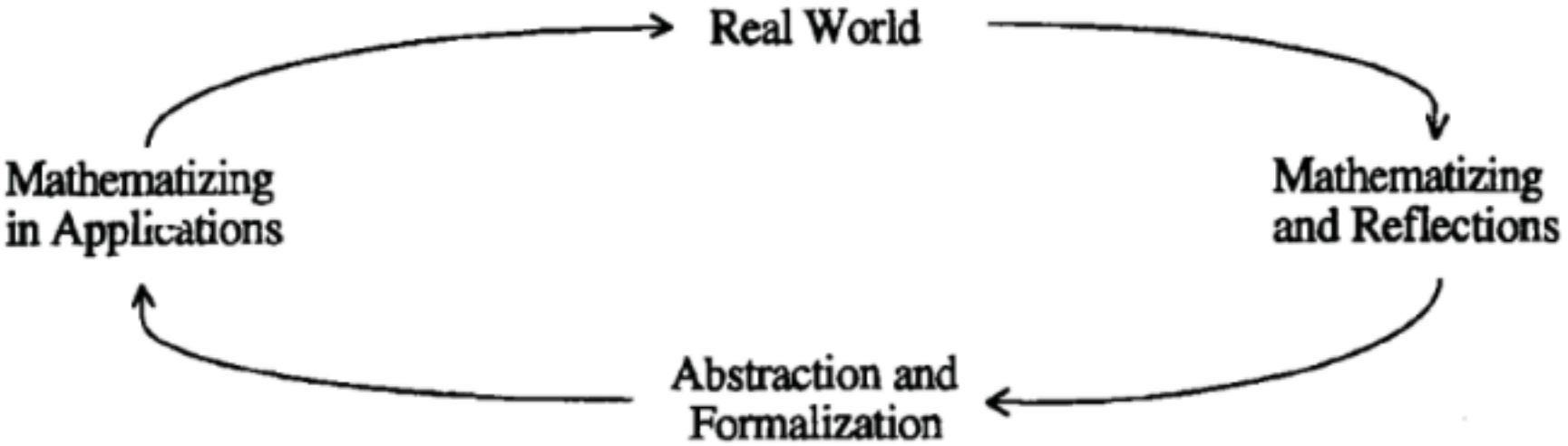
but REALISTIC [‘zich realiseren’] also means that students are presented problem situations that they can “imagine” and “experience as real”

Realistic

- *extra-mathematical contexts*
- *intra-mathematical contexts*

extra-mathematical contexts

"conceptual mathematization" (1987)

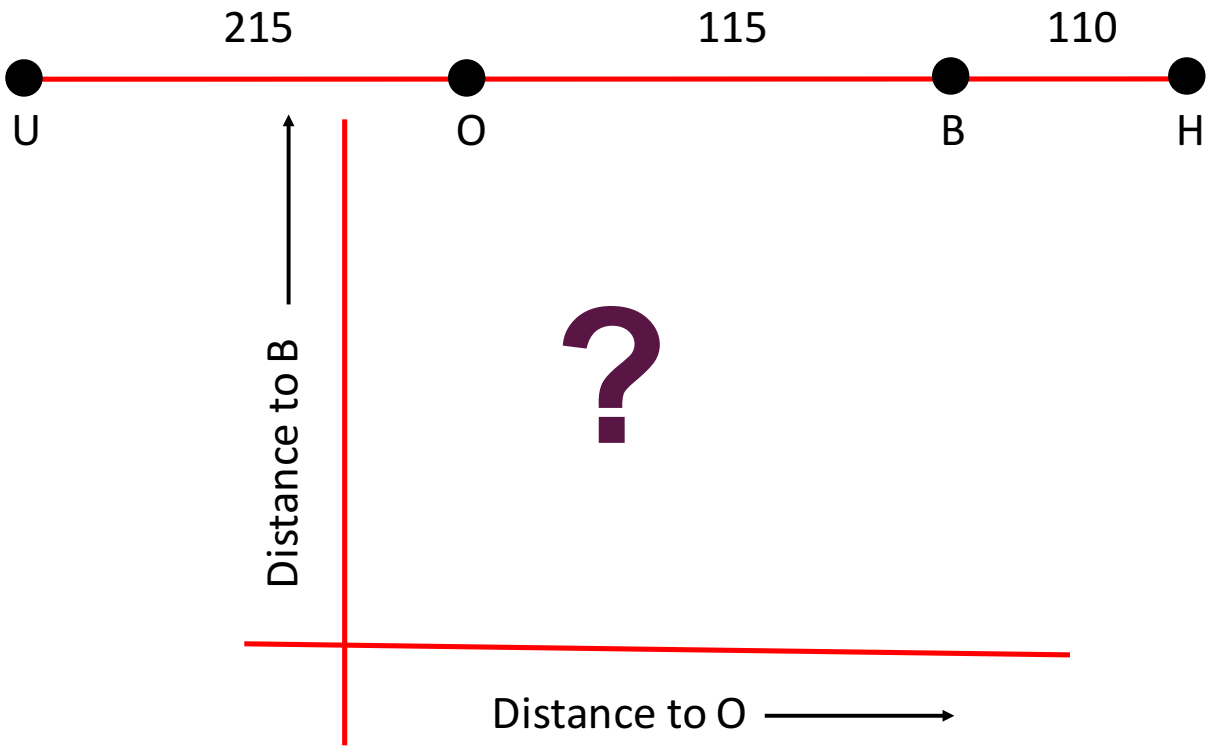


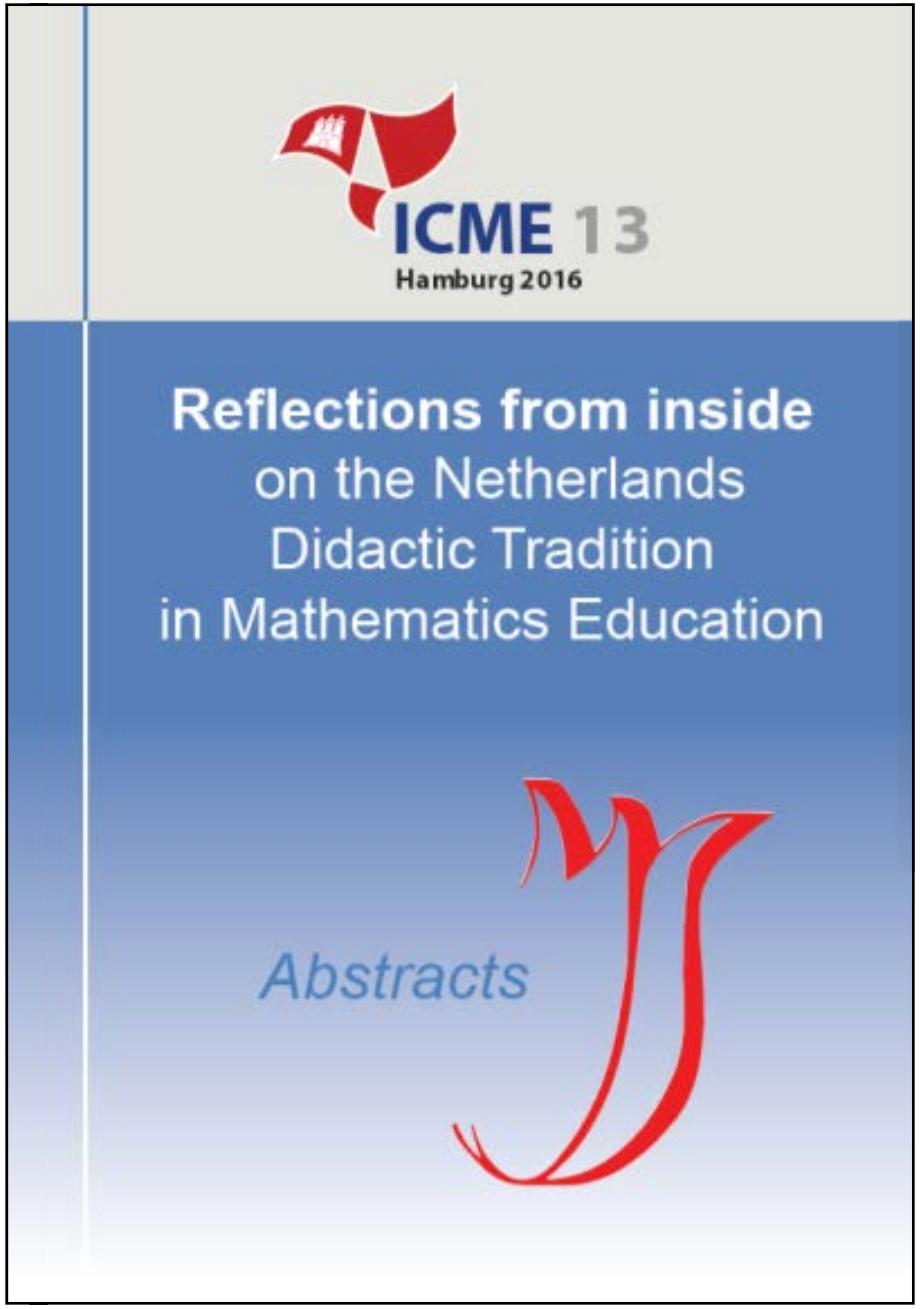
intra-mathematical contexts

- relevance of own productions
- presenting problems that have not been taught yet
  context-based solution ➔ progressive mathematization
- challenging students with (classical) mathematical puzzles (*extra-mathematical contexts*)

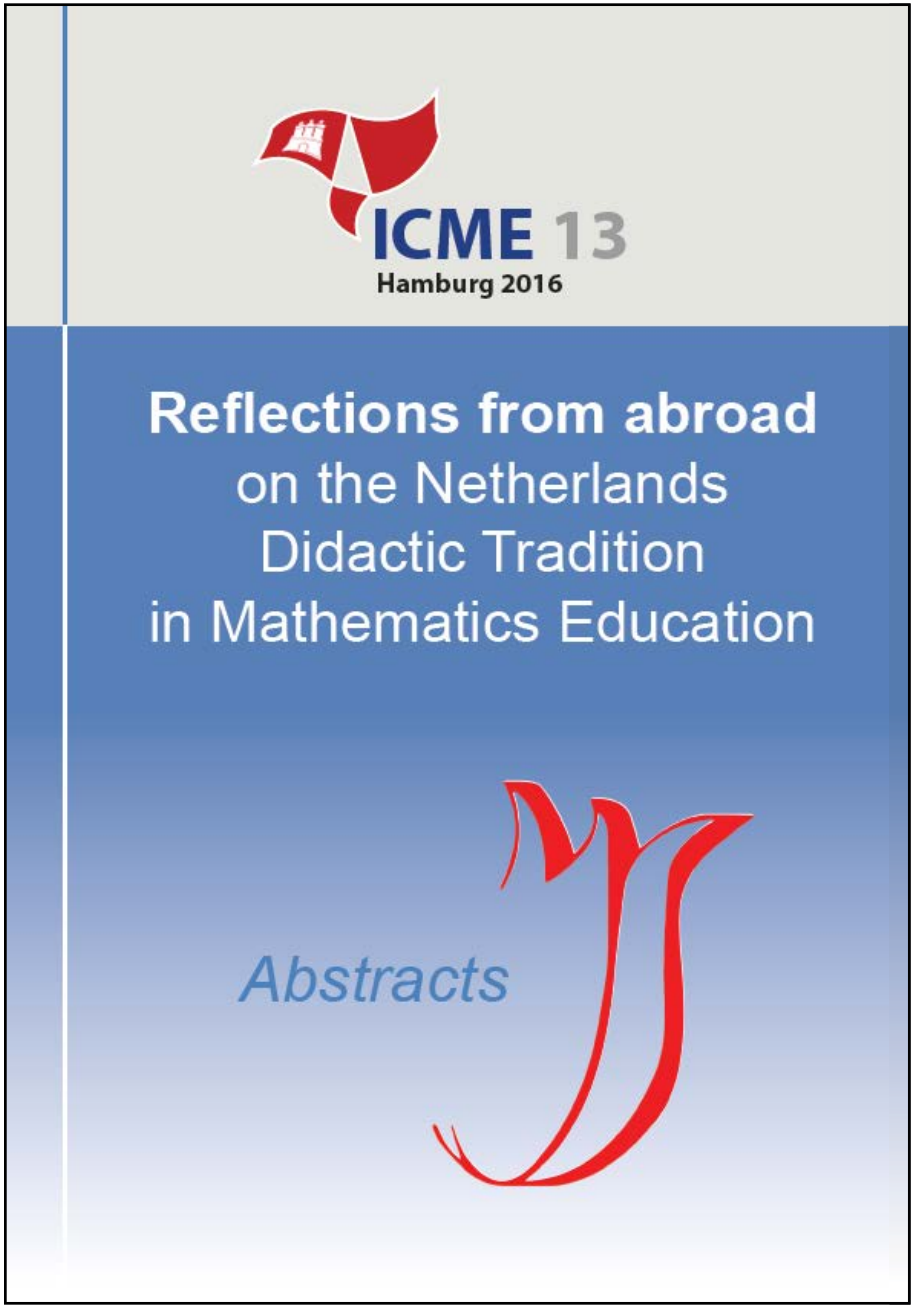
- making use of children's curiousity and triggering students' mathematical thinking

working on a mathematical problem yourself





Monica Wijers, Dede de Haan, Pauline Vos, Rijkje Dekker, Harm Jan Smid, Adri Treffers, Marja van den Heuvel-Panhuizen, Marjolein Kool, Mar van Zanten, Koeno Gravemeijer, Wil Oonk, Ronald Keijzer, Joke Daemen, Ton Konings, Theo van den Bogaart, Paul Drijvers, Martin Kindt, Kees Hoogland, Iris van Gulik-Gulikers, Jenneke Krüger, Jan van Maanen, Michiel Doorman, Aad Goddijn, Ed de Moor, Wim Groen, Floor Scheltens, Judith Hollenberg, Ger Limpens, Ruud Stolwijk, Jan de Lange



David Webb, Frederick Peck, Dirk De Bock, Wim Van Dooren, Lieven Verschaffel, Erich Wittmann, Cyril Julie, Faaiz Gierdien, Abraham Arcavi, Berinderjeet Kaur, Wong Lai Fong, Simmi Naresh Govindani, Petra Scherer, Betina Zolkower, Ana María Bressan, Silvia Pérez, María Fernanda Gallego, Xiaotian Sun, Wei He, Dirk De Bock, Johan Deprez, Dirk Janssens, João Pedro da Ponte, Joana Brocardo, Christoph Selter, Daniel Walter, Dor Abrahamson, Elisa Stone, Kyeong-Hwa Lee, YeongOk Chong, GwiSoo Na, JinHyeong Park, Omar Hernández-Rodríguez, Jorge López-Fernández, Ana Helvia Quintero-Rivera, Aileen Velázquez-Estrella, Mogens Niss, Zulkardi, Ratu Ilma Indra Putri, Ariyadi Wijaya, Usha Menon, Paul Dickinson, Frank Eade, Steve Gough, Sue Hough, Yvette Solomon

Dutch strand of the “European Didactic Traditions”(16:30-17:30)
2nd hour

- **Experiences with RME in ...**

David Webb
USA


Zulkardi &
Ratu Ilma Indra Putri
Indonesia

Sue Hough
England &
Cayman Islands

- **Critical friends**

Dirk De Bock
Belgium

Cyril Julie
South Africa

13th International Congress on Mathematical Education
July 24 – 31, 2016 in Hamburg / Germany

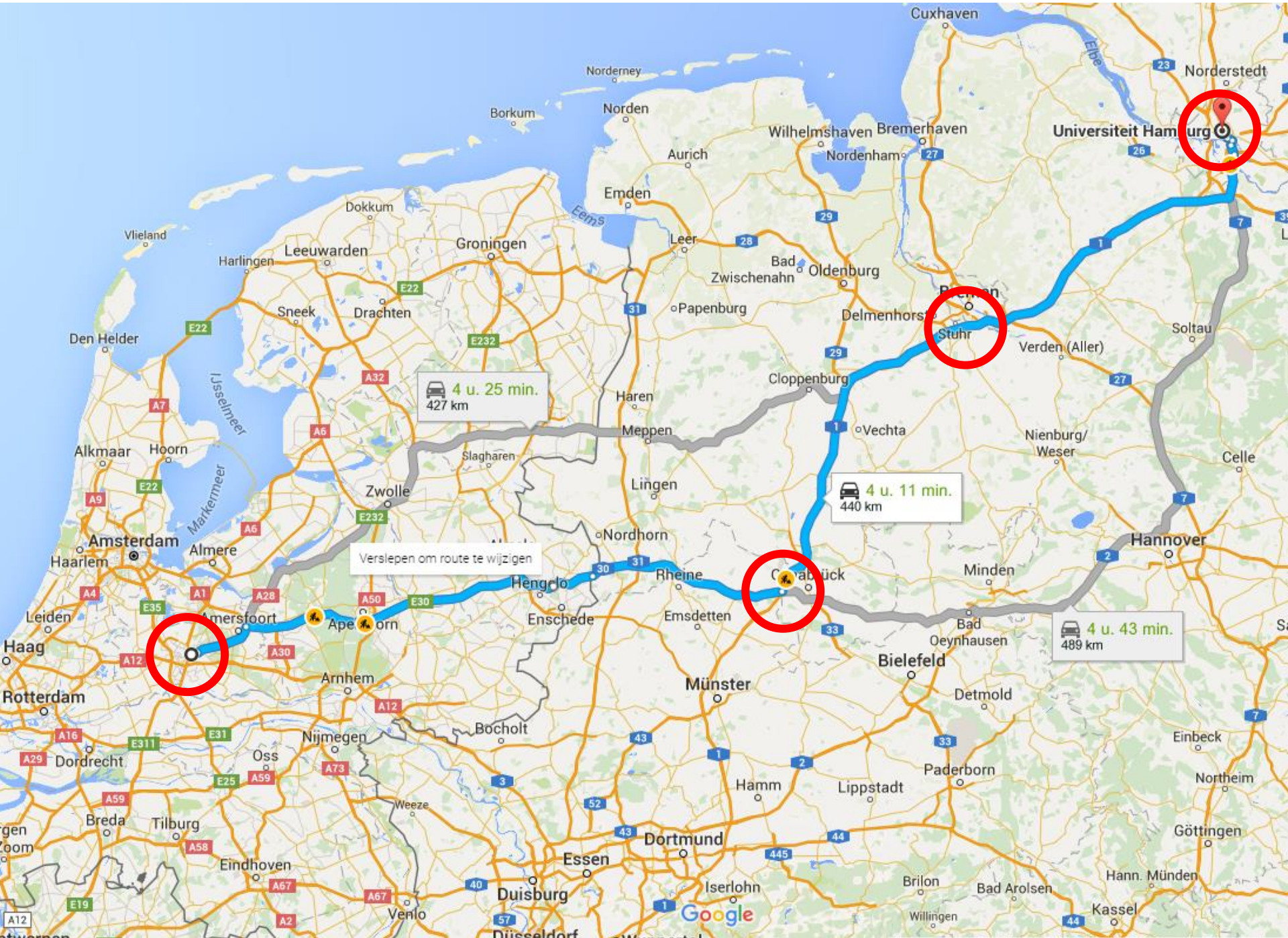
Driving to Hamburg

Thematic Afternoon European Didactic Traditions: the Netherlands

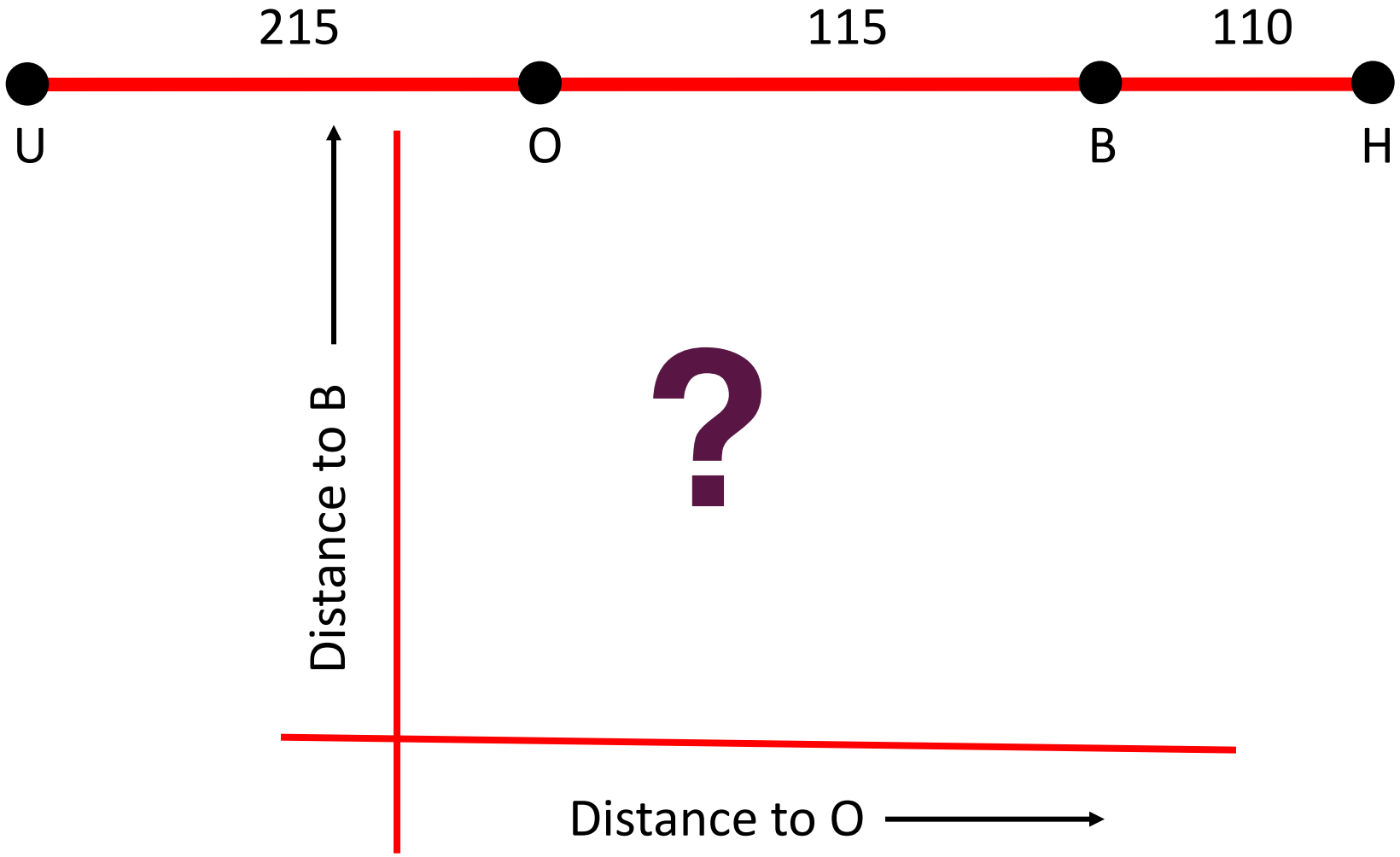
Paul Drijvers, Freudenthal Institute
p.drijvers@uu.nl

Universiteit Utrecht

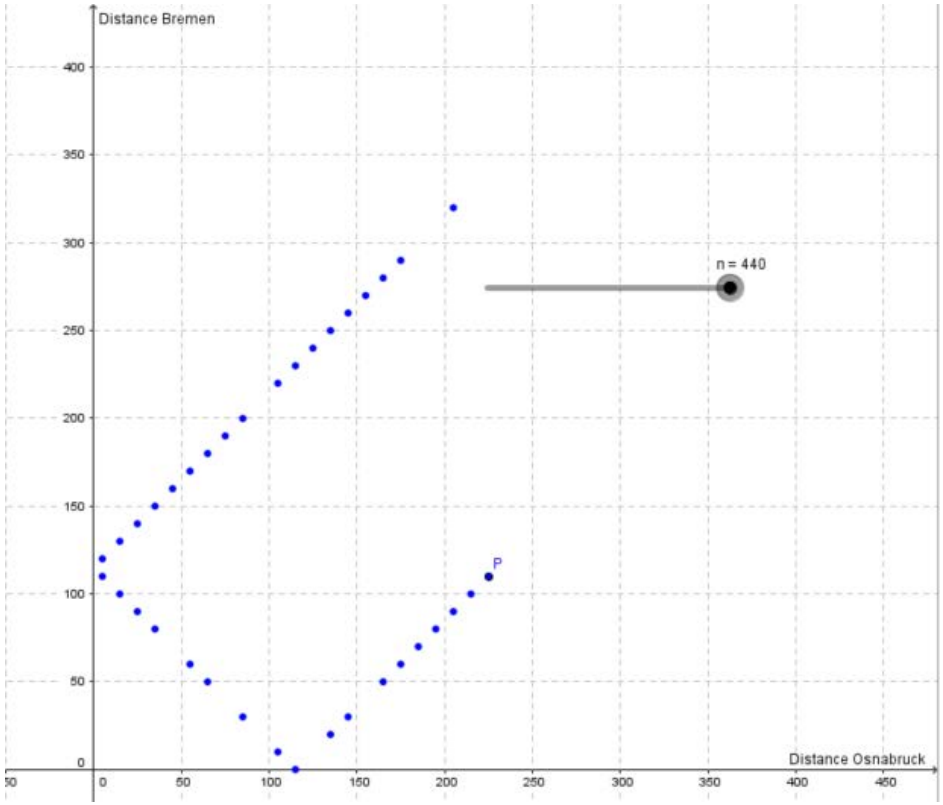
[Faculteit Bètawetenschappen
FISME Freudenthal Institute for Science and Mathematics Education]


4 u. 25 min.
427 km
4 u. 11 min.
440 km
4 u. 43 min.
489 km
Verslepen om route te wijzigen
Universiteit Hamburg
Amsterdam
Rotterdam
Haag
Arnhem
Nijmegen
Münster
Dortmund
Essen
Duisburg
Bielefeld
Hannover
Bremerhaven
Oldenburg
Groningen
Leeuwarden
Enschede
Osnabrück
Google

Driving to Hamburg: schematizing



Driving to Hamburg: animation



Driving to Hamburg: model

Problem orientation:

- Starting point in U: $(O, B) = (215, 330)$
- End point in H: $(O, B) = (225, 110)$

Model:

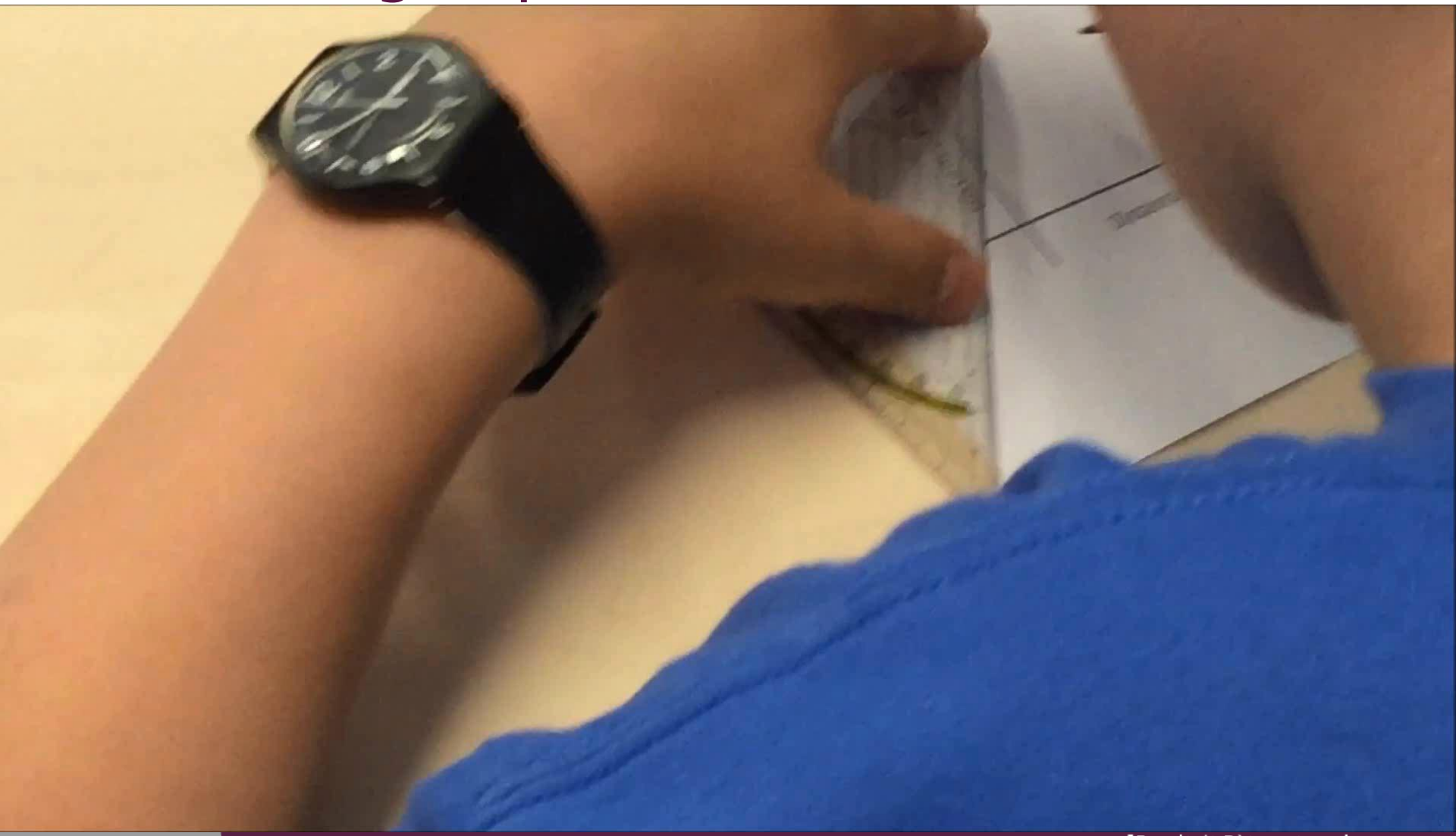
- u = distance to Utrecht (independent variable)
- $O(u)$ = distance to Osnabruck = $|u - 215|$ (dependent)
- $B(u)$ = distance to Bremen = $|u - 330|$ (dependent)
- $P(u) = (O(u), B(u))$

-> *So we have a parametric curve!*

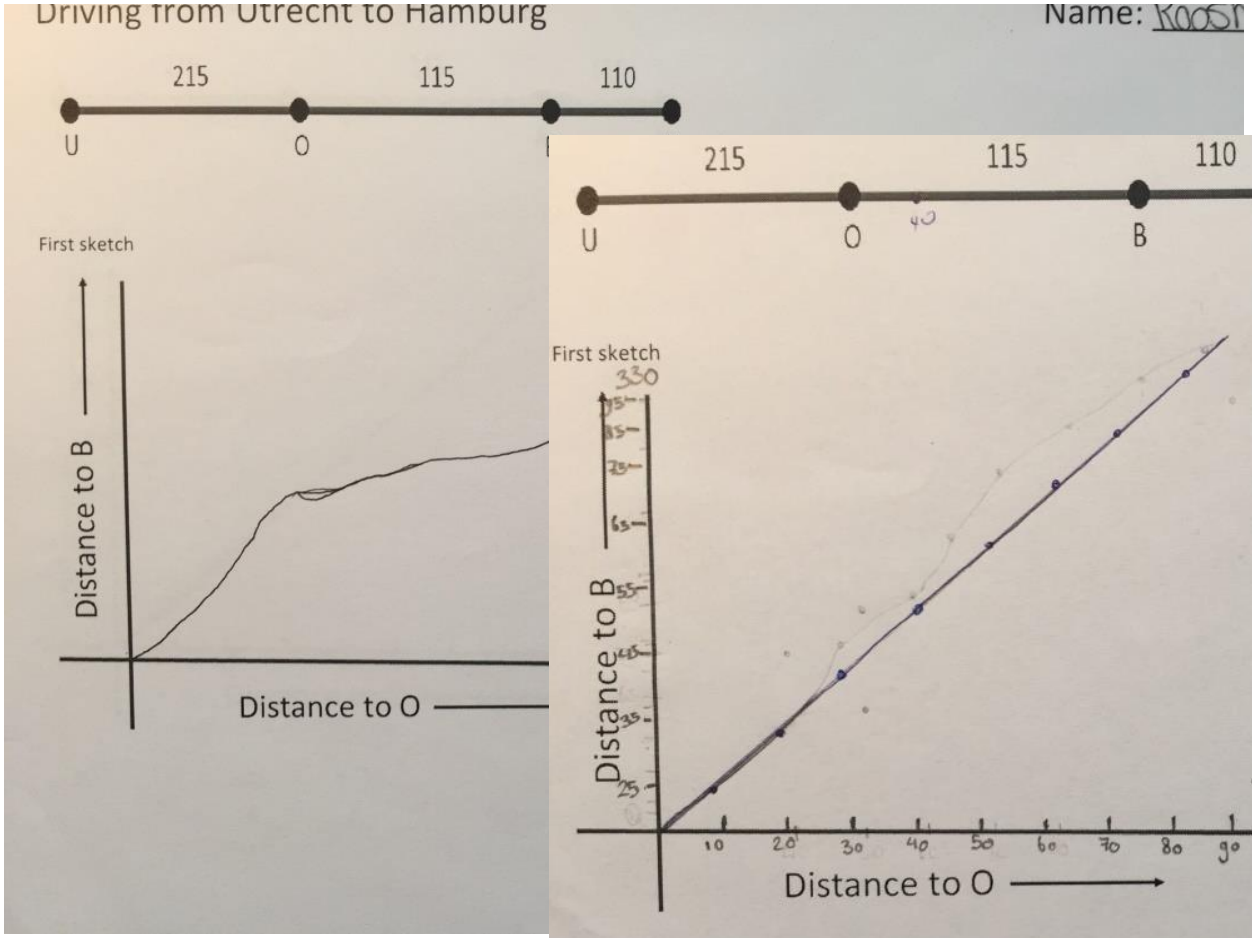
Teaching experience

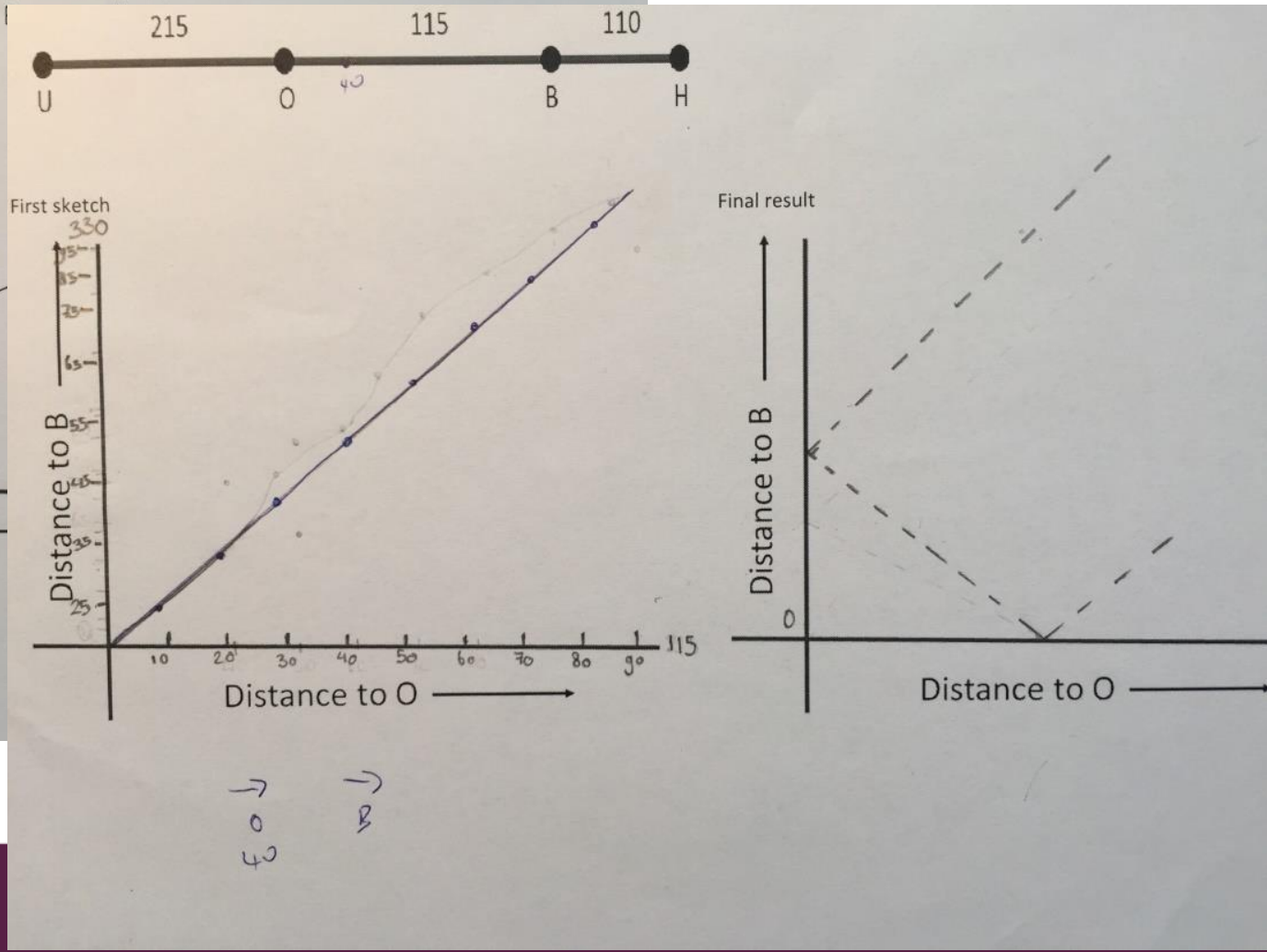
- 13-14 year old
- High achievers
- Bilingual stream

Teaching experience



Teaching experience





Conclusion: activity features

What makes this a nice problem?

- Realistic, meaningful context as point of departure
- Unconventional, non-routine problem (no time-distance, but distance-distance graph)
- Different types and levels of approaches and solutions
- Input from students, interaction between students and between teacher and students

These aspects are core in NL math ed tradition.

To design such productive tasks is our challenge!

Thank you!

Thematic Afternoon
European Didactic Traditions

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2016-07-27

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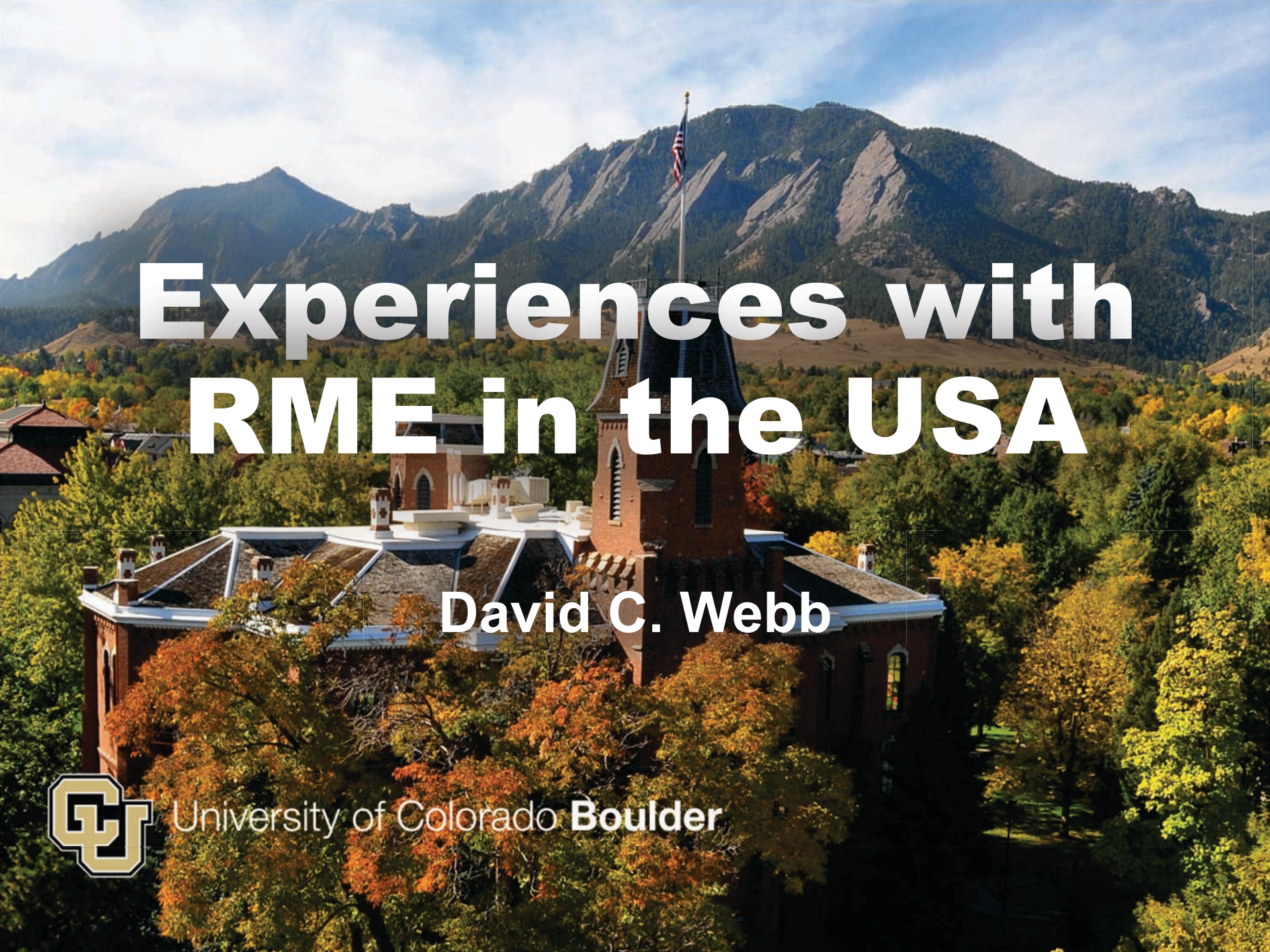
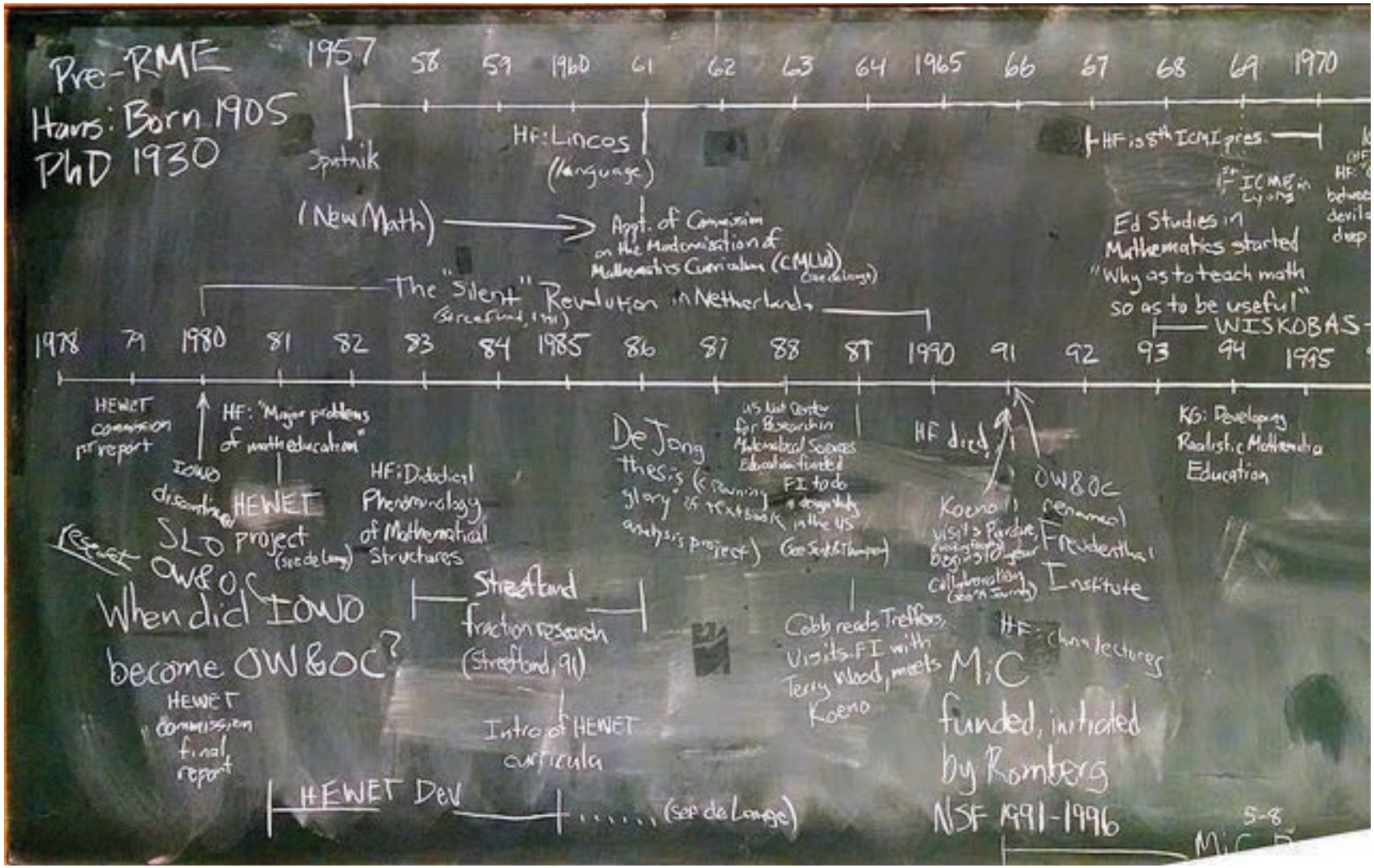
Universiteit Utrecht

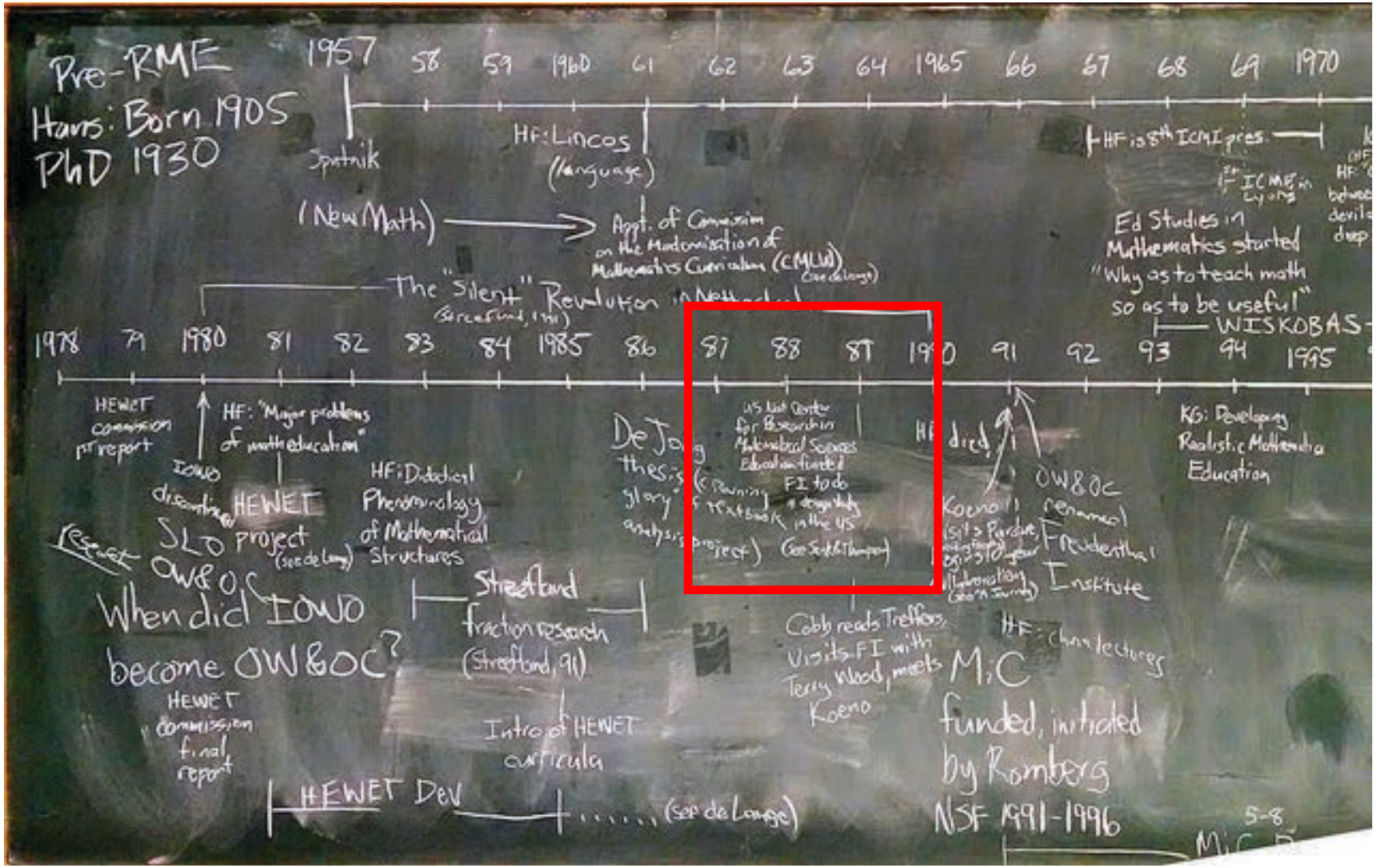
[Faculteit Bètawetenschappen
FISME Freudenthal Institute for Science and Mathematics Education]


Experiences with
RME in the USA
David C. Webb
University of Colorado Boulder

RME Timeline Global



RME enters the US



Thomas A. Romberg

Jan de Lange

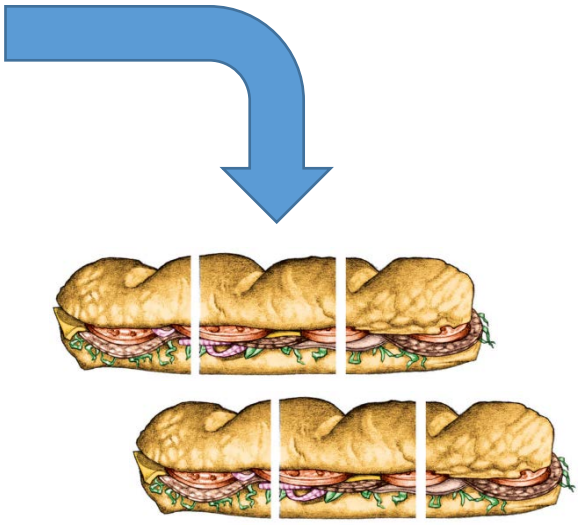
From proof of concept to large scale projects

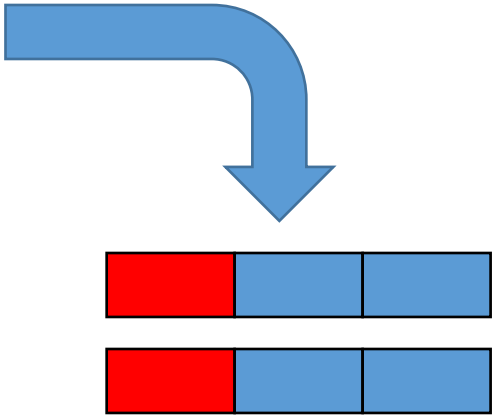
- Whitnall study → Mathematics in Context
  - de Lange, van Reeuwijk, Burrill et al
- Mathematics in the City
  - Fosnot, Dolk et al
- Statistical reasoning
  - Gravemeijer, Cobb et al
- ARISE/COMAP: Assessment Tasks → Curriculum
  - Garfunkel, van der Kooij et al
- Assessment: RAP → CATCH → BPEME
  - Abels, Dekker, de Lange, Feijs, Querelle, Webb

US Attraction to RME

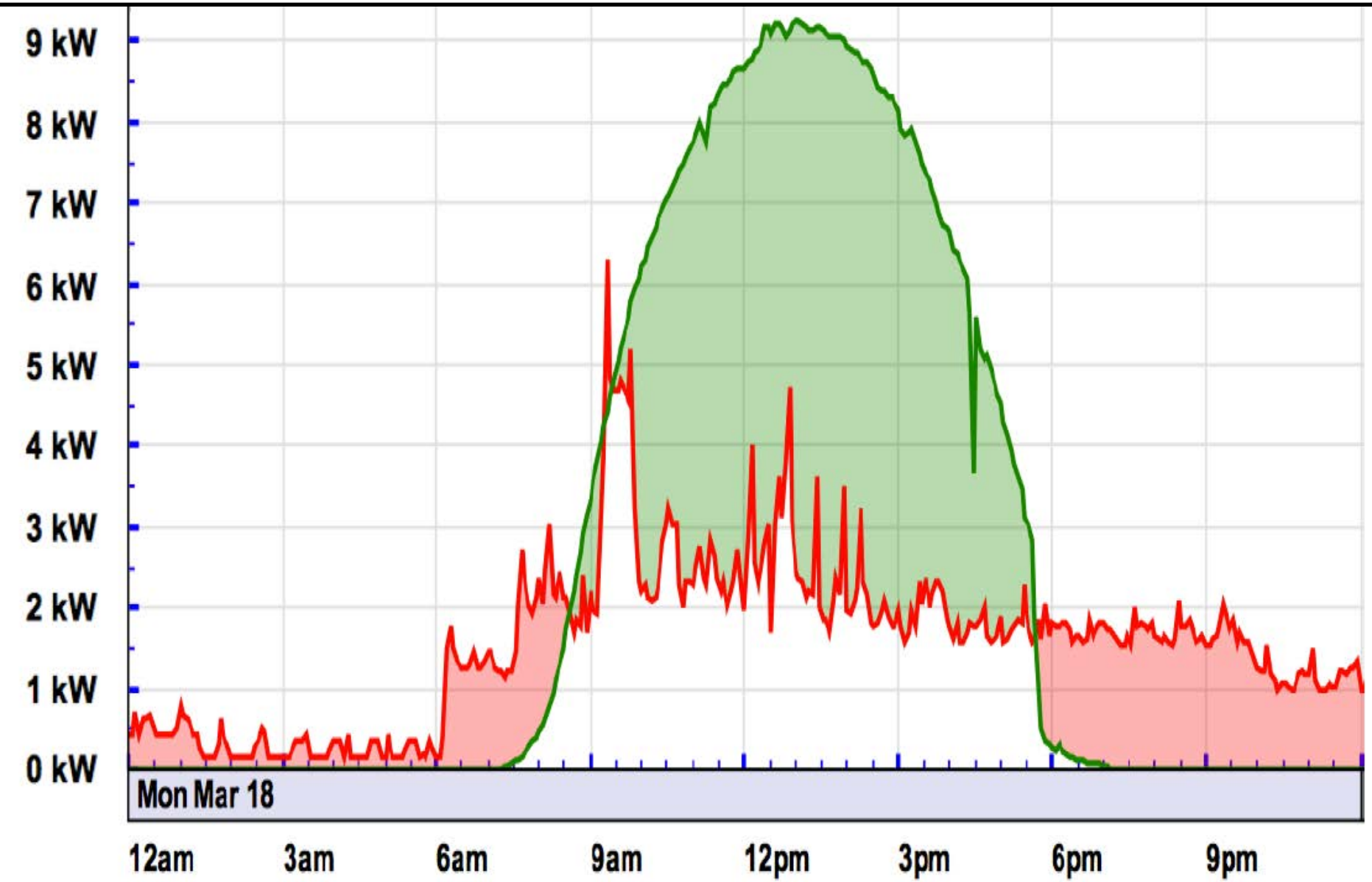
- Unique context-first approach
- Preformal models and tools
- Robust approach to assessment

Context first approach





1. Consider the bell-shaped (green) curve $g(t)$. What quantity does the area under this curve, between two points $t = a$ and $t = b$, represent? What are the units for this quantity?



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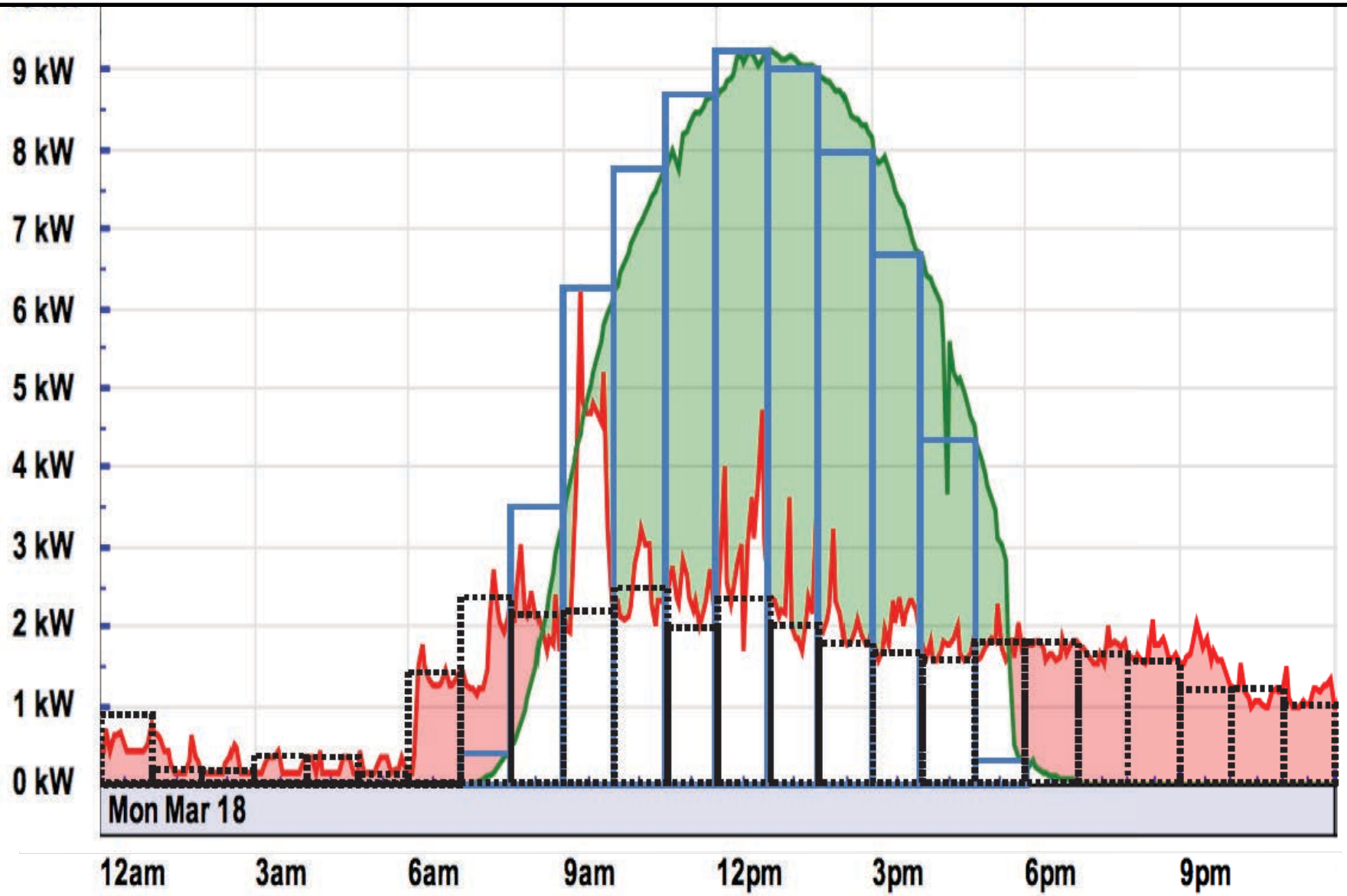
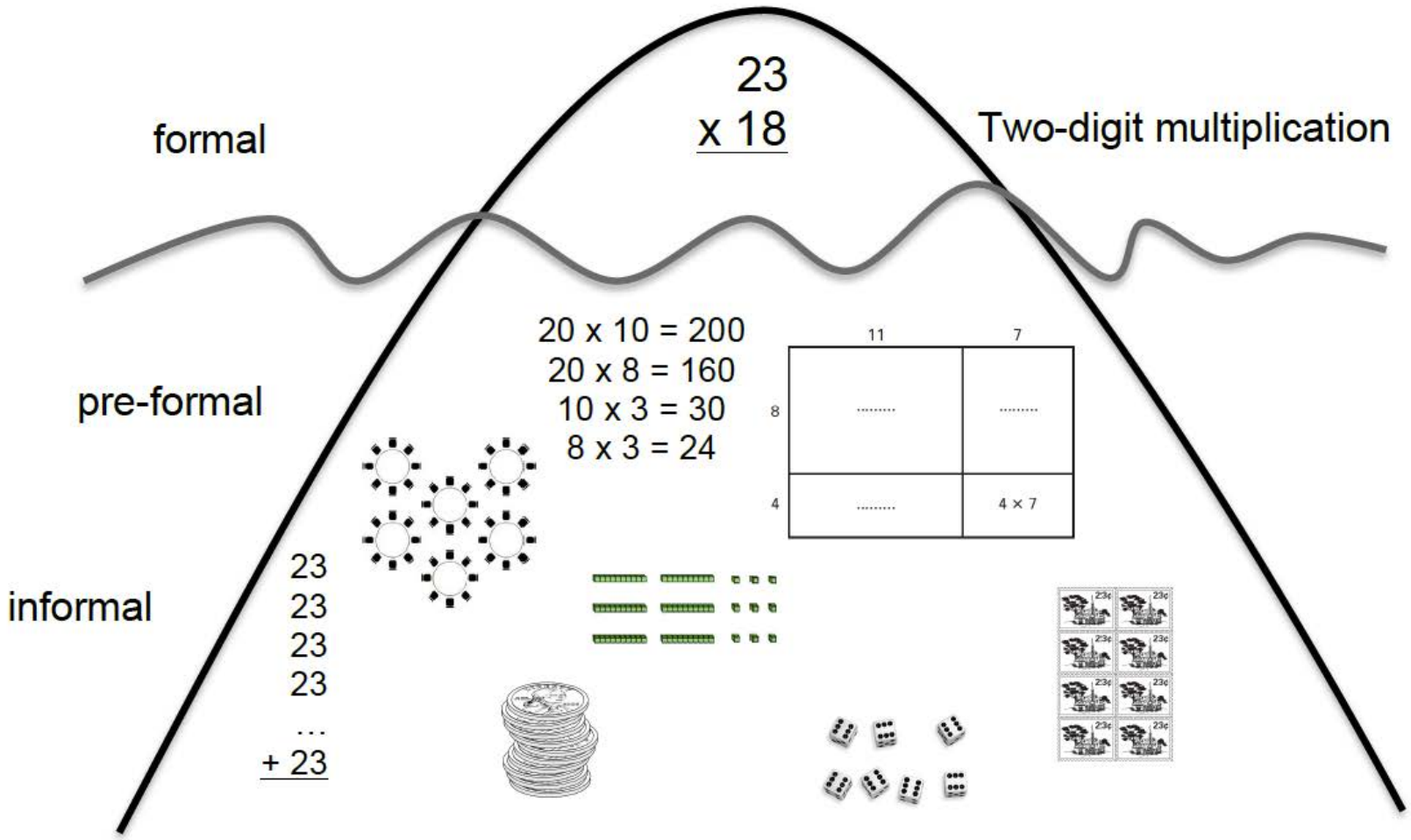


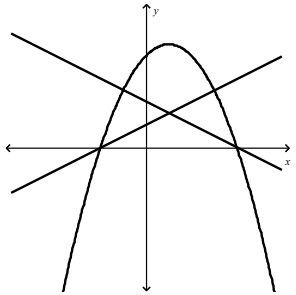
Pre-formal models and tools

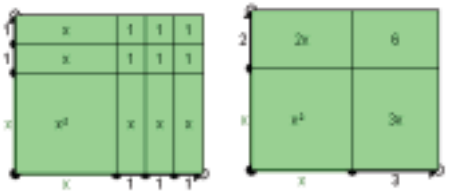


$y = (2x + 4)(-3x - 8)$

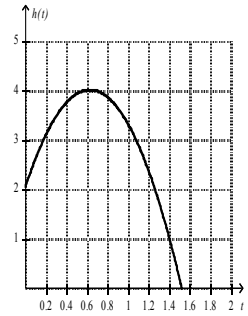
$(x + 3)(x + 2) = x^2 + 5x + 6$

$h(t) = -5t^2 + 3t + 2$





x	y



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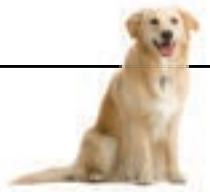
w

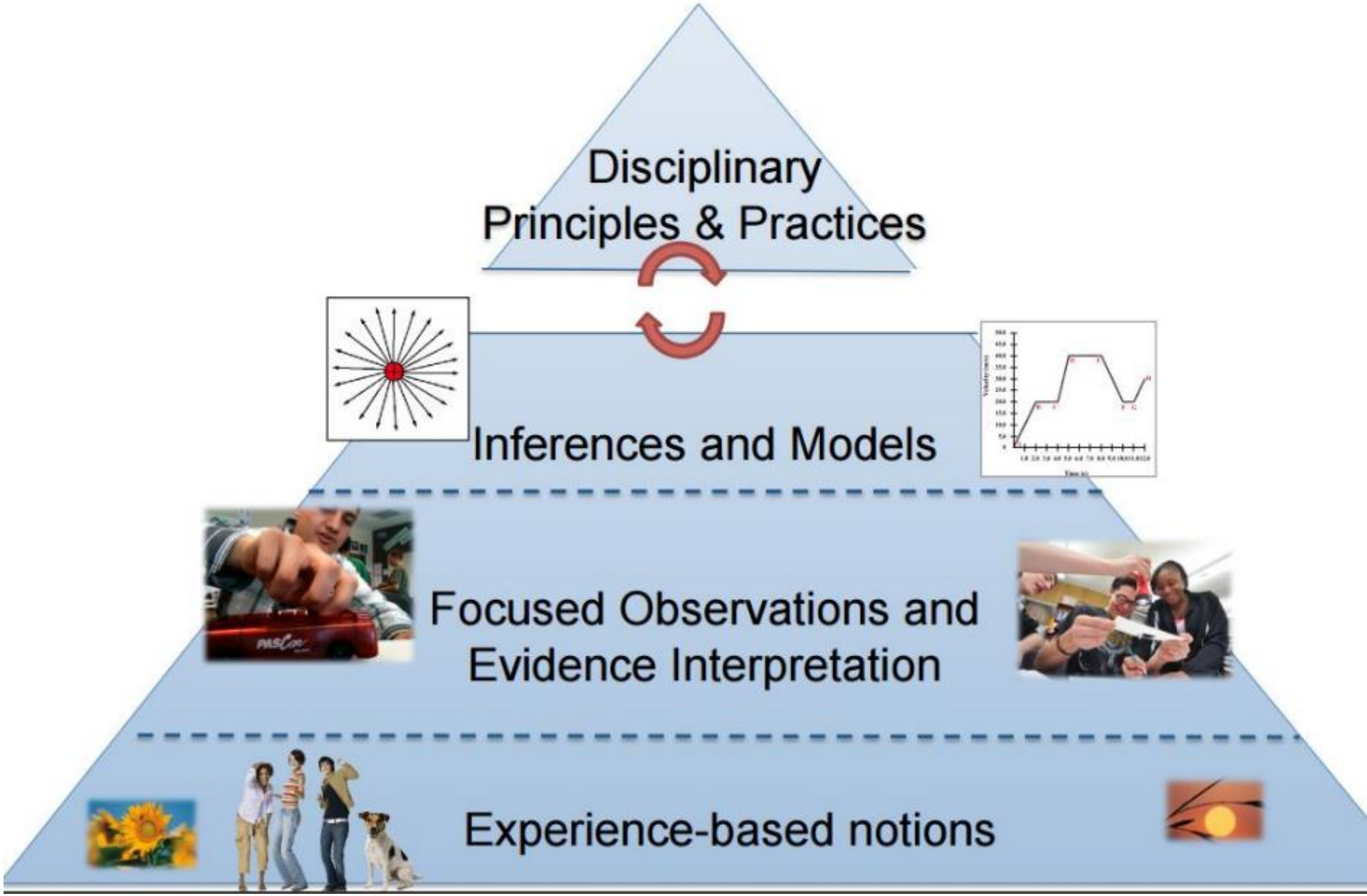


Formal

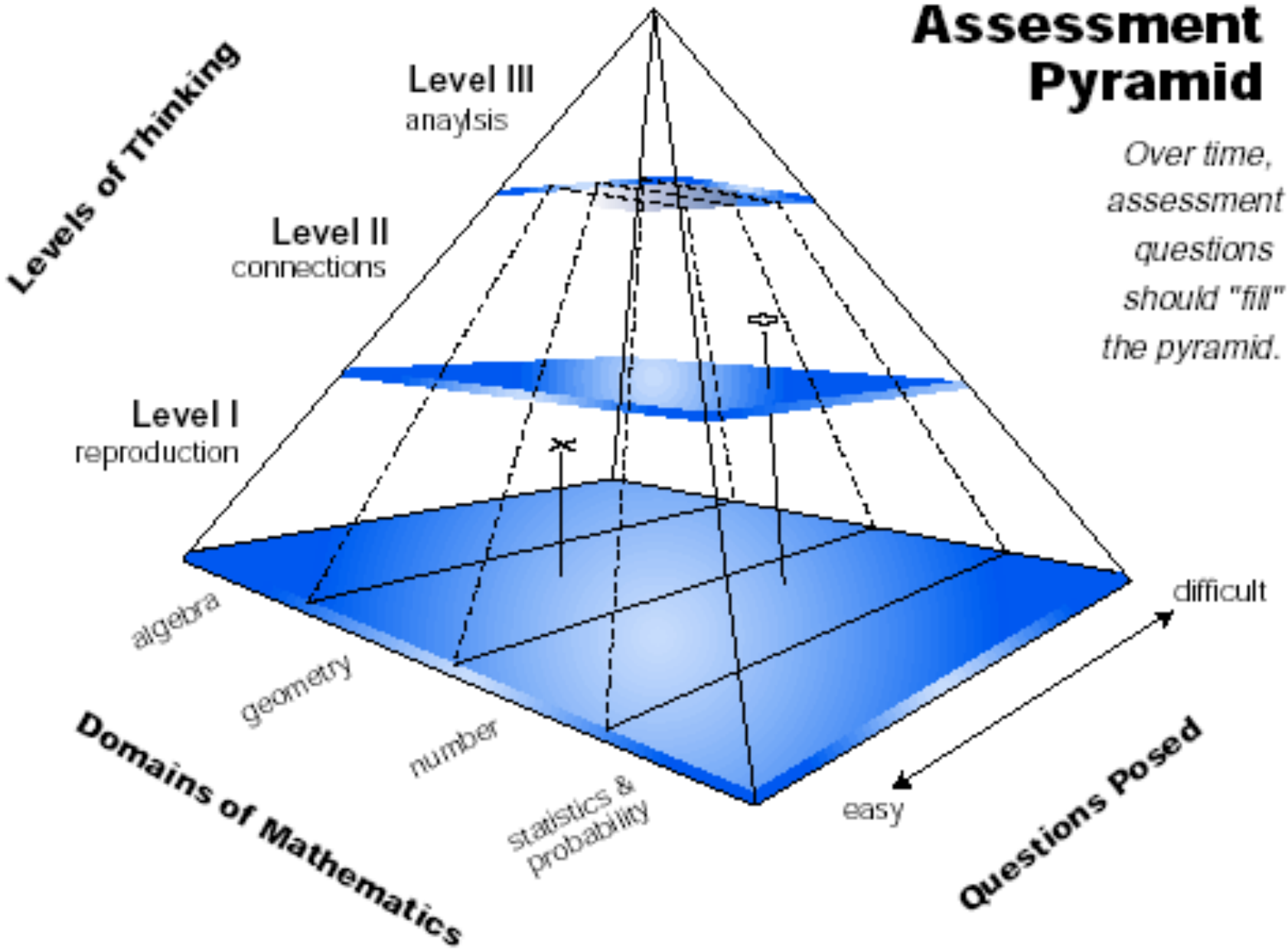
Pre-formal

Informal

A model – how disciplinary knowledge is learned

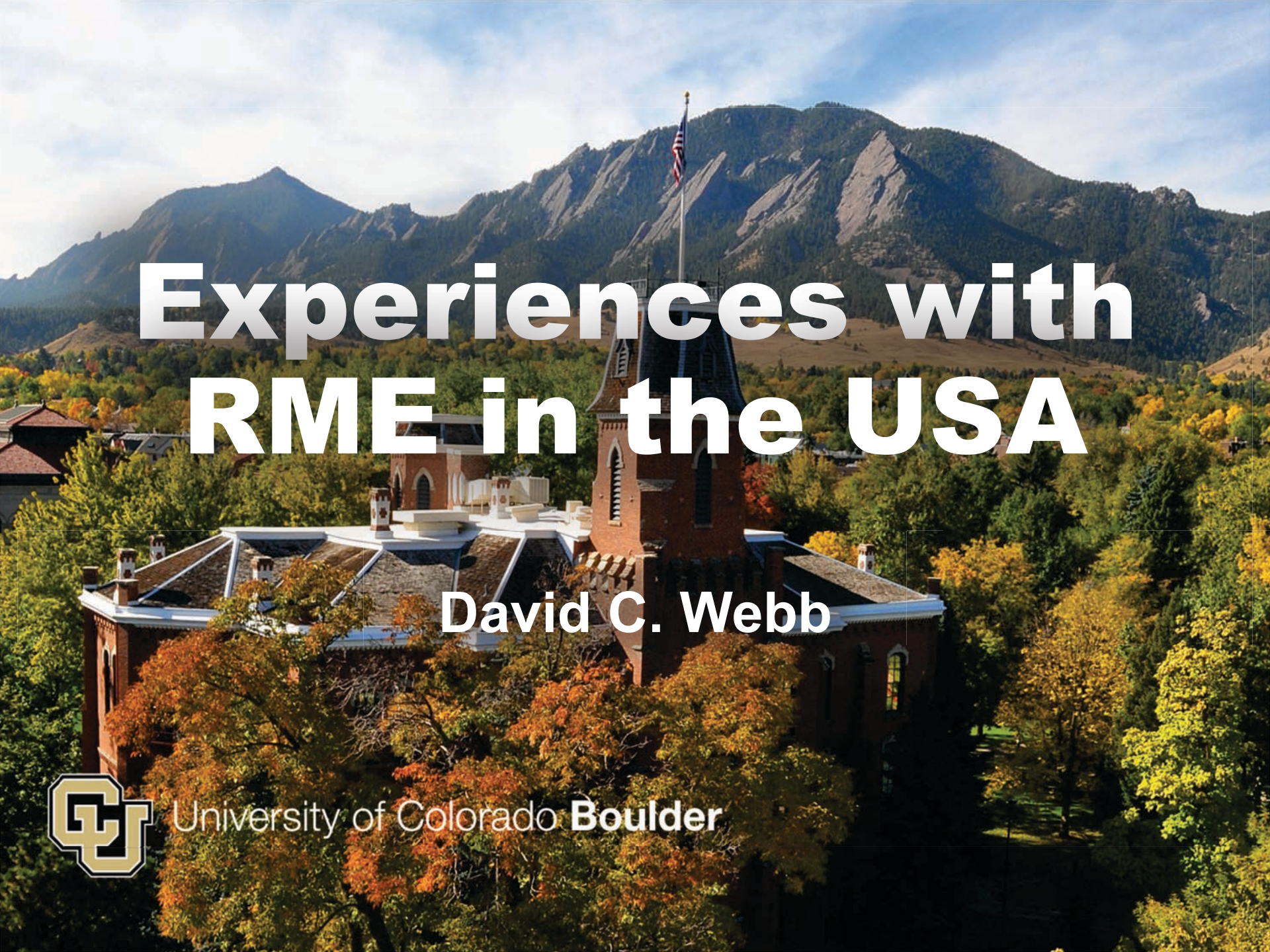


Comprehensive assessment



5th International Realistic Math Education Conference: September 18 – 20, 2015

University of Colorado Boulder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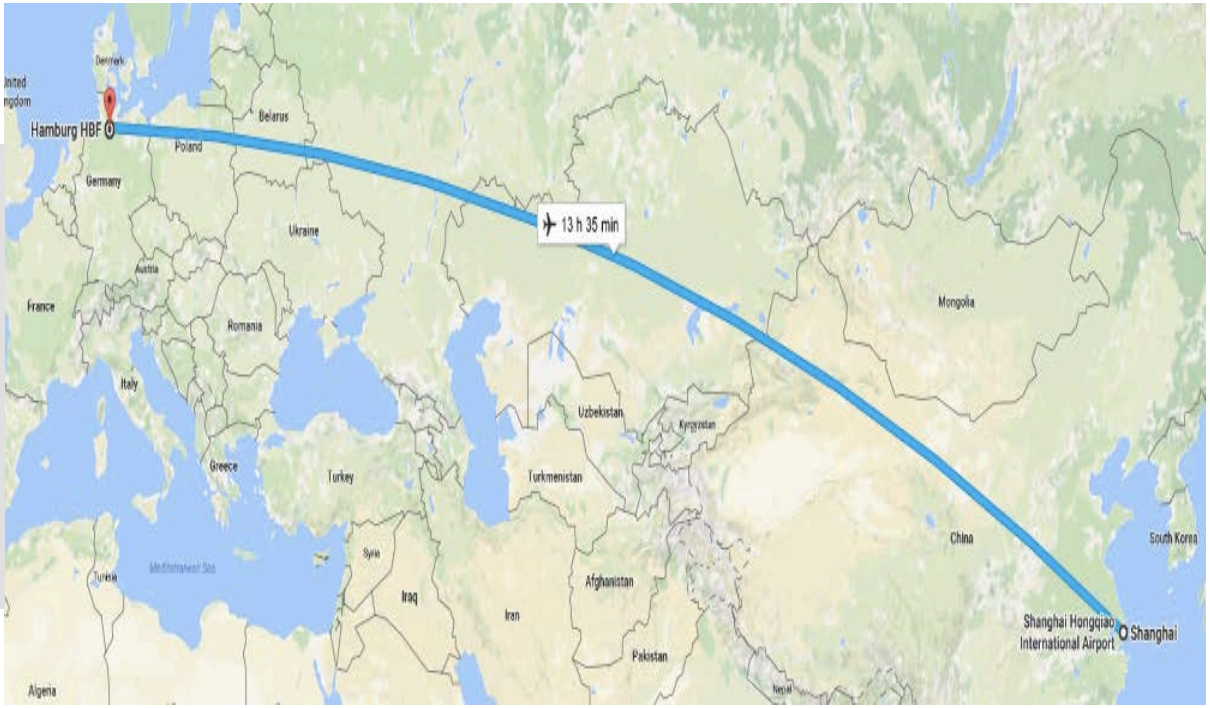
13th International Congress on Mathematical Education
July 24 – 31, 2016 in Hamburg / Germany
ICME13 Hamburg 2016

Thematic Afternoon: European Didactic Traditions – Netherlands
Wednesday, 16.00-18.00

TWO DECADES OF RME IN INDONESIA:
FROM ICMI SHANGHAI 1994 TO ICME HAMBURG 2016

Zulkardi,
Ratu Ilma Indra Putri
Sriwijaya University
Aryadi Wijaya
Jogyakarta State University,
Indonesia



OUTLINE

- Mathematics reform using RME in Indonesia
- The development of PMRI
  - Initiation
  - Implementation
  - Dissemination
- PMRI growth beyond project
  - Development of a web portal on PMRI
  - SEA-RME Course
  - Founding an Indonesian Journal on (R)ME
- PMRI continues

Math Reform using RME in Indonesia

In 1994, Jan de Lange- Director of Freudenthal Institute presented his keynote in ICMI International seminar, in Shanghai.

Prof. Sembiring saw Jan's presentation about RME. As Dikti's team who search what the best math. education to change math. modern in Indonesia, Sembiring invited Jan to come to Indonesia in order to help Indonesia reforming mathematics education.

Sembiring

The ‘culture’ decision of Indonesia to adapt RME to Indonesian context

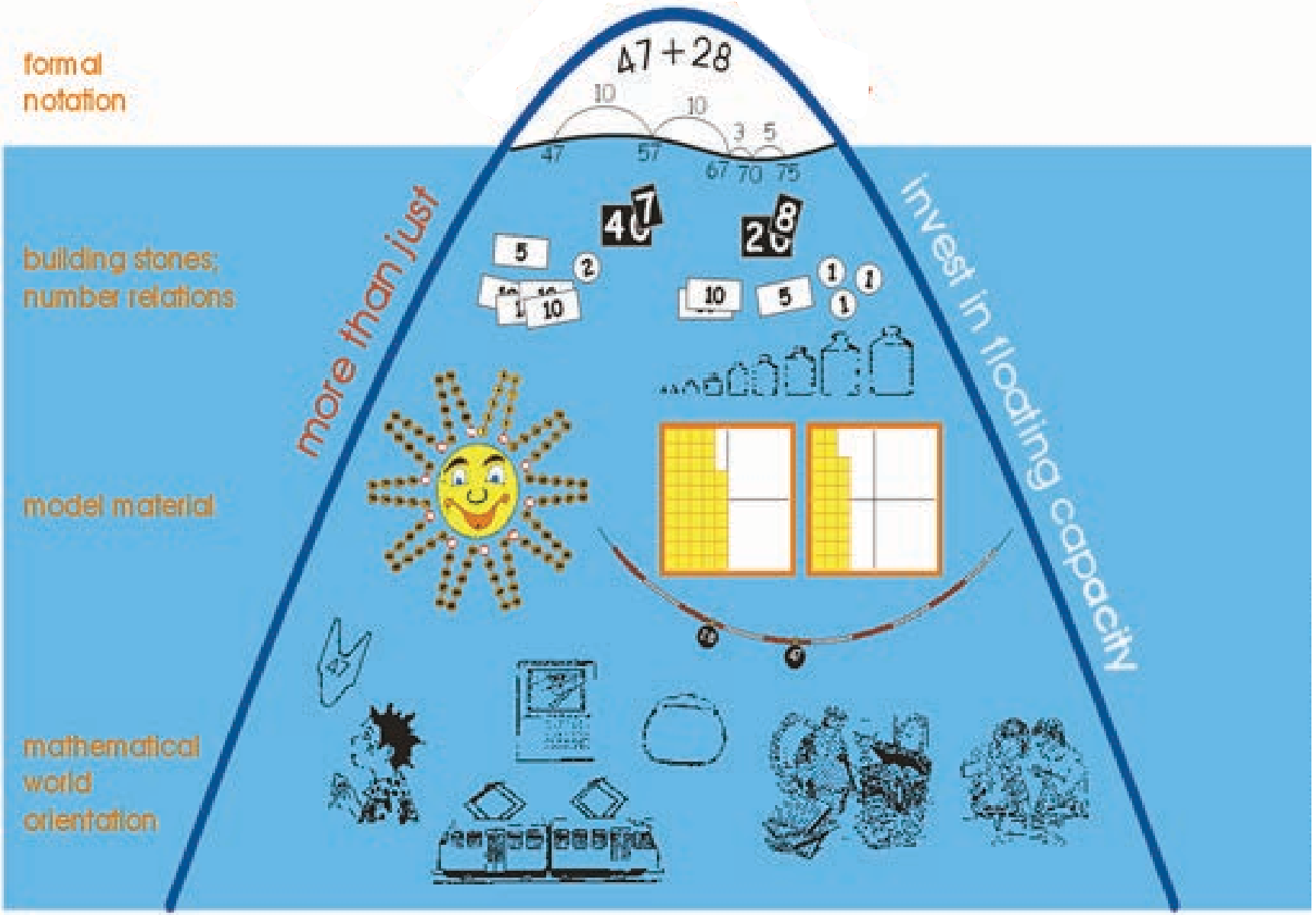
In 1998, Jan de Lange came to ITB, Bandung visited pak Sembiring. With Prof. Tjeerd Plomp (UTwente) and Annie Kuipper, they conducted a workshop on RME with 20 candidates and selected 6 Ph.D.

Indonesian Government sent and funded the six students to do a Ph.D. research on RME in the Netherlands. The six followed the ‘sandwich PhD. Program” between UTwente and Utrecht University. Finally, they got PhD. on RME in December 2002. All of them are professor. (Prof. Sutarto is also here now!)

THE DEVELOPMENT OF PMRI

Year	Reform Movement of PMRI
1994	Initiation: Prof.Sembiring Prof. Jan de Lange in ICMI Shanghai
1998-2002	Six PhD candidates sent to the Netherlands to learn RME
2001	Implementation: PMRI is started and Small Project PMRI was started in 3 cities in Java
2006-2011	Dissemination of PMRI (DO-PMRI) Project funded by Dutch Government about 25 out of 34 Provinces and linked by a Web Portal P4MRI.net
2008	Starting a Joint Master program on RME-PMRI (IMPOME) among Unsri Palembang-Utrecht University and Unesa Surabaya
2010	Starting Mathematics Literacy Contest (KLM) and Journal on (R)ME
2011	Starting SEA-DR Conference on Design Research on
2012	SEA-RME for Teacher from ASEAN countries
2016	Starting PhD program on PMRI and National Center in Unsri Palembang

PMRI


formal notation
building stones; number relations
model material
mathematical world orientation
more than just
invest in floating capacity
47 + 28
10
10
3
5
47
57
67
70
75


08.11.2013

A decade of PMRI in Indonesia

Edited by

Robert Sembiring
Kees Hoogland
Maarten Dolk

Table of contents

A decade of PMRI in Indonesia: Story form some chapters

1. Introduction to: A decade of PMRI in Indonesia

Robert Sembiring, Kees Hoogland, Maarten Dolk

3. My story of realistic mathematics in Indonesia

Lee Peng Yee

5. Realistic mathematics education theory as a guideline for problem-centered, interactive mathematics education

Koeno Gravemeijer

19. The future of PMRI

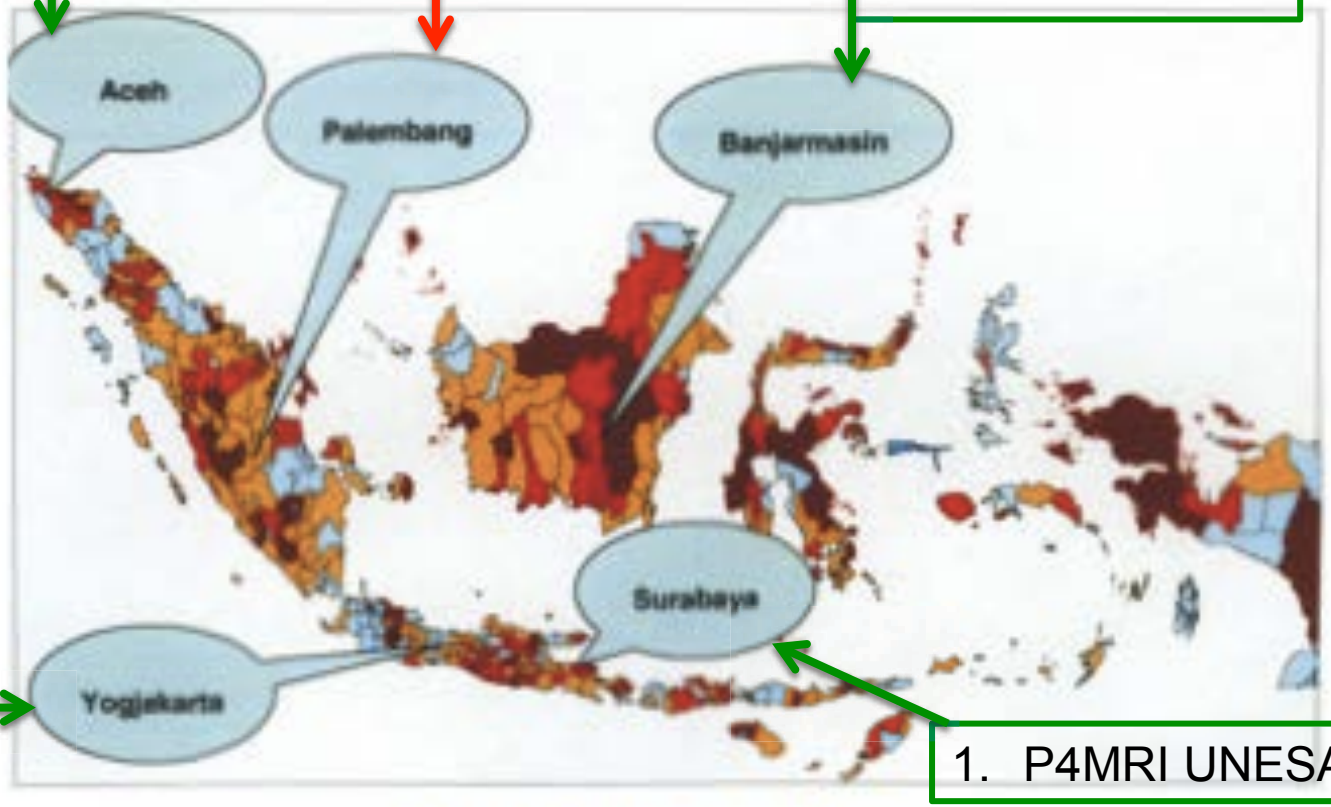
Robert Sembiring, Sutarto Hadi, Zulkardi, Kees Hoogland

THE ACTUAL INFRASTURUCTURE OF PMRI (Sembiring, et.al ,2011)

1. P4MRI UNSRI
2. IMPOME (MASTER PMRI)
3. KLM
4. SEA-DR
5. P4MRI.NET
6. JOURNAL (R)ME
7. INDOME (DR. PMRI)

1. P4MRI ACEH

1. P4MRI ULM



1. P4MRI UNY

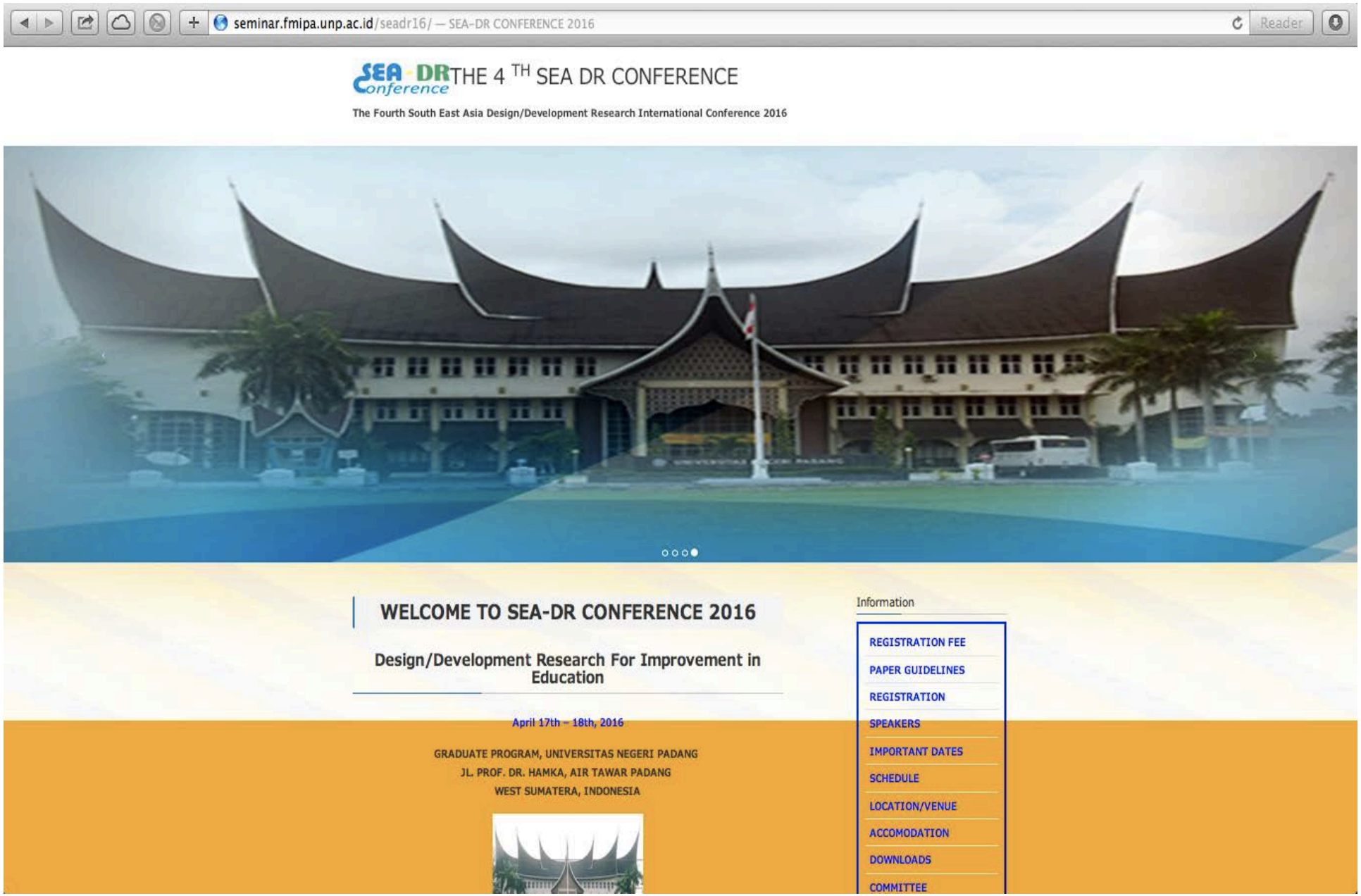
1. P4MRI UNESA

Figure 4: five P4MRIs, the actual infrastructure

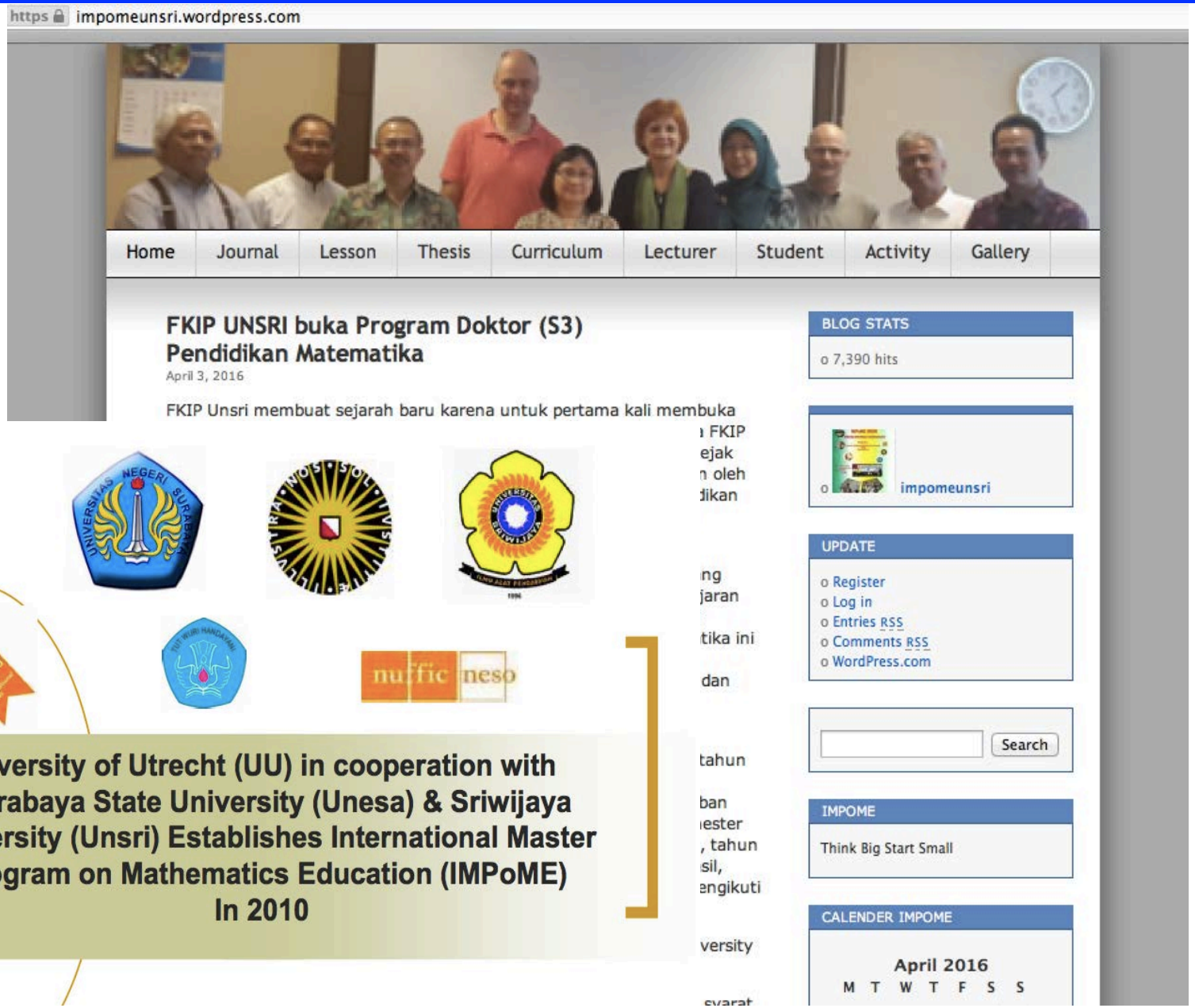
Portal PMRI www.p4mri.net

https://sites.google.com/site/kondriunsri2013/

http://seminar.fmipa.unp.ac.id/seadr16/

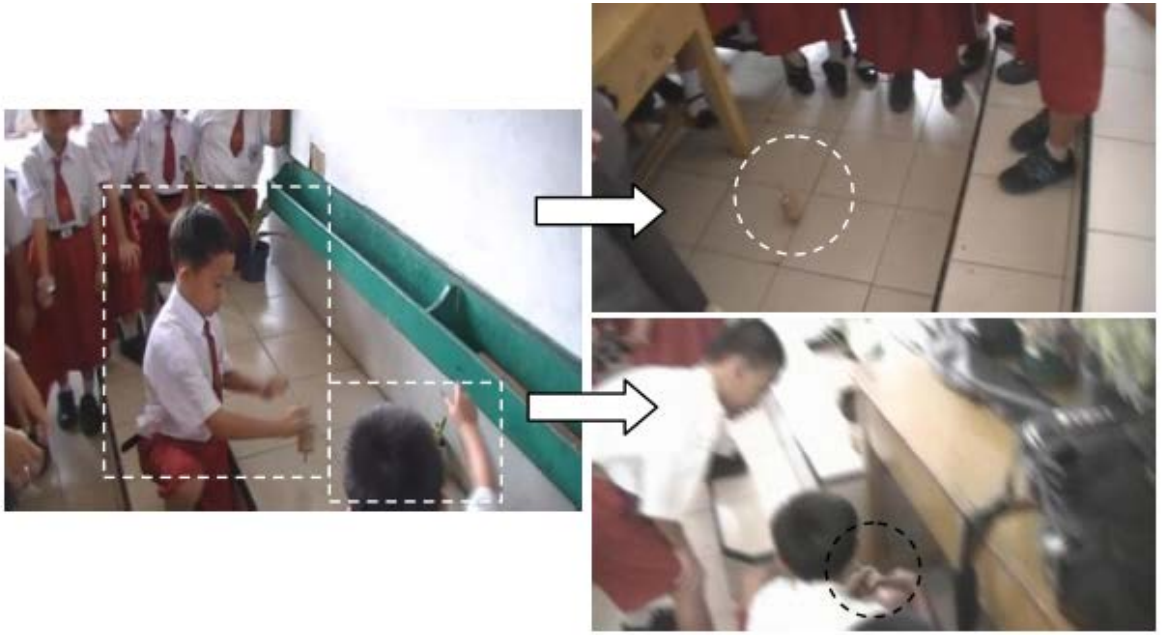


impomeunsri.wordpress.com

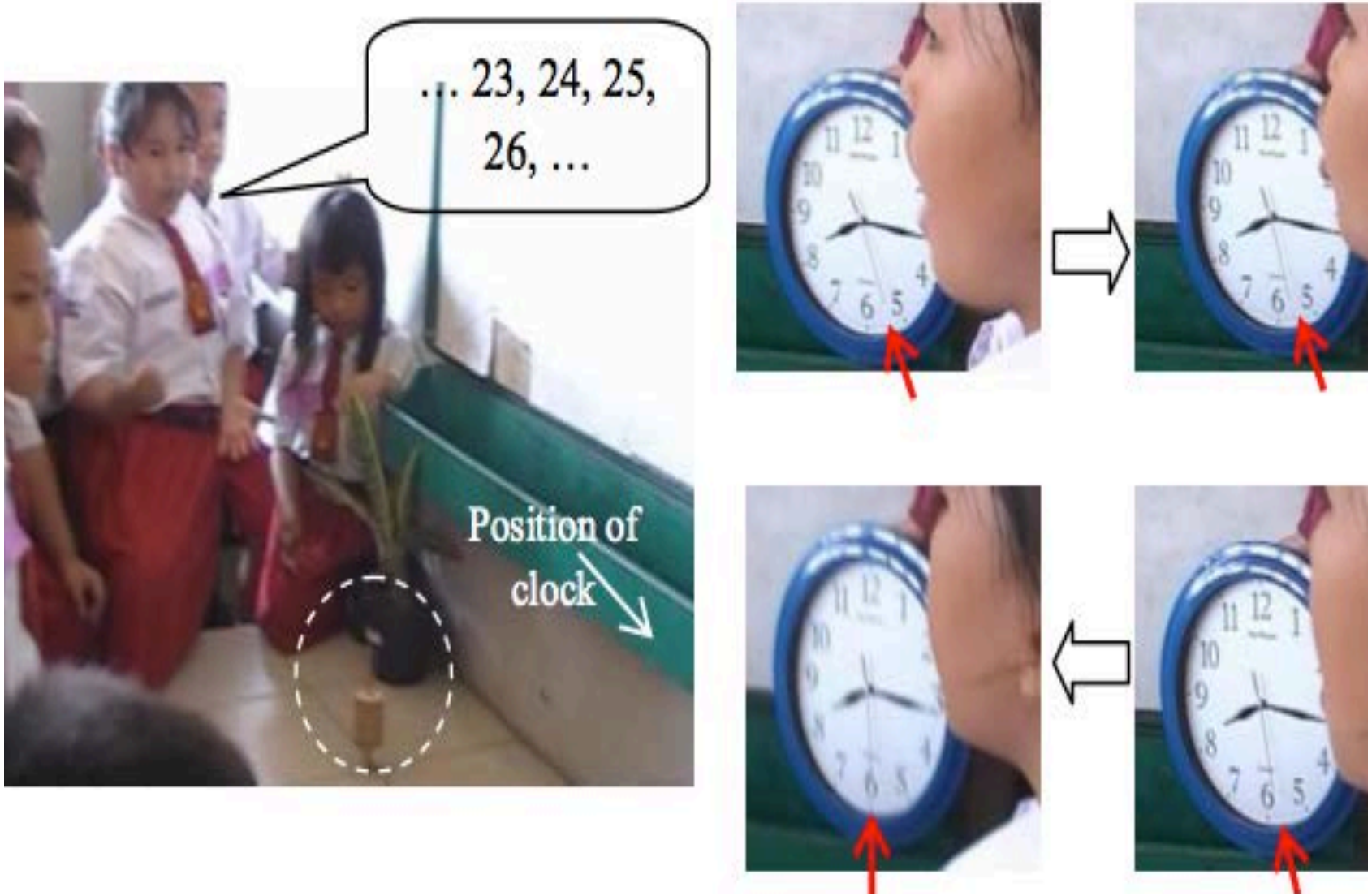


Example of research using PMRI

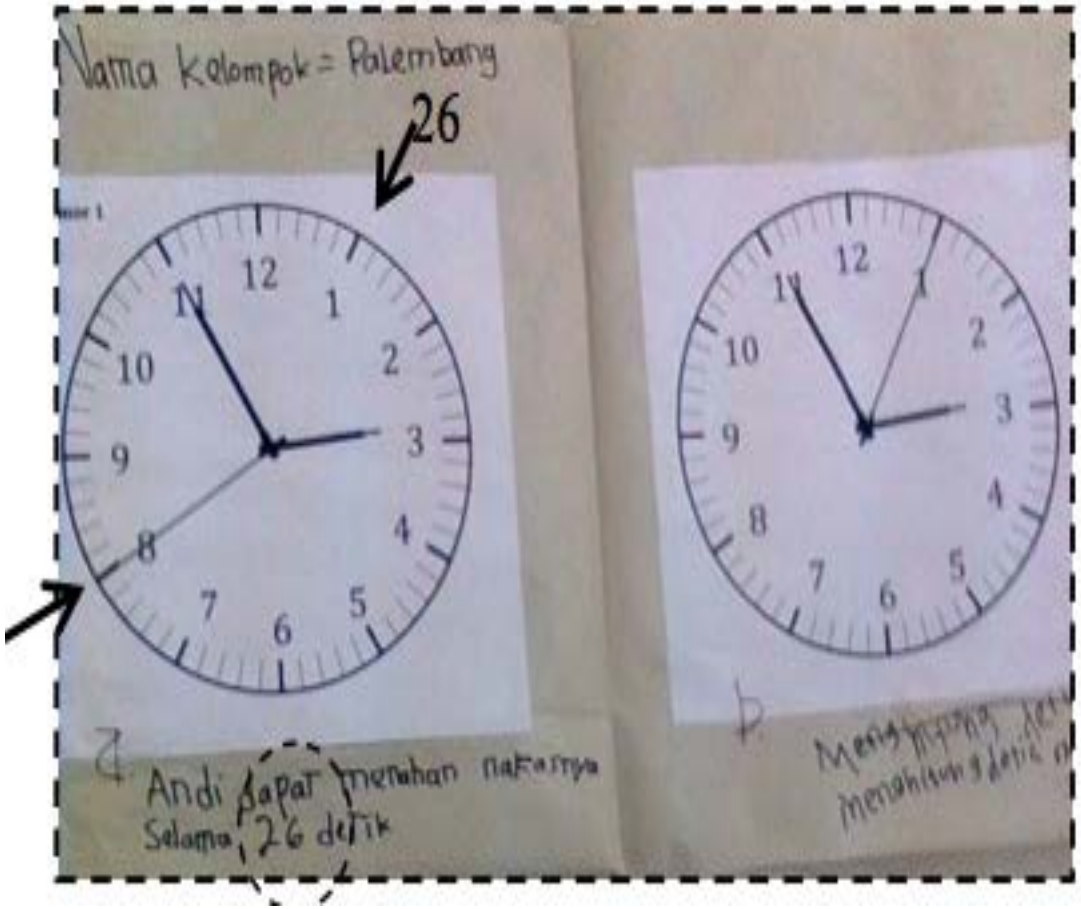
1. Counting orally during Gasing is moving



2. Students count orally in conjunction with the movement of clock hand



3. Counting the strip and the space of the strip of second interval on the clock



SEAMEO Regional Centre for
QITEP in Mathematics
Learning Mathematics Joyfuly and Meaningfuly

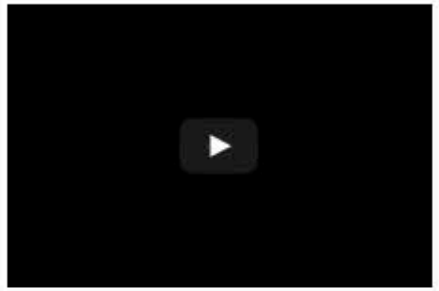
Course on Southeast Asia Realistic Mathematics Education for Junior Secondary School Matehmatics Teachers, 8-28 October 2012

Wed, 30 Apr 2014

Prof. Subanar, Ph.D., Director of the Southeast Asian Ministers of Education Organization - Regional Centre for Quality Improvement of Teachers and Education Personnel in Mathematics (SEAMEO QITEP in Mathematics), officially opened a three-week course on Southeast Asia Realistic Mathematics Education (SEA-RME) for Junior Secondary School Mathematics Teachers on October 8, 2012, in the Auditorium of PPPPTK Matematika, Yogyakarta, Indonesia. The course ran until October 28, 2012. The opening ceremony was also attended by Prof. Masami Isoda from University of Tsukuba-Japan, Prof. Allan L. White from the University of Western Sydney-Australia, and Prof. Sutarto Hadi from Lambung Mangkurat University-Indonesia.

OUR FAMILY

PROFILE



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Publisher

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The Profile of JME in Google Scholar

- Total Citations in Google Scholar (2013-2014): **88**
- Total Articles in Google Scholar (2013-2014): **31**
- h-index: **6**
- i10-index: **5**
- Total Impact Factor Google Scholar (2015): 88/31 = **2.8387**

JME Gallery

Journal on Mathematics Education
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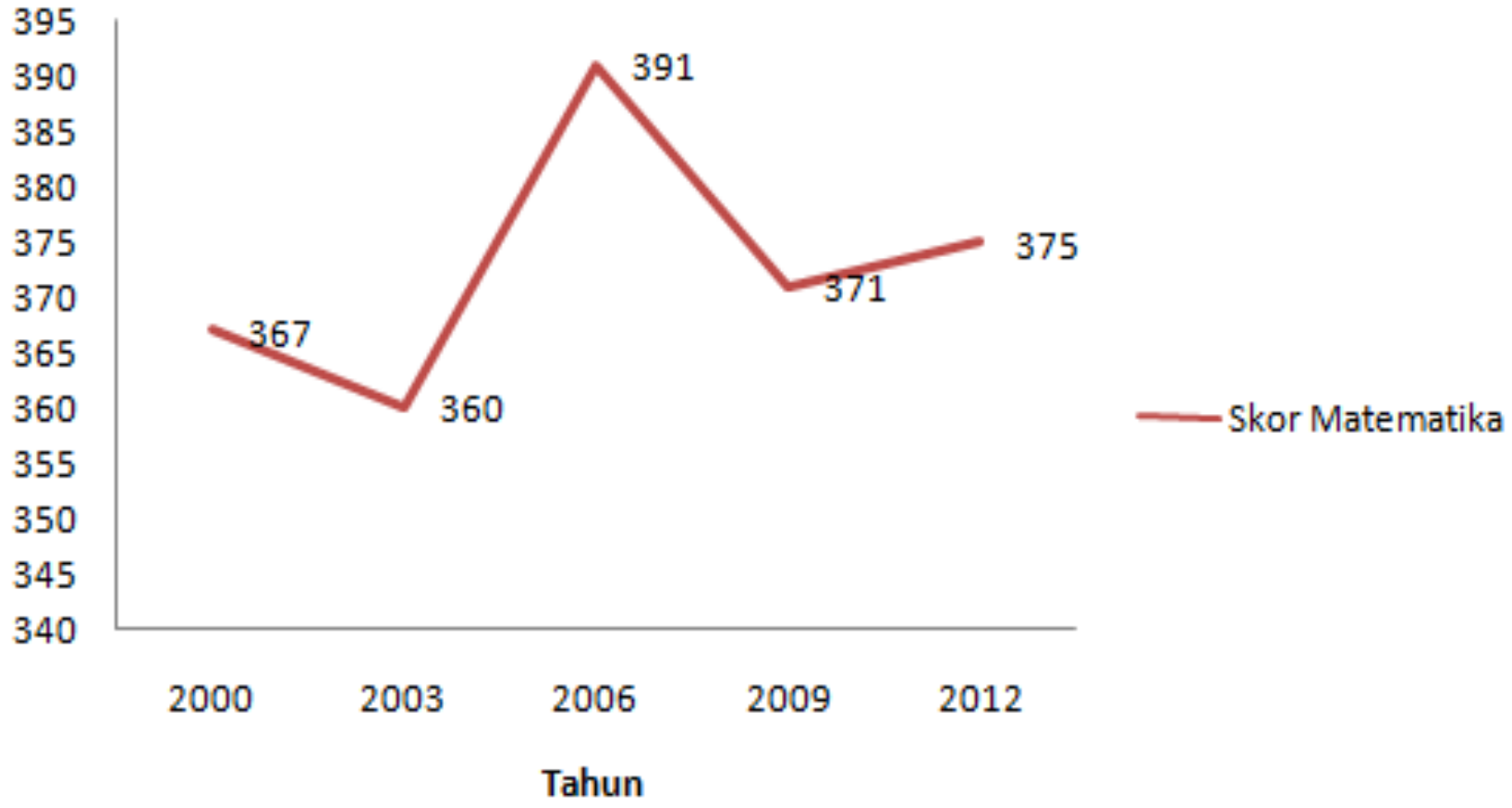
Scores PISA Mathematics of Indonesian Students in Year 2000 to 2012



(IDN) Students' difficulties with context-based (PISA) problems

(Wijaya, 2015)

Four types of errors in solving context-based problems:

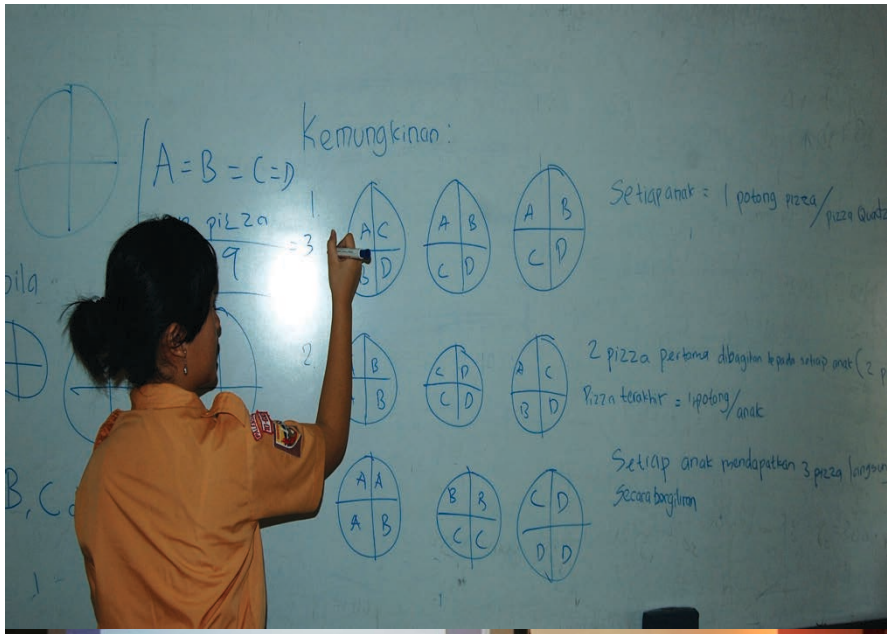
- Comprehension: 38%
- Transformation: 42%
- Mathematical processing: 17%
- Encoding: 3%

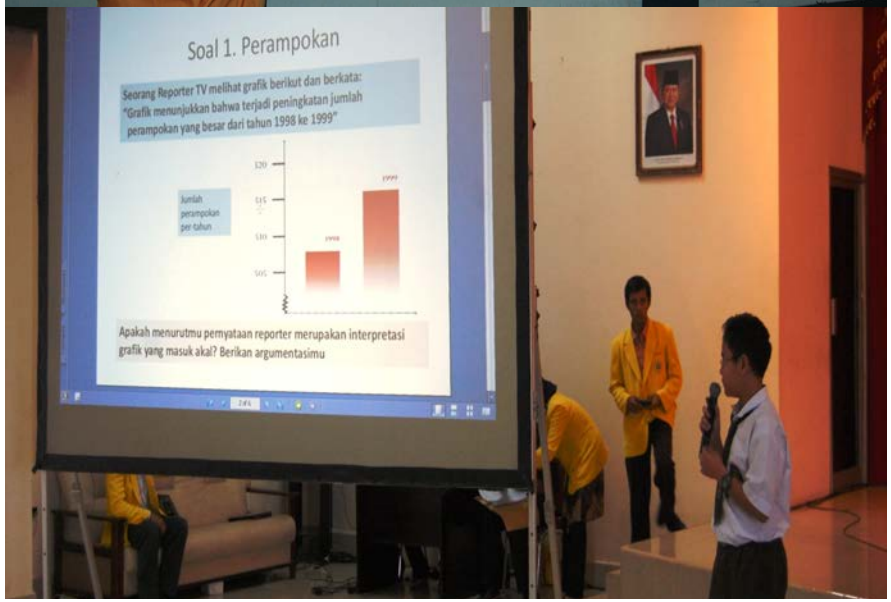
Conclusion:

IDN students' mainly have difficulty in comprehending a context-based problems and in transforming them into mathematical problems.

Kontes Literasi Matematika-KLM
(A national contest on Mathematics Literacy)

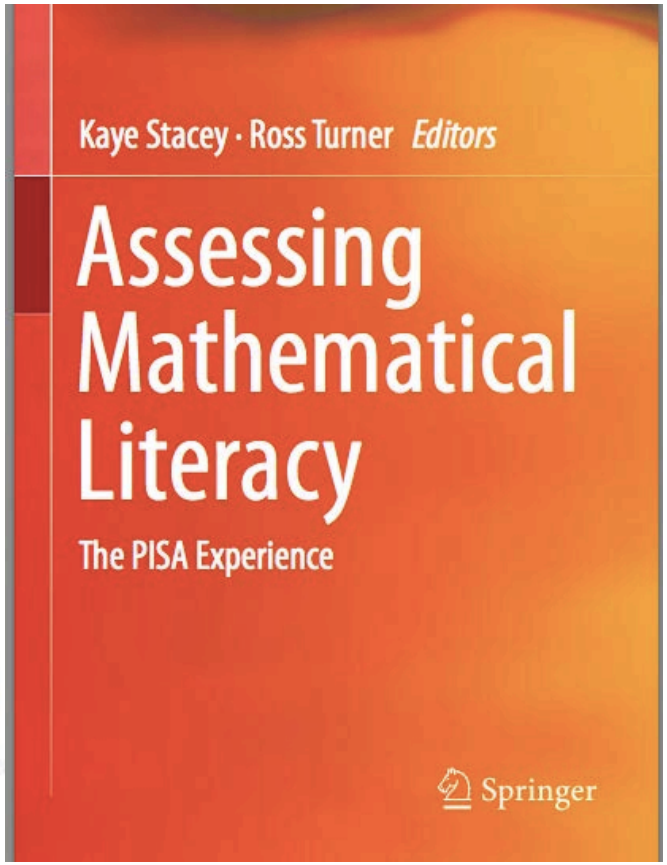
- Since 2010 in Unsri Palembang
- Since 2015 at 12 Teacher educations in 11 provinces in Indonesia
- A workshop on PISA for teachers is also conducted
- Followed by thousand students
- Three levels of contest:
  - Qualification: Written Test (level 1, 2 and 3 PISA tasks)
  - Semi-final: Written test in the white board (level 3 & 4)
  - Final: Presentation (HOTS level 5-6 of PISA)





Web blog PISA Indonesia (www.pisaindonesia.wordpress.com)

New Math Literacy book including PISA actions in Indonesia (Stacey & Turner, 2015)



FUTURE OF PMRI: CHALLENGES

- LOWEST PISA SCORE & HAPPIEST IN THE SCHOOL
- SUSTAINABLITY OF PMRI INFRASTRUCTURE
- CENTER OF EXCELLENCE OF PMRI
- ….

MARS PMRI

MARS PMRI By: Dr. Mulyardi

TELAH HADIR DI BUMI PERSADA
INOVASI PEMBELAJARAN MATEMATIKA
MENGGUNAKAN BUDAYA INDONESIA
PMRI ITU NAMANYA

DENGAN PMRI GURU JADI KREATIF
DENGAN PMRI SISWA MENJADI AKTIF
MANFAATKANLAH WAHAI PENDIDIK BANGSA
AGAR MATEMATIKA DISENANGI SISWA

REFF. PMRI CARA YANG BIJAKSANA
MENYAPA SISWA DENGAN RAMAH
PMRI SUATU INOVASI
MENGGUNAKAN BUDAYA ANAK NEGERI

DENGAN PMRI SISWA JADI CERIA
DENGAN PMRI KELAS JADI GEMBIRA
HAI PENDIDIK BANGSA LAKSANAKAN SEGERA
DEMI PEMBELAJARAN MATEMATIKA

Kembali ke Reff.

Thank you for coming

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Blog : www.p4mri.net;

Facebook : facebook.com/zulkardiharun;

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REFLECTIONS FROM ENGLAND AND THE CAYMAN ISLANDS

Sue Hough, Paul Dickinson, Steve Gough, Yvette Solomon, Frank Eade

Manchester Metropolitan University

ICME conference

July 2016

BRIEF OUTLINE

- Dealing with clashing educational ideologies
- RME based projects in the UK
- RME based projects in The Cayman Islands

CLASHING IDEOLOGIES

- 1999 – a team from Manchester Metropolitan University visit the Netherlands

- 13 year old Dutch students are asked to say which is larger $\frac{2}{3}$ or $\frac{3}{4}$

- Use of a variety of strategies: mediating quantity, percentage, decimal, comparison to the whole, drawings

CLASHING IDEOLOGIES

- English teachers' own knowledge of mathematics and their expectations of mathematics to be taught is often limited to an emphasis on the acquisition of procedures

- Work out $\frac{6}{16} \times \frac{8}{18}$

- How do you know you are right?

b) $\frac{6}{16} \times \frac{8}{18} = \frac{48}{288}$ or $5\frac{44}{288}$

$6 \times 8 = 48$

$$\begin{array}{r} 16 \\ \times\ 18 \\ \hline 128 \\ 160 \\ \hline 288 \end{array}$$

because times top and time the bottom gives answer to a fraction multiplication

b) $\frac{6}{16} \times \frac{8}{18}$

$\frac{6 \times 8}{16 \times 18} = \frac{48}{288}$ ÷2 $\frac{24}{144}$ ÷2 $\frac{12}{72}$ ÷2 $\frac{6}{36}$ ÷6 $\frac{1}{6}$

÷2 ÷2 ÷2 ÷6

16 x 10 = 160

16 x 8 = 80

6 x 8 = 48

288

1

I know because 6x8=48 and 16x18 is 288. and it cancels down to $\frac{1}{6}$.

HOW DO YOU KNOW YOU ARE RIGHT?

'I know it's right because that's how I was taught to do it'

'I was taught this method and just accepted it'

'I know they're right because I've been doing it for years and have checked my answers'

HOW DO YOU KNOW YOU ARE RIGHT?

‘I wasn’t 100% sure I was right, this is a regurgitation of a procedural method’

‘I can’t think of why I am right’

‘I have no concept of what these answers mean in terms of the actual question, and no idea if they’re even sensible. If I made a mistake a wouldn’t notice’

$$\frac{6}{16} \times \frac{8}{18}$$

- 100% used a procedure to answer
  - 71% used a procedure to justify a procedure
  - 0.5% used estimation
  - 22.5% said they didn't know why they were right.

CLASHING IDEOLOGIES

- Radical differences between RME and English education system in terms of :
- Student and teacher expectations about the nature of mathematics and mathematics classrooms

As seen in our

curriculum

textbook resources

classroom cultures

assessment systems

accountability structures

'TEACHING TO THE TEST'

6 Here is a list of numbers.

11 8 11 14 11 15 13 14

(a) Find the mode.

(a)..... [1]

(b) Find the range.

(b)..... [1]

(c) Find the median.

- Under RME assessment problems should be:
- Accessible and worth solving
- Unfamiliar, giving opportunities for students to formulate their own constructions
- Facilitate ownership and decision making on the part of the student

(Van den Heuvel-Panhuizen, 2005)

RME TRIALS IN ENGLAND

Trial 1 – 2004 - 2007

Over 400 project students aged 11- 14 across 12 schools

Project teachers taught using the 'Mathematics in Context' textbook series developed by the University of Wisconsin in collaboration with the Freudenthal Institute

Project teachers attended 6 days training per year, supported by Manchester Metropolitan University and the Freudenthal Institute

TRIAL 1 - OUTCOMES

- Similar performance for project and control students on traditional examination questions

- Performance on unfamiliar type problems was significantly better for project students compared with control (36% correct v 17% correct in the case of 'lower attainment' range)

- 'Evidence that project pupils' approach to problem solving changed and this influenced how they understood the mathematics'

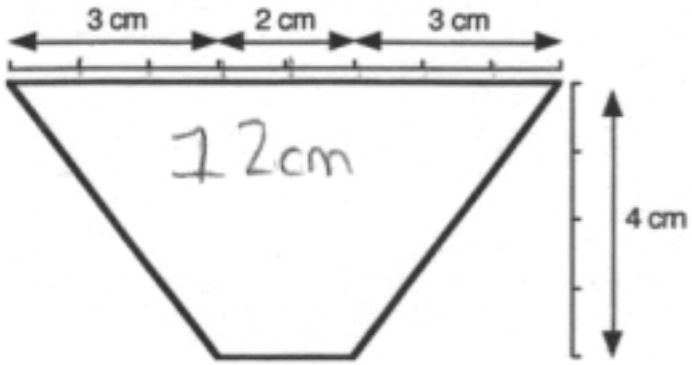
(Searle and Barmby, 2012)

TRIAL 1 – A DIFFERENT APPROACH TO SOLVING PROBLEMS

Questions 1 *continued*

(b) Find the area of the shape shown below.

Show carefully how you worked it out.



I got this because 3x2x3x4=48 and the 1 divided by 4 because there a 4 numbers.

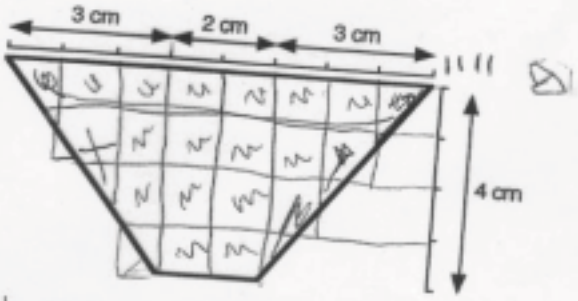
TRIAL 1 – A DIFFERENT APPROACH TO SOLVING PROBLEMS

Questions 1 *continued*

(b) Find the area of the shape shown below.

Show carefully how you worked it out.

4½



15 whole squares.

I divided the shape into squares and counted how many whole squares there was there was 15. I then added pieces to other pieces to make them whole and I got 4½ I added this to 15 so it was the aread of 19½

TRIAL 1 – A DIFFERENT NOTION OF PROGRESS

- A shift from ‘doing something with the numbers’ to ‘making sense of the problem

- A recognition of progress in understanding the concept of area as exemplified by the use of a ‘model of’ the situation and how this develops into a ‘model for’

(Streefland 1985, 1993)

- An appreciation of the iceberg model and the ‘landscape of learning’

(Webb et al 2008, Fosnot & Dolk, 2002)

RME – a different view of progress

The formal

'Model for'

'Model of'

Context

RME TRIALS IN ENGLAND

Trial 2 – 2007 - 2010

Smaller scale, around 240 project students aged 14 – 16, across 10 schools

Project teachers taught using the 'Making Sense of Maths' resources developed at Manchester Metropolitan University in collaboration with the Freudenthal Institute

Project teachers attended 6 x 2 hour twilight training per year, on a **voluntary** basis, supported by Manchester Metropolitan University.

TRIAL 2 - OUTCOMES

- Similar performance for project and control students in National GCSE examinations.
- Performance on unfamiliar type problems was significantly better for project students compared with control in the middle to lower attainment range
- Publication by Hodder Education of ‘Making Sense of Maths’ as a textbook series.

RME TRIALS IN ENGLAND

Trial 3: 2012 - 2016

Working with students aged 16 and over who have **not** gained the National GCSE examination to an acceptable standard.

Success rates for these re-sit students are traditionally very low (In 2012-2013 only 9.3% of students went onto improve their GCSE grade.)

(Department for Education, 2014)

Very short intervention, 12 hours on Number, 9 hours on Algebra, out of their 9 month resit course.

TRIAL 3 - OUTCOMES

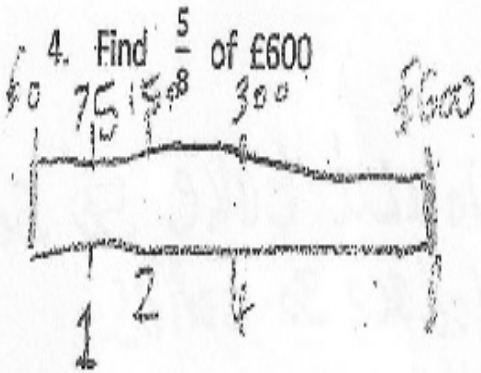
- Small but significant gains for the project group on Number
- For some students using a variety of contexts which led to use of the bar model helped them to see connections across elements of the curriculum
- Some students used the bar model as an algorithmic strategy, rather than as a ‘model for ‘ making sense of a problem

PRE AND POST-TEST APPROACHES TO FINDING OF $\frac{5}{8}$ OF £600

4. Find $\frac{5}{8}$ of £600

£450

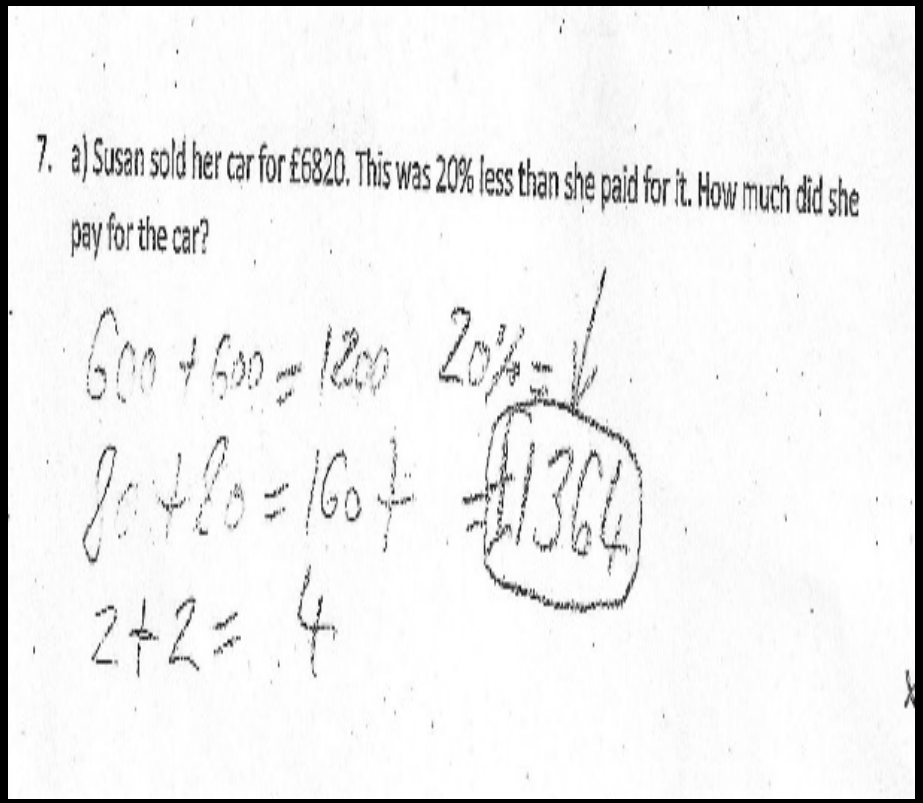
4. Find $\frac{5}{8}$ of £600

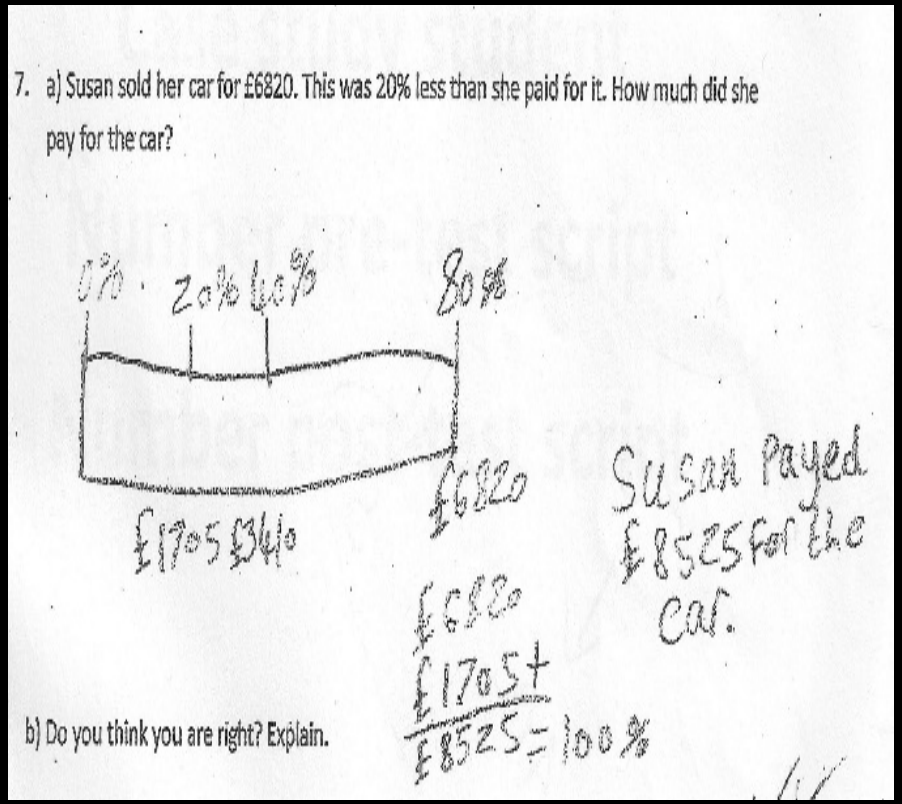


£300 + £75 = £375

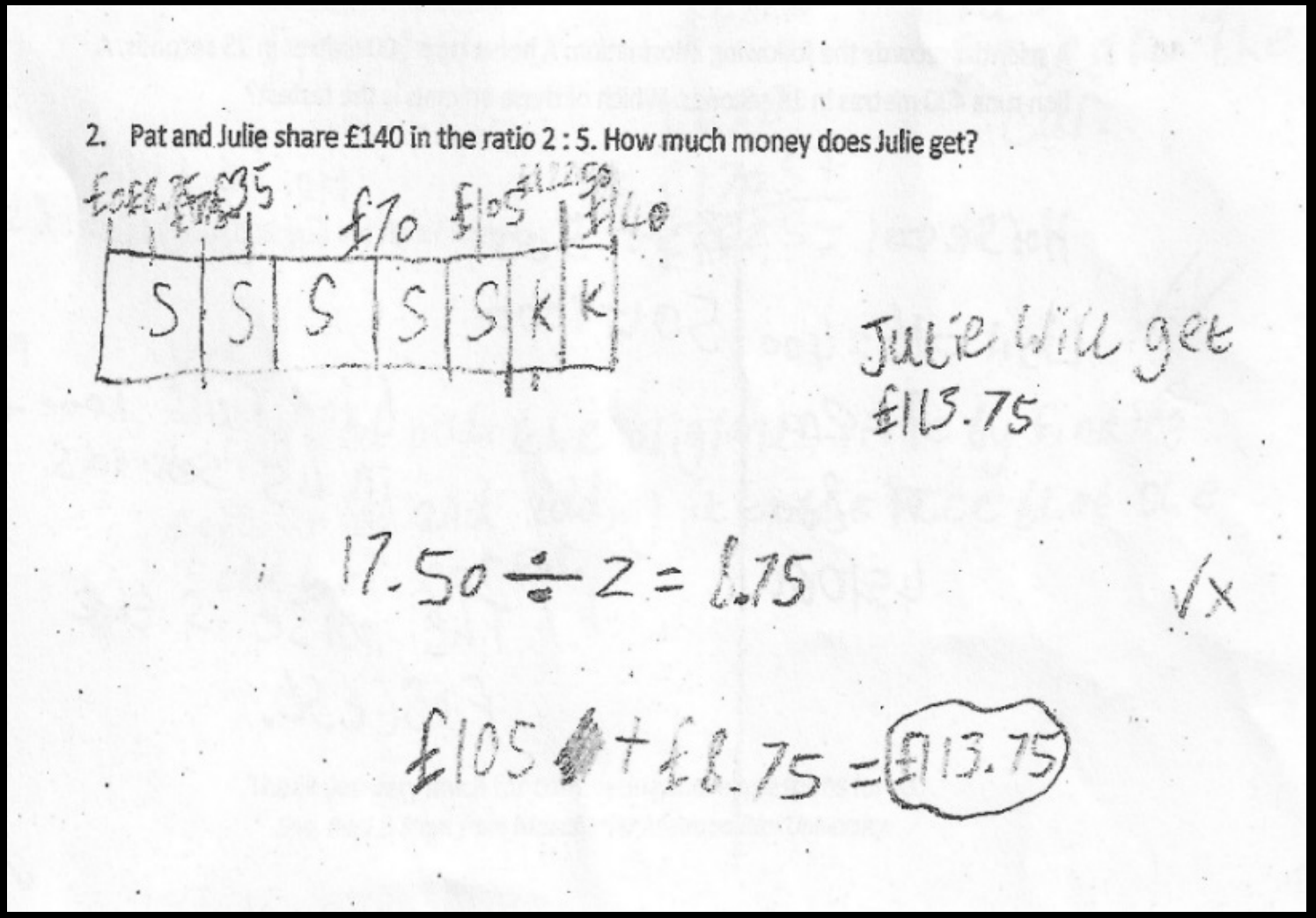
$\frac{5}{8}$ of £600 = £375

PRE AND POST-TEST APPROACHES TO FINDING SUSAN'S ORIGINAL CAR PRICE





PRE AND POST-TEST APPROACHES TO SHARING £140 IN THE RATIO 2 : 5



TEACHER IMPACT

- ‘The students show much more confidence….much less of a correct/ incorrect environment compared to a traditional approach and so the students are not frightened to ‘have a go’ as much

- ‘They stopped asking ‘What is the point in this?’ They stopped saying ‘can we do something different?’ I stopped replying ‘you have to do it because you have an exam in it’ Energy levels were higher in the room, a lot more discussion took place

TEACHER IMPACT

- ‘the underlying understanding being really important, so I’m pulled in two ways….I really like what you do and buy into it, and the other side of me is saying’dam with this group, I’ve still got to this, this and this, and when am I going to do it?’

RME BASED PROJECTS IN THE CAYMAN ISLANDS

- 2011, Frank Eade from MMU becomes Mathematics Advisor for the Cayman Islands

- Primary students in the Cayman Islands were expected to learn rules given to them by their teachers, many were behind, participated little in class and constantly asked for help, afraid to take mathematical risks

- Very little use of context, models or imagery. Even the context of money was not well understood.

RME BASED PROJECTS IN THE CAYMAN ISLANDS

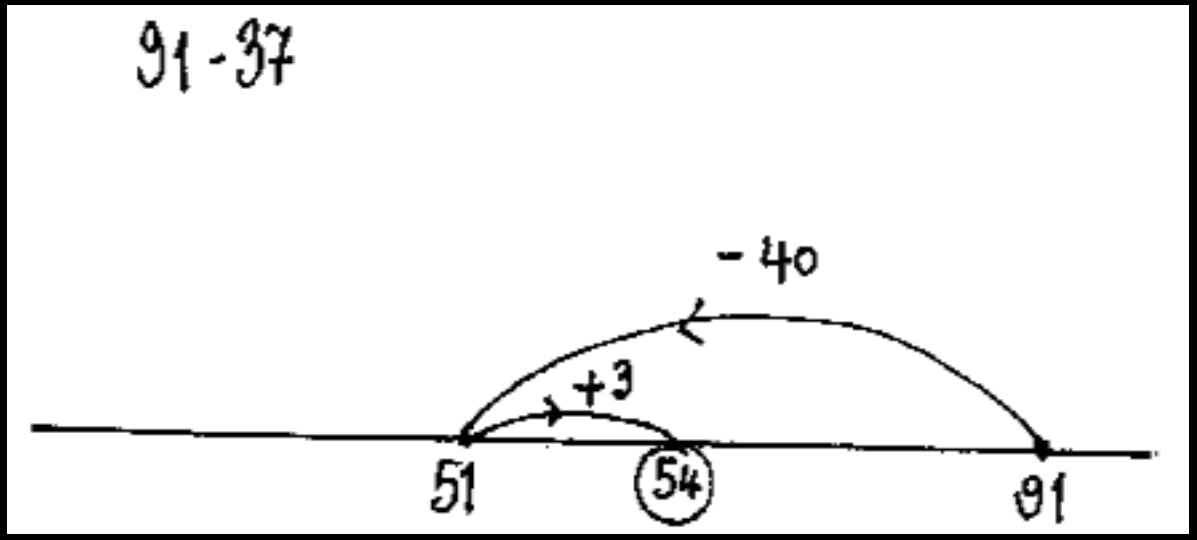
- Introduced 'Mathematics Recovery' training (Wright et al, 2014). Extensive use of models such 10-frames, the 100 bead bar and arrays… but little use of contexts

- A group of Primary teachers were trained in use of the empty number line and how to use contexts as a point of entry

CAYMAN ISLANDS - OUTCOMES

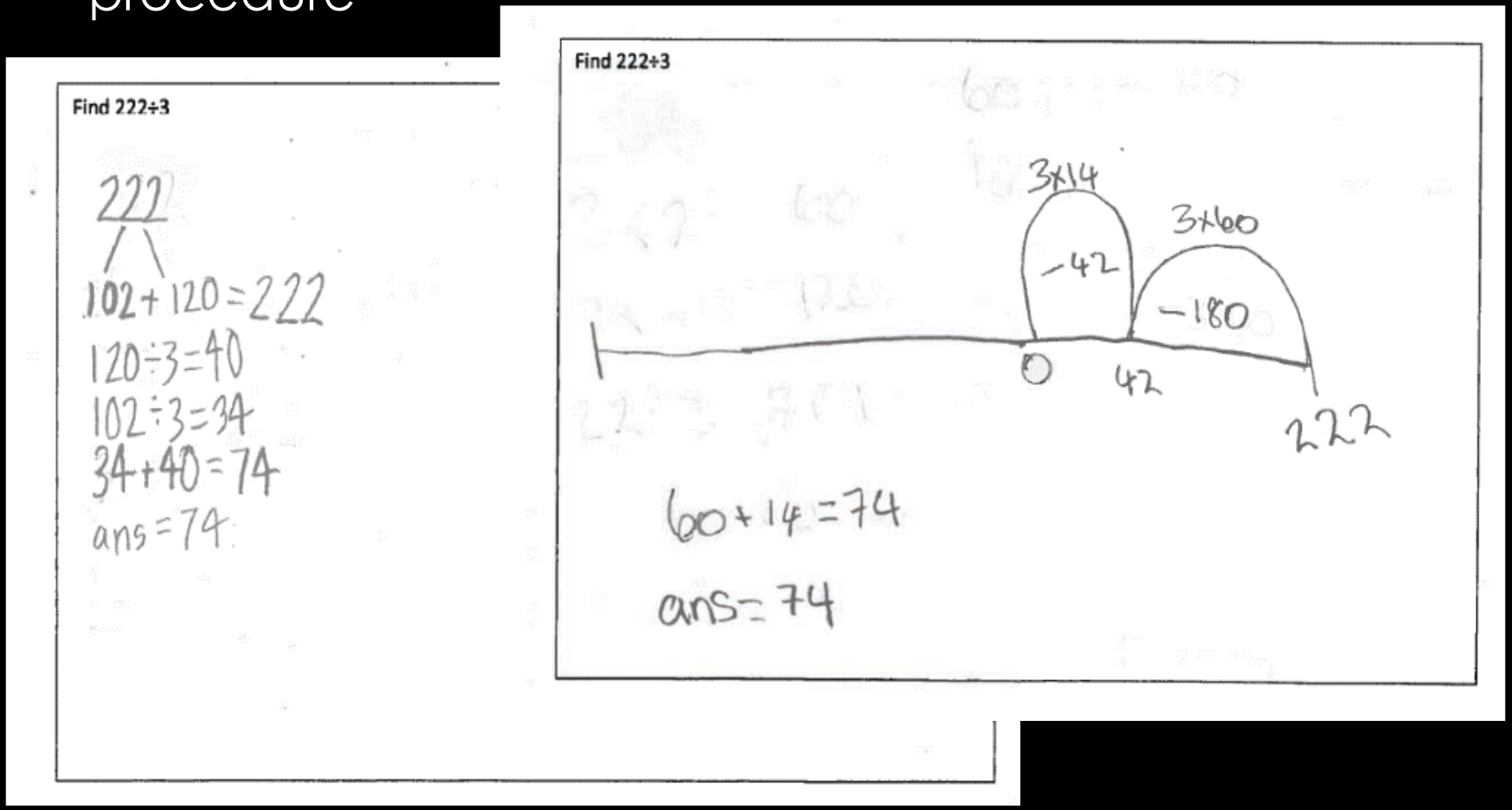
- Increased awareness in students of how calculations like 91 – 37 can be represented on a number line

(In 2011, 16% of Year 6 could explain compared with 46% in 2013.)



CAYMAN ISLANDS - OUTCOMES

- Increased use of a range of informal strategies for division problems like 222 ÷ 3, rather than following a procedure



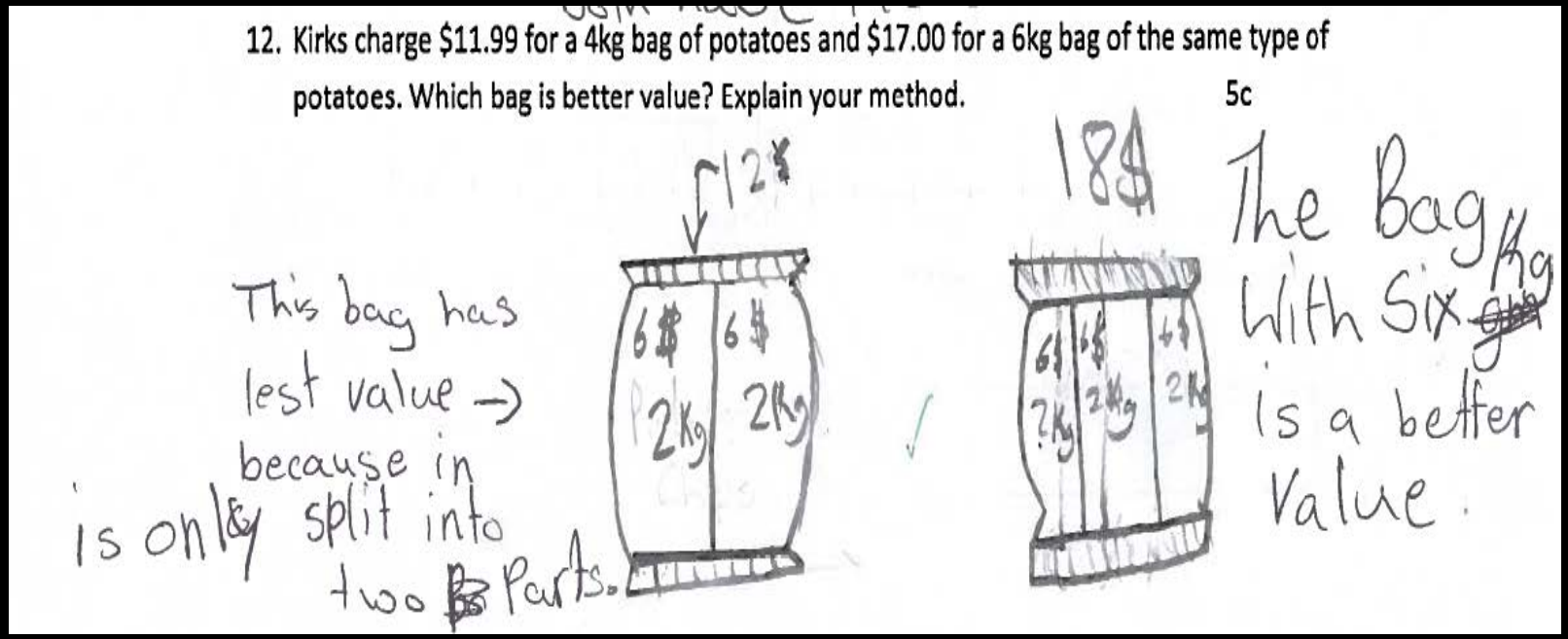
CAYMAN ISLANDS - OUTCOMES

- At Secondary age group, under guidance from Frank, teachers began trialling the 'Mathematics in Context' textbooks.
- Pre and post-test mean scores for a 13 question test are shown below

	Pre-test mean	Post-test mean
Set 1	44%	45%
Set 2	14%	26%
Set 3	5%	15%

CAYMAN ISLANDS - OUTCOMES

- Students showed initiative using a range of solution strategies:

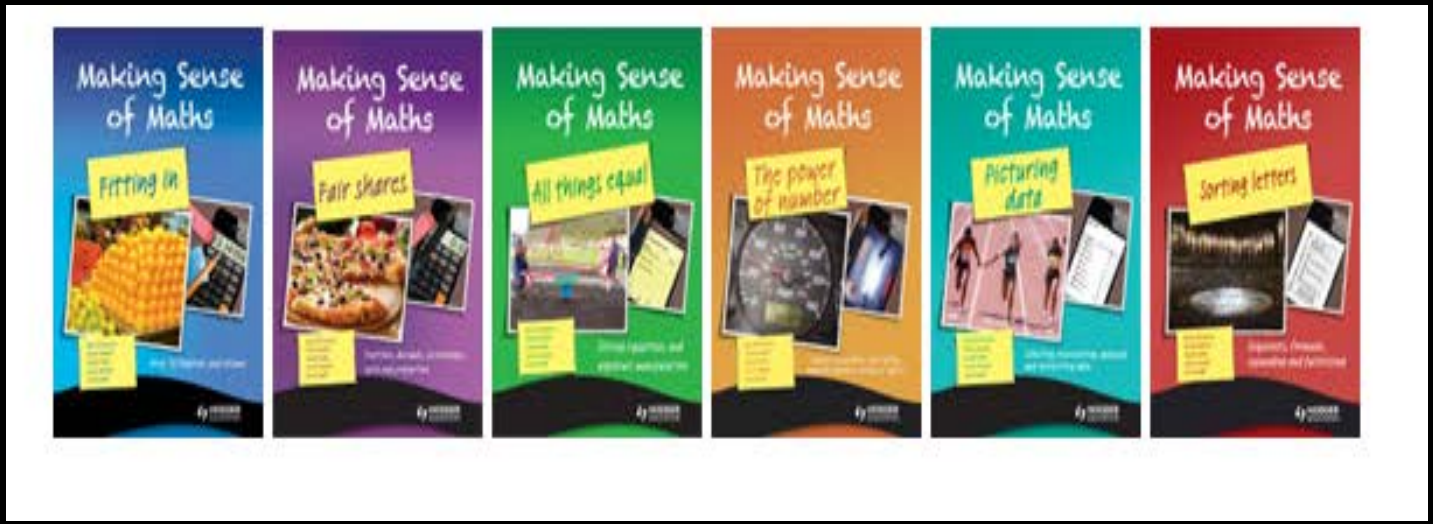


CHINKS OF LIGHT

- Where teachers receive substantial training in the use of RME, they experience significant shifts not only in the way they see mathematics but in the way they operate within their classrooms
- The latest version of an English curriculum does stress the need to develop mathematical reasoning and the ability to problem solve as well as procedural fluency
- National public examinations in England to include more problem solving with open ended questions set in real life contexts

(OCR, 2014)

‘Making Sense of Maths’ series published by Hodder



EXAMPLES WE HAVE TRIALLED…

KU LEUVEN

The influence of RME on Belgian school mathematics

ICME 13, Hamburg, 24-31 July 2016

Thematic Afternoon: European Didactic Traditions - Netherlands

Dirk De Bock, KU Leuven (Belgium)

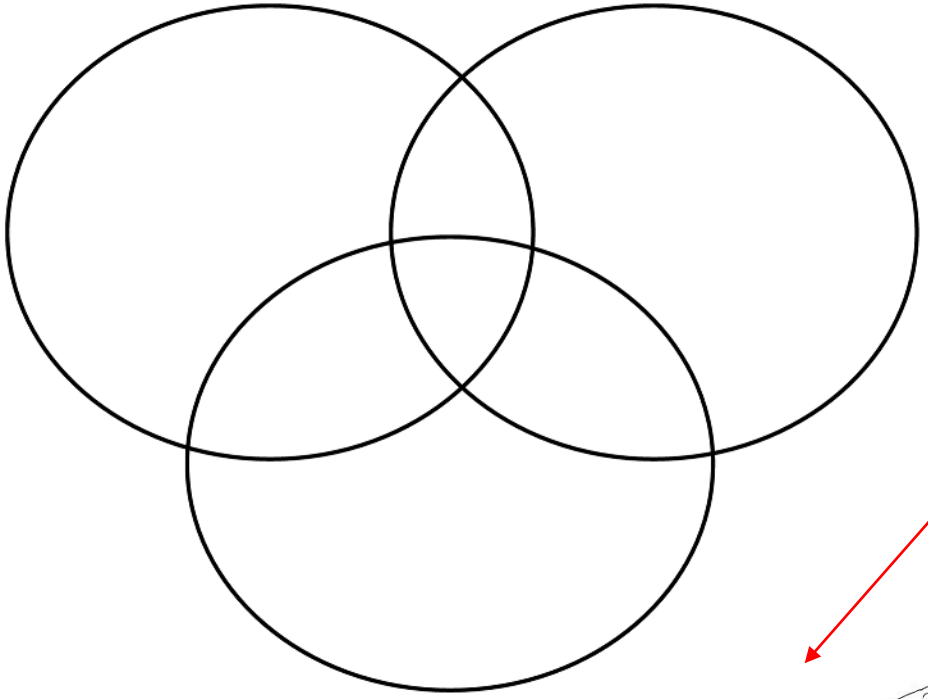
Papy and Freudenthal…

~~H~~ HF

Mechanistic, Structural, Realistic

M

Belgium/
the Netherlands/
…
≤ 1950s



R

the Netherlands/
≥ 1970s

S Belgium/…/
1960s-1970s

KU LEUVEN

In search for alternatives…

Raf Feys

1982

When evaluating the renewed mathematics education, we should not only compare with the old mathematics, but also with alternatives like the ones that are, e.g., developed in the Netherlands by Wiskobas. We need the courage to examine the alternatives thoroughly. (…) We opt for an alternative reform along the lines of the Wiskobas approach of the IOWO, complemented, however, with a strong emphasis on the social-societal aspect of mathematical world orientation

KU LEUVEN

Mathematics programs of the late 1990s

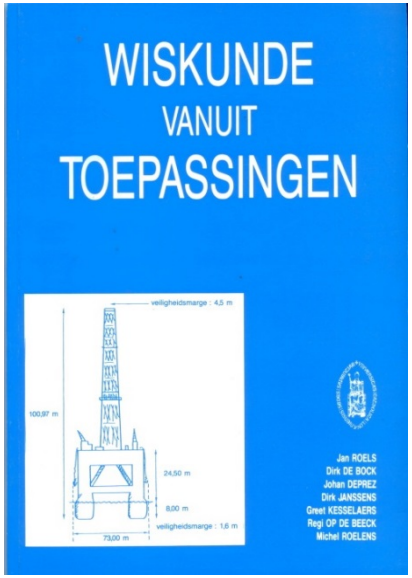
Mathematics in primary school should focus on mathematizing reality. It is therefore necessary to set mathematics education into a natural context.

Children learn to describe situations derived from their own living environment in the language of mathematics.

Mathematics starts from real problems, problems that are experienced as ‘real’ by the pupils themselves.

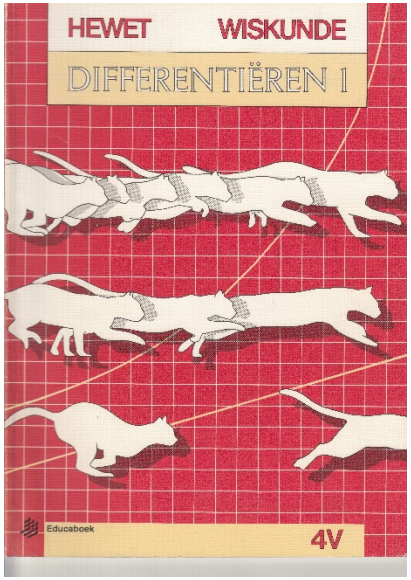
At the secondary level

Since 1984



1990

Strong inspiration from the HEWET cahiers (de Lange and Kindt), e.g. the idea of “conceptual mathematization”



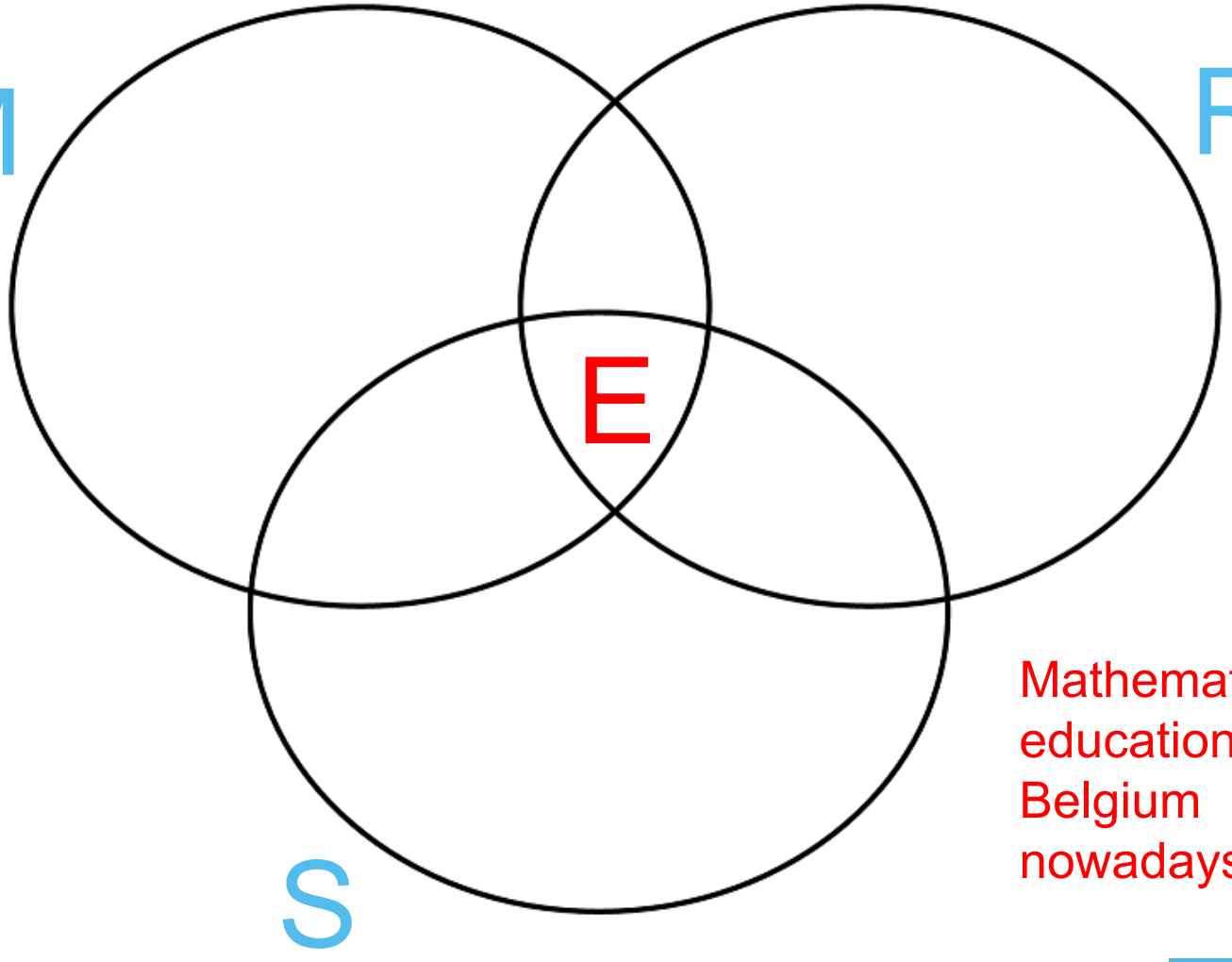
1984

KU LEUVEN

Some topics: A true metamorphosis!

- Exponential and logarithmic functions:
  models for exponential (or cumulative) growth
- Trigonometry: models for periodic phenomena
- Matrices: modelling with “blocks of numbers”
- Derivatives and integrals

An Eclectic approach?



Mathematics education in Belgium nowadays?

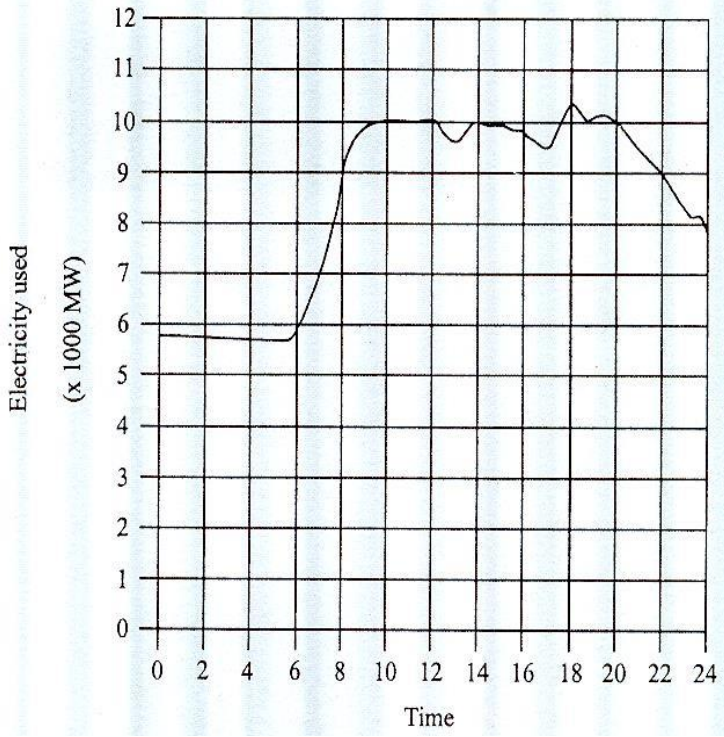
Realistic Mathematics Education
Some remarks by a critical friend

Cyril Julie
University of the Western Cape, South Africa

RME has demonstrated itself as a viable approach for incorporating some of its ideas in the state-mandated school mathematics curriculum in South Africa

ELECTRICITY

Read the following extract from a local newspaper article and use the information together with the graph shown below to answer the questions that follow.



- Use the graph to describe how the electricity consumption changed over the 24 hour period.
- From the graph, which time of the day is the electricity consumption the highest? Why do you think so?
- Read the newspaper article above. When will be the cheapest tariff for electricity?
- Explain the difference between the electricity consumption from 06:00 to 08:00 and that from 10:00 to 12:00 on the graph.
- Do you think that it is practically possible for a household to lower their electricity payments with the new tariff structure? Why or why not?
- Use the slide given below to determine during which hour of the day the greatest change in electricity consumption occurred.

The broader South African school mathematics context

High-stakes mathematics examination at the end of 12-13 years schooling

Solve for x and y simultaneously:

$$y + 1 = 2x$$
$$x^2 - xy + y^2 = 7$$

Given the quadratic sequence: $-1\ ;-7\ ;-11\ ;p\ ;\ \ldots$

3.1.1 Write down the value of p.

3.1.2 Determine the n^{th} term of the sequence.

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms.

Table 11.1: Overall achievement rates in Mathematics

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2012	225 874	121 970	54.0	80 716	35.7
2013	241 509	142 666	59.1	97 790	40.5
2014	225 458	120 523	53.5	79 050	35.1
2015	263 903	129 481	49.1	84 297	31.9

Improvement of these results a priority for schools, parents, politicians and higher education institutions.

Teacher: When I taught the stuff they could do it. When I saw the results I wonder whether I was teaching them.

Learner: When we did the work in class I understood and could do it, but in the examination it was as if I went blank

RME is low on incorporation of strategies and tactics to develop procedural and other fluencies to address what I call the “forget problem”

Operating with powers (I)

$a \times a \times a \times b \times b \times c = a^3 \times b^2 \times c^1 = a^3b^2c$

The **exponents** of *a, b,* and *c* are 3, 2 and 1.
(*note*: the exponent 1 is mostly not written)

With the exponents 3, 2 and 1 and the letters *a, b* and *c* also can be made other products, for example: ab^3c^2.
There are six different products that one can make using ***a, b, c*** and the exponents 1, 2, 3 .

- Write the other four products.
- Multiply the six products to each other.
  The result can be written in the form $a^{\cdots} b^{\cdots} c^{\cdots}$
  Which exponents do you get?

Positive Algebra , Martin Kindt

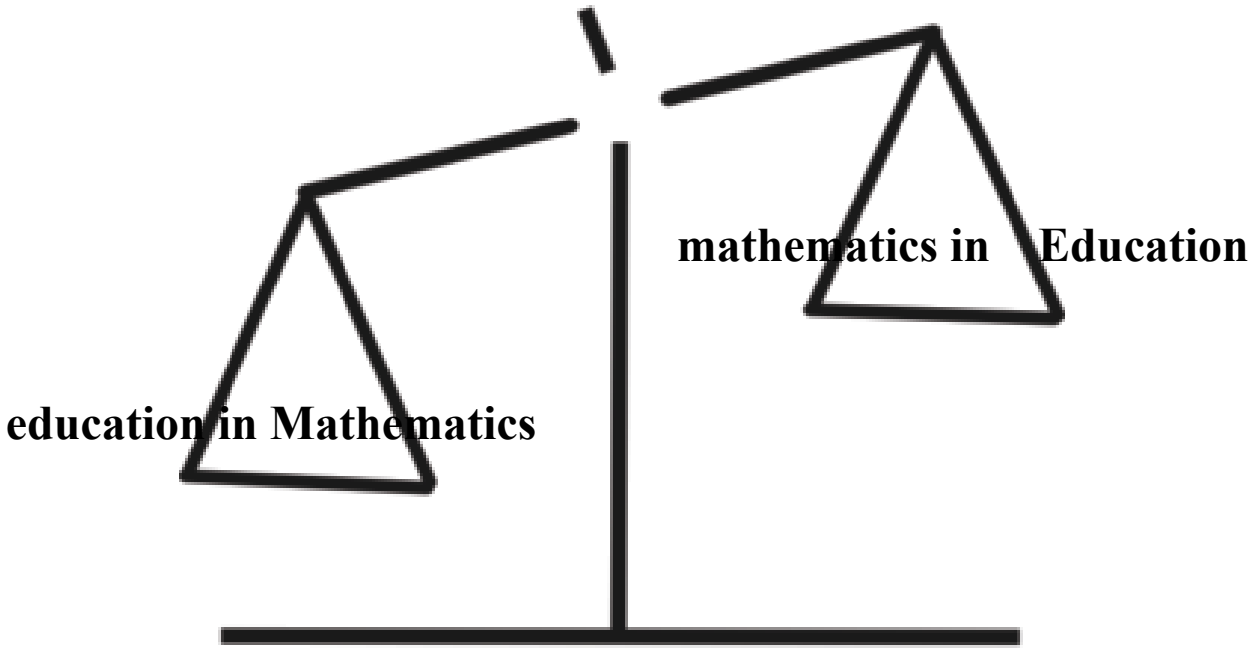
education in Mathematics vs mathematics in Education

education in Mathematics

Induction into the ways mathematicians work and the practice of doing mathematics (Seymour Papert's "Teaching children to be mathematician vs teaching them mathematics")

mathematics in Education

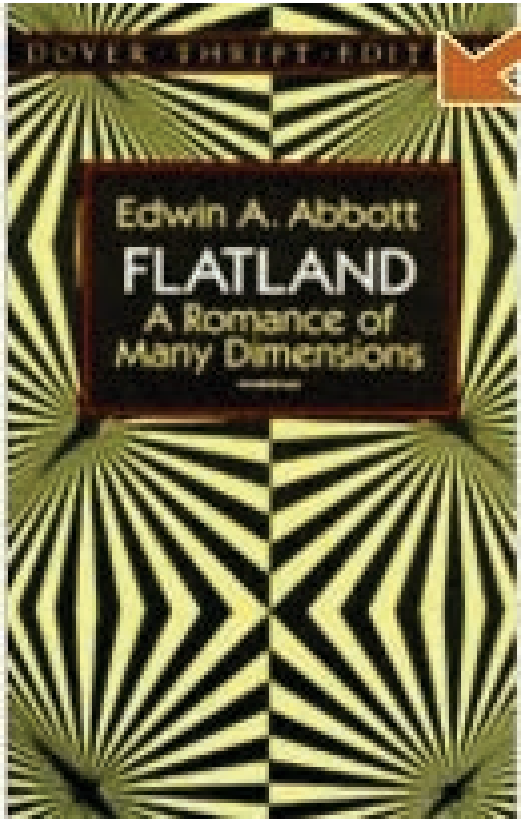
Development and nurturing of the global and responsible citizen who has a sense of the contributions Mathematics, as part of the totality of humanity's knowledge heritage, have made to the current human condition



RME low on “mathematics in Education”

Mathematics in general recreational matters low

Mathematics as metaphor to ‘story’ societal issues



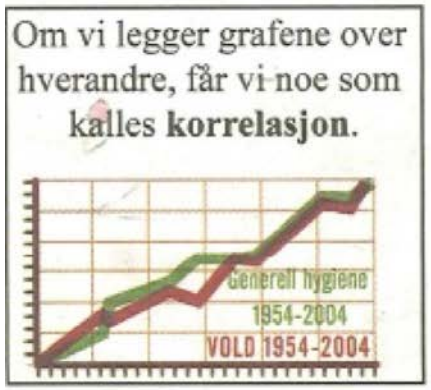
Mathematics as backdrop to crime (and possibly other) novels

Hacking, AI, encryption, decryption, large prime numbers, etc

Global Information

Violence increased since the 50's

Health increased since the 50's



Better health contributes towards the increase in violence or vice-versa

Currently a father is 25 years older than his child. In seven years he will be 5 times as old as his child.

Not strictly RME but Dutch

What is the father doing now?

Years	Child	Father
Now	P	P + 25
7 years later	P + 7	P + 32

$$5(P + 7) = P + 32$$
$$P = -\tfrac{3}{4} \text{ years}$$

THANK YOU