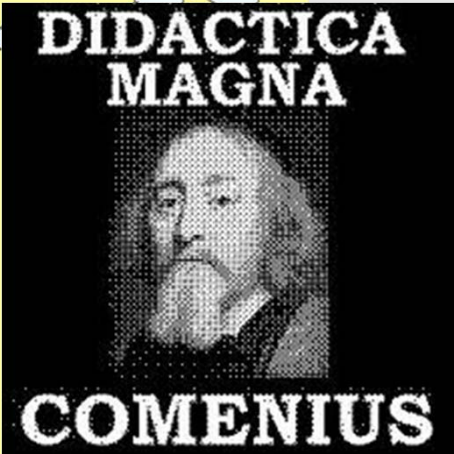


Thematic Afternoon: European Didactic Traditions



July 27, 2016, 15:00 – 18:30

Chair: Werner Blum, University of Kassel (Germany)



Map not to scale
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Four important *common features* of various European didactic traditions:

- **strong connection with mathematics and mathematicians**
- **key role of theory**
- **key role of design activities for learning and teaching environments**
- **firm basis on empirical research**

Other features (such as important role of proofs and proving or of interplay between mathematics and contexts) integrated

Four *selected cases*:

- **France** *responsible*: Michèle Artigue
- **Netherlands** *responsible*: Marja vandenHeuvel-Panhuysen
- **Italy** *responsible*: Maria Alessandra Mariotti
- **Germany** *responsible*: Rudolf Sträßer





Schedule today:

- **15 – 16 plenary**
- **16 – 16:30 coffee break**
- **16:30 – 18:30 four parallel groups according to cases**



First hour 15 – 16:

- **15 – 15:05 introduction and overview**
- **15:05 – 15:40 key features:**
 - **role of mathematics and mathematicians (Mariotti)**
 - **role of theory (Artigue)**
 - **role of design activities (vandenHeuvel)**
 - **role of empirical research (Sträßer)**
- **15:40 – 15:55 preview on second and third hour:
France/ Netherlands/ Italy/ Germany**
- **15:55 – 16 closing**



Role of Mathematics and Mathematicians in four European didactic traditions



Maria Alessandra Mariotti
Department of Information Engineering and Mathematics
University of Siena

Mathematicians and educational issues

We intend to highlight the role played by mathematicians in :

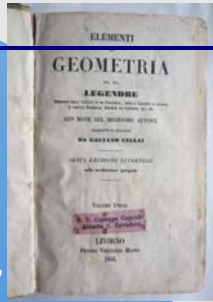
- focusing educational issues;
- addressing educational problems: designing curricula and designing teachers' formation, ...
- the development of mathematics education as a research field.

A diversity and similarity will be highlighted in order to make evident how a common European tradition emerges, beyond historic and cultural differences

The European tradition

The tradition of investment of mathematicians in educational issues

- At the very beginning
 - Crucial political events: French revolution, the Unification of the Italian kingdom, the ... leading to administrative reorganization of the Instructional system and direct involvement of professional mathematicians
- The turn of the twentieth century – the political engagement in educational issue
 - In France, in Italy, In the Netherlands ... in Germany :
 - The influence of Klein
 - The start of an international net
- The period of the New Math reform – engagement in innovation of curriculum
 - Innovation of the curricula
- The development of Math Education as a scientific discipline



dition

Textbooks:

Lagrange, Laplace, Monge, ...Geometry books
Legendre, Amiot, Serret



Luigi Cremona, senator of the Kingdom, Minister, author of books ...

The Committee of public instruction: Condorcet

beginning

political events: French revolution
unification of the Italian kingdom



Debate about how to run the preparation for University: Felix Klein

- in Germany
- ... the exception. in Germany professional mathematicians (beginning) not play a formal role. mathematicians were interested in mathematics education

f the twe

Reform 1902:
reinforcement of
scientific education:
Borel, Darboux

Mathematicians involved in
Committees: 1906 legal status
of teachers, but only in 1914,
math teachers gain an equal
status:
Enriquez, Castelnuovo

of the twentieth
engagement in edu

- in France,
- in Italy,
- in Germany
- the influence of Klein
- The sta of

The evolution of the
previous debate:
Functional thinking
(Funktionales
Denken)

Influence of Klein

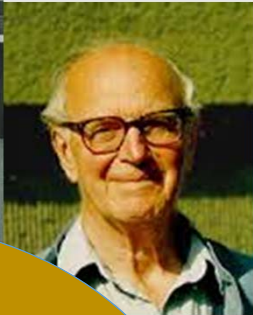
The influence of Klein

- In Italy:
 - Early translation of Klein's ideas into the Italian curriculum, highlighting the debate and tension between traditional and modern pedagogical stances
 - The creation of University courses for the formation of teachers, titled *Matematiche elementari da un punto di vista superiore*
 - Her discussion group became the Mathematics Working Group, 1936; did not get much response from mathematicians

Later ...

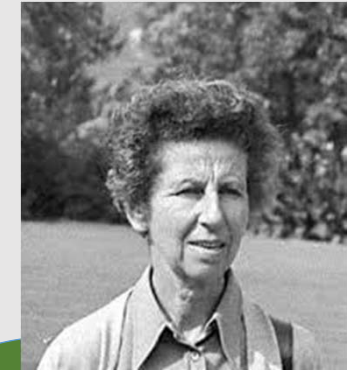
ICMI

The period of the new Math reform



Commission
Lichnerowicz

...
Books: Choquet
& Dieudonné



Government
Commission for the
Modernization of
Math curriculum
(Professional
Mathematicians and
teachers):
Freudenthal

Innovation of
Mathematics
Instruction

UMI-Elaboration
of new curriculum
offered to the
Ministry
Innovation in the
school practice:
Emma
Castelnuovo



From innovation to research

Analogies and differences in the four countries

France and Italy

From innovation to research: France and Italy

IREMs
1969 Paris, Lyon and Strasbourg ...

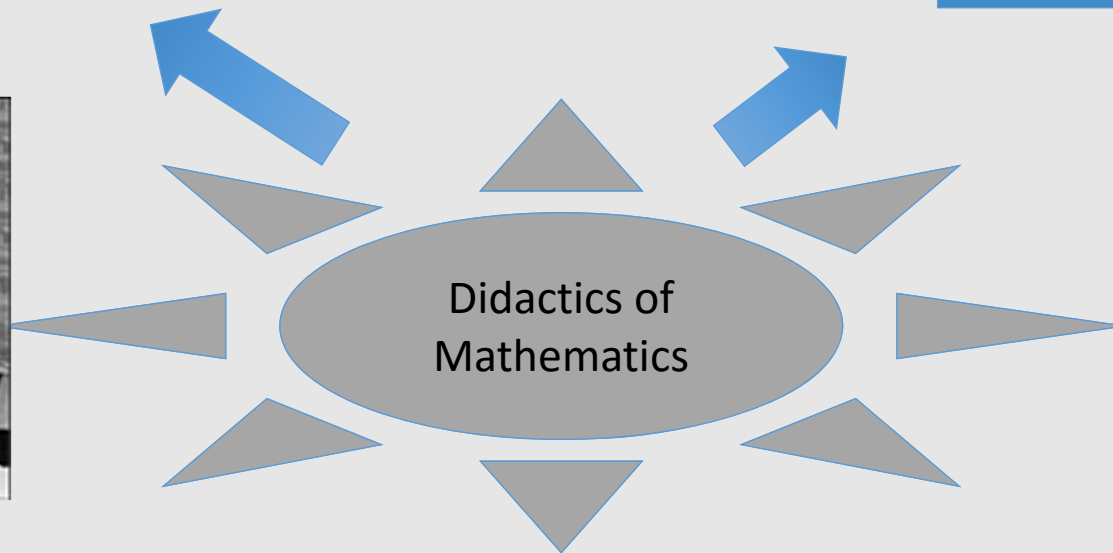
1974
CNR- Research Groups



André Revuz



Giovanni Prodi



The development of Didactics of Maths as a scientific discipline

Key features in France and Italy

- The first generation of didacticians was for the main part made of academics affiliated to Mathematics Departments
- The specific structure of the IREMs and of the Italian Research Groups, mixing professional mathematicians, teachers and didacticians.

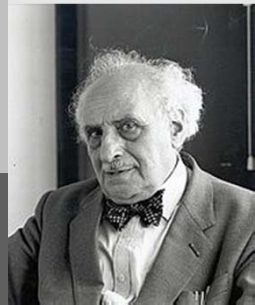
From innovation to research

Analogies and differences in the four countries

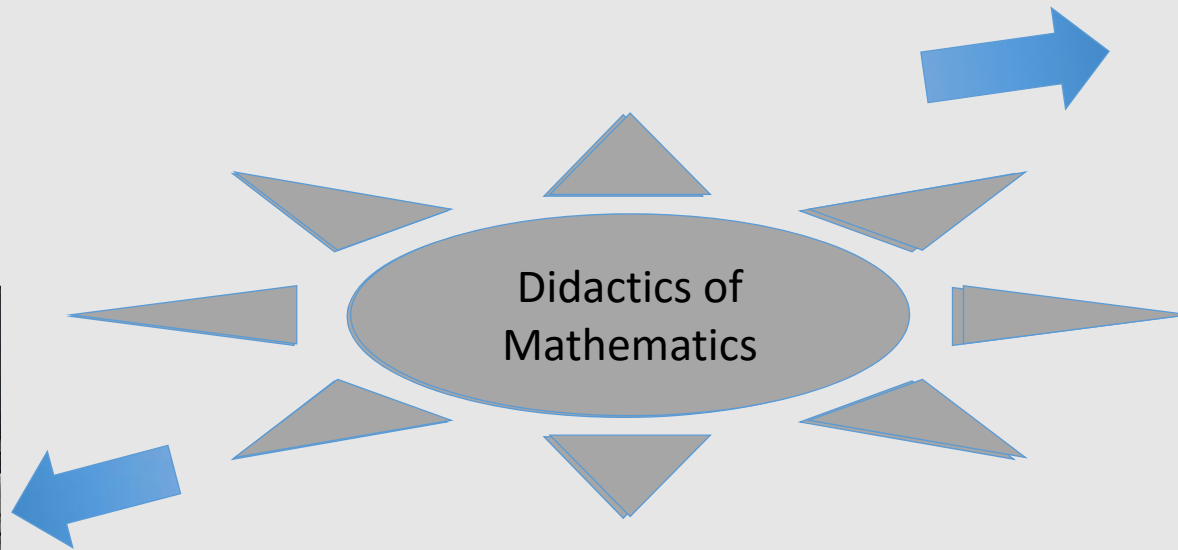
Germany and The Netherlands

Differently from Italy and France the emergence of Didactics of Mathematics as a scientific discipline does not see mathematicians directly involved

From innovation to research: Germany and The Netherlands



IOWO (1971) →
Freudenthal Institute
(1991)



Journal -JMD
IDM- Bielefeld



Mathematics in the didactic tradition

A common essential feature

strong epistemological concern

- **The case of proof as a theme of research**

The dialectic of
validation ...
Balacheff , Arsac,
Legrand, Duval,
Durand-Guerrier

Argumentation
and proof in
the classroom
Boero,
Mariotti, ...

Producing proofs
Reiss-group

To conclude

A diverse and heterogeneous experience emerging from different histories and educational cultures.

- The key role played by some of the mathematicians in the emergence of Didactics of Mathematics as a scientific discipline.
- But also sometimes the existence of tension between mathematicians and didacticians

The legacy undertaken by the European society of research in mathematics education (ERME) to build up an European community of research ... and the engagement in consolidating the relationship with the Mathematicians community.



Theoretical developments and their roles in four European didactic traditions



Michèle Artigue

LDAR, Université Paris Diderot – Paris 7

Theories in mathematics education

A word used to denote a diversity of objects:

- from very local constructs and distinctions to systems of concepts organized in coherent structures;
- from constructions mostly 'home-grown' to constructions mostly 'borrowed and adapted' from other fields;
- from constructions having emerged decades ago to quite recent elaborations.

Objects mediating research practices in different ways, according to the role they are mainly given: tools for understanding / tools for action.

A diversity also evident in the four didactic traditions on which this thematic afternoon focuses, despite their geographical proximity.



The French tradition

A particular importance attached to the building and consolidation of theoretical foundations for the field.

The ambition to develop the didactics of mathematics as a genuine scientific field of research, having its specific 'problématiques', theoretical tools and methodologies, and whose main object is the understanding of teaching and learning processes taking place in all kinds of didactic institutions.

Three main pillars



The Theory of
Conceptual Fields

The Theory of
Didactical Situations



The Theory of
Didactical
Transposition



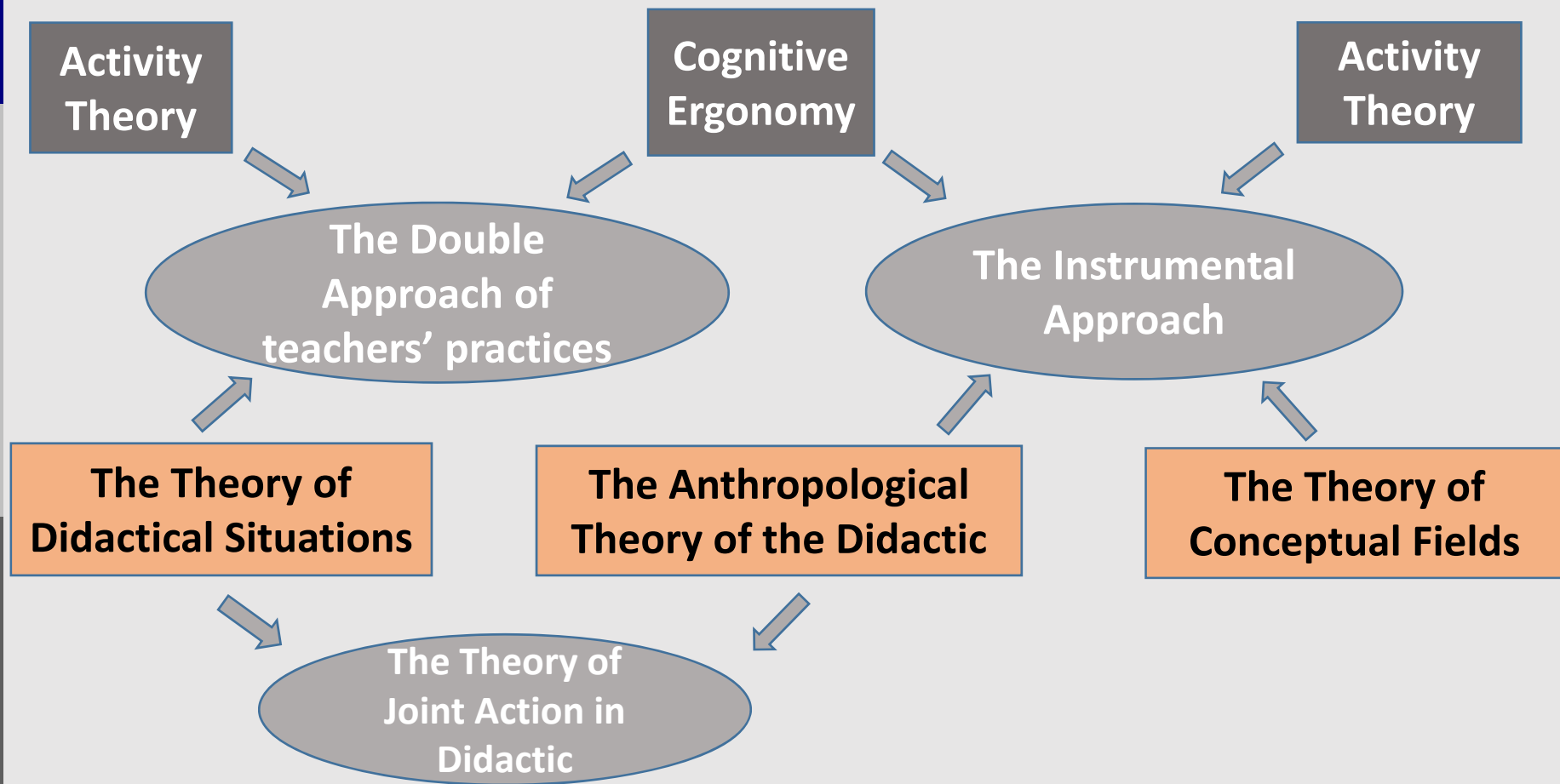
The Anthropological
Theory of the
Didactic



Some essential characteristics

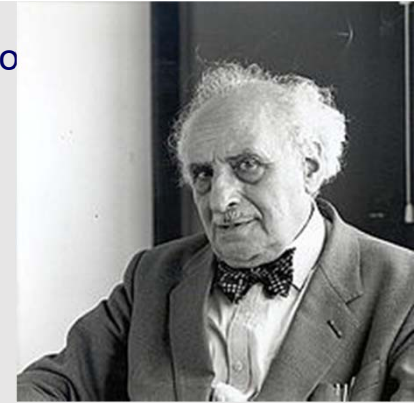
- A **systemic vision of the field**, in which basic objects are didactic situations and institutions that shape what students are given the possibility to learn or not.
- A **strong epistemological sensitivity** expressed through different constructs: conceptual fields, epistemological obstacles, fundamental situations, reference epistemological models...
- **Theories mainly conceived as tools for understanding**, through the identification of didactic phenomena, such as the phenomenon of didactic contract which has been incorporated in the theory itself.
- The privileged role given to methodologies giving access to the functioning of didactic systems (didactical engineering, institutional analyses...).

More recent theoretical developments





The Netherlands tradition

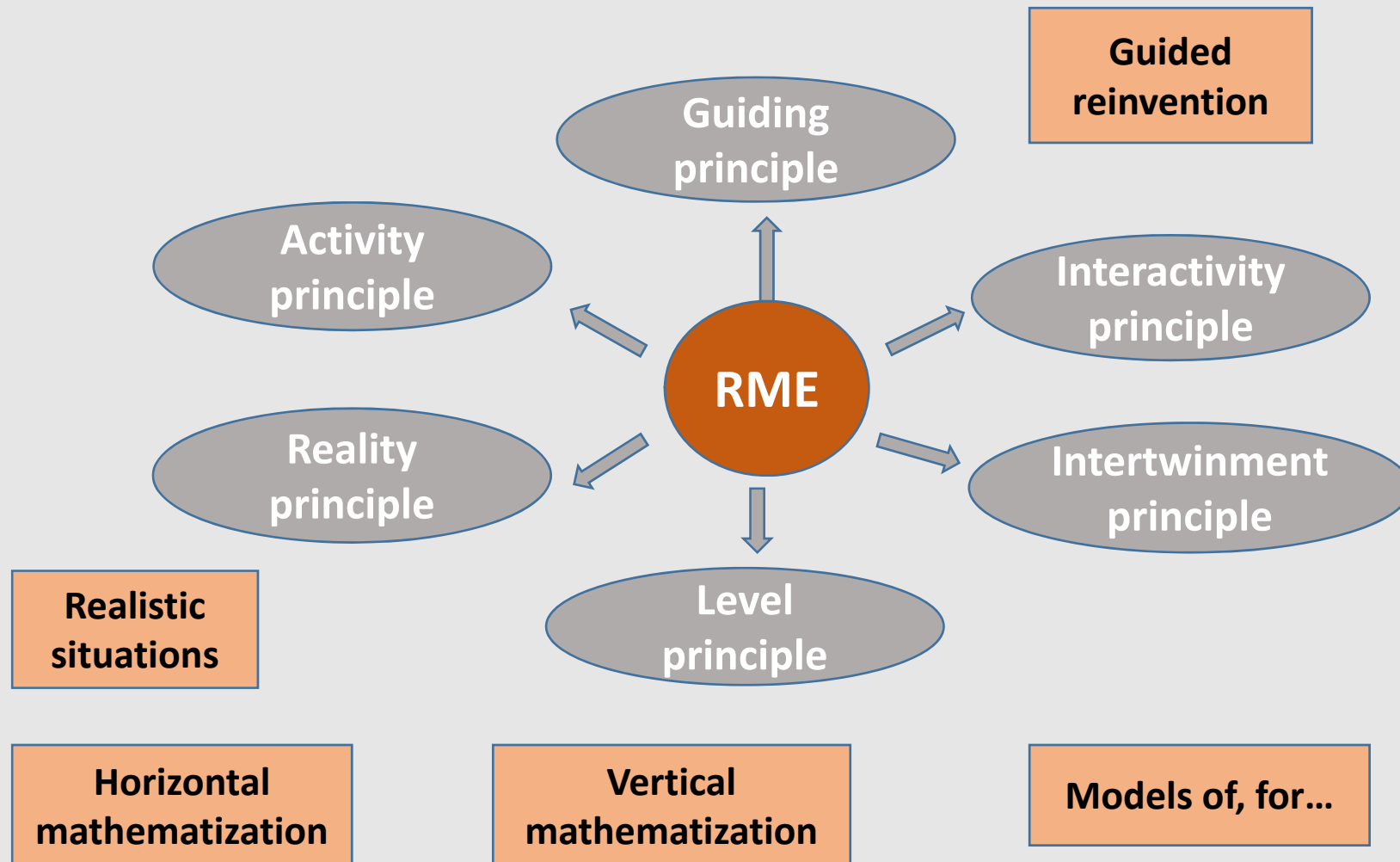


A long term theoretical development but a much more homogeneous landscape due to the preeminence of **Realistic Mathematics Education**.

At the source of RME, a deep epistemological reflection carried out by Hans Freudenthal, strongly oriented by its didactical goals and resulting in a **didactical phenomenology of mathematical structures**.

A theory oriented towards educational design, thus the fundamental role played by IOWO and the Wiskobas group around Treffers in its emergence and consolidation.

A theory organized around six general principles



A theoretical landscape progressively enriched

- By the refinement of the theory.
- By the development of **Local Instruction Theories** in line with RME principles on a diversity of mathematical topics.
- By the connections established with other approaches such as Socio-constructivism, or more recently the Instrumental approach or Embodied cognition.



The Italian tradition

A long term tradition of Action-research and collaborative work between mathematicians and teachers, but the more recent emergence of a theoretical expression of this tradition in terms of **Research for innovation**.

The resulting development of specific theoretical frameworks combining theoretical constructs from different sources, such as:

- the notion of **Field of Experience** due to Paolo Boero;
- the **theory of Semiotic Mediation** due to Mariolina Bartolini Bussi and Maria Alessandra Mariotti;
- the notion of **Semiotic Bundle** and the **Approach of Action, Production and Communication (APC)** due to Ferdinando Arzarello.



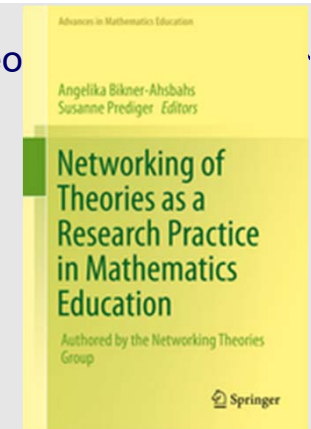
The German tradition

The ambition, early expressed by some scholars, to create the field of mathematics education as a scientific discipline.

The efforts made by H.-G. Steiner to establish an international debate on theories of mathematics education, and the underlying philosophies and epistemologies of mathematics.

Influential theoretical constructions such as the Interactionist approach of mathematics classrooms developed by Bauersfeld, Krummheuer, and Voigt, or the idea of modelling cycle.

However, a theoretical landscape not so easy to synthesize as researchers use and combine a diversity of theoretical constructs from different origins.



To conclude

A diverse and heterogeneous landscape shaped by different histories and educational cultures.

The efforts undertaken by the European society of research in mathematics education (ERME) to build connections through the 'networking between theories'.

The leading role played by researchers from these traditions in this enterprise.



Role of design of teaching and learning environments in four European Didactic Traditions



Marja van den Heuvel-Panhuizen
Freudenthal Institute, Faculty of Science
Freudenthal Group, Faculty of Social and Behavioural Sciences
Utrecht University

Design of mathematics education

- **can involve the design of**
 - Tasks
 - Lessons
 - Teaching sequences
 - Textbooks
 - Curricula
 - Assessments
 - ICT-based material
 - Programs for teacher education
- **can be done by**
 - Teachers
 - Teacher educators
 - Textbook authors
 - Curriculum developers
 - Assessment developers
 - ICT designers
 - Researchers/didacticians
- **can be done *ad hoc* or *research-based***
- **forms the meeting point of theory and practice, which through the design influence each other reciprocally**



The French tradition

Design (of mathematical tasks, situations or sequences of situations)

- based on theoretical frameworks underlying research
- an **essential component** of research work

This is reflected in

- the concept of **Didactical Engineering** in the **Theory of Didactical Situations** (*since early eighties*)
 - Epistemological and mathematical preliminary analyses
 - Search for situations which capture the epistemological essence of the mathematics to be learned
 - Optimization of the potential of these situations for students' learning
- the concept of **Study and Research Path** in the **Anthropological Theory of Didactics** (*last decade*)
 - The paradigm of “questioning the world”
 - The dialectics between *media* and *milieu*



Where?

- **Design activities naturally take place especially in the research actions developed in**
 - IREMs (Institute of research on maths teaching)
 - IFÉ (French Institute for Education)

Dissemination

- **Through various channels:** publications of the IREM network, journals for teachers, curricular resources, and some textbooks
- **But, up to recently, dissemination or up-scaling was only the focus of a few research projects; e.g.:**
 - ACE (Arithmetic and understanding at elementary school)
 - Second generation didactic engineering (Perrin-Glorian)



The German tradition

Till 1970/80s

Design for direct use in teaching and learning mathematics

Design activities took place in the context of Stoffdidaktik

- long tradition
- preoccupation with content
- course development for teaching mathematics; aim to make mathematics understandable
- evaluation was often left to incidental evidence

Empirical turn in 1970s

Design for studying effects of didactical variables

- **Studies on**
 - the effect of courses
 - general aspects of teaching
 - cognitive development of learners
- **Course development was less prominent**

Wittmann et al. counterbalanced this by defining the didactics of mathematics as a 'design science'



Current situation

Design for direct use in teaching and learning mathematics

- Course development is often left to experienced teachers, textbook authors, and government committees
- Innovative courses are published in teacher-oriented journals

Design for studying effects of didactical variables

- Only recently, researchers in the didactics of mathematics seriously embrace engineering in terms of a research on systematically controlled didactical variables



The Italian tradition

mid 1960s – mid 1980s

Concept-based didactical approach & Innovations in classrooms

- Main focus on epistemological analysis as base for didactical innovations
- The theoretical reflection on didactical suggestions and realization in school was weak
- The design approach pragmatic
- The work of E. Castelnuovo generated eloquent examples of this pragmatic approach towards innovations

mid 1980s – present

Overcoming pragmatic approach & Development research paradigm


- Innovations as research and not just actions in classrooms
- Development of teaching experiments aimed at validating specific curricular innovations, concerning both mathematical content and didactical methods
- The design of teaching and learning environments are at the same time the objective and the means of the experimentation



The Dutch tradition

Emphasis on design

- Making things, looking for pragmatic solutions and being creative and innovative are typical features of Dutch culture
- The design aspect of mathematics education is the most significant characteristic of the Dutch didactic tradition in the past half century
- The reform towards RME started (end of the 1960s) with *designing new education* as an alternative for mechanistic education



Practice & theory

- Initially, design activities were practice-oriented
- Theory development (RME) resulting from these activities
- Later, theory (RME) as a guide for designing education and design activities for further theory development

Implementation of the designs

- Strong infrastructure (conferences, journals, networks):
new ideas immediately used in teacher education
- Textbooks as important change agents

Multifaceted educational designs

- Contexts • Didactical models • Longitudinal teaching-learning trajectories • Textbook series • New programs and examinations • Mathematics events • ICT-based designs



Empirical research in four European didactic traditions



Rudolf Sträßer, University of Gießen



Empirical Research

**In various institutional settings and varying visibility
for all four cases (France, Germany, Italy, Netherlands)**

**Designing learning opportunities
in Mathematics was important**

This created the need for legitimizing these environments.

One way to legitimize:

Show the effectiveness

by means of empirical research

(whatever effectiveness means)

Empirical Research

Because of the complexity of the field Didactics of Mathematics,
a cause-effect research (\approx classical natural science research)
is difficult, if not impossible.

Partly as a „fall-out“ from the need of design of LE:

Variety of questions – aims – topics – research methods

- **statistical analysis** (tests / questionnaires)
 - **content analysis** (of curricula / textbooks) /
 - **classroom analysis** (by videographs / observation sheets)
 - **transcript analysis** (e.g. with concepts from linguistics)
- and triangulation and „mixed methods“**

Empirical Research in the four cases

is developing in different types / with different purposes

Large Scale like COACTIV

„The COACTIV study was designed to investigate teacher competence as a key determinant of instructional quality in mathematics.[R]esearch questions ...

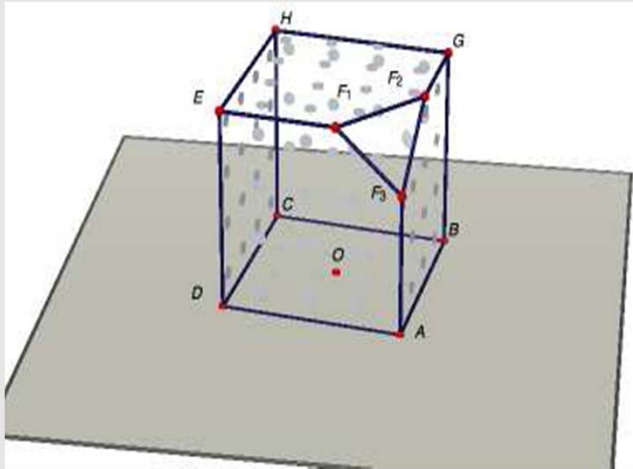
- ...teacher competence ... identified empirically
- ...teacher competence [as] influence [on] classroom practice

- ... effective outcomes
- Why c

data sources

<u>Teachers</u>	<u>Instruction</u>	<u>Students</u>
<ul style="list-style-type: none"> - Knowledge tests - Questionnaires <ul style="list-style-type: none"> - written - computer-based 	<ul style="list-style-type: none"> - Teacher survey - Student survey - Analysis of tasks set <ul style="list-style-type: none"> - for homework - in lessons - class tests 	<ul style="list-style-type: none"> - Achievement tests (PISA) - Questionnaires

Case study like MITHALAL on 3D-geometry



Question: „how space geometry situations could incite students to use axiomatic geometry?“

More specific: How to destabilise students' trust in iconic 3D-representations and motivate an axiomatic approach?

Framework:

DUVAL's „déconstruction dimensionnelle“ and the Theory of Didactical Situations („TDS“)

Methodology:

„qualitative approach to analyse the students dealing with“
[the reconstruction of a drawing showing a 3D-configuration].

Results:

In/with a specific software-environment (Cabri-3D) a 'milieu' can be created where dimensional deconstruction and reference to axioms is necessary.

Distinctions

■ Large scale versus medium/small scale

Large scale studies tend to make differences between a sample an argument while

Small/medium scale studies tend to make specificities of the „case“ an argument

■ Quantitative versus qualitative methods

Quantitative studies tend to use (sophisticated) statistical techniques to arrive at general 'laws'

while

Qualitative studies tend to use techniques from content analysis to better understand the phenomena (->heuristics)

Purposes 1

■ prescriptive versus descriptive

Prescriptive studies tend to show how things *should* be
while

Descriptive studies tend to give the best possible description
and understanding of the domain under study, *not to change* it.

■ developing versus illustrating theory

Experimental studies may be undertaken to
Develop, elaborate a theory, put it to a test
(*Italy, France, Netherlands*)

or

Illustrate an existing theory ('existence proof')
(*France, Germany, Italy, Netherlands*)

Purposes 2

■ action versus fundamental research

Action research (*Italy, Netherlands*) is deeply involved with the phenomena and persons under study;

main aim: improve actual teaching&learning

while

Fundamental research (*France, Germany, Netherlands*) tends to prioritize understanding of/from the phenomena under study;

major aim: improve the theoretical concepts

■ political interest versus development of science

Empirical research can be driven by political interest

(*e.g.: Germany*)

in addition to an interest in scientific progress and curriculum development.

Second hour 16:30 – 18:30:

- **the case of France**
- the case of the Netherlands
- the case of Italy
- the case of Germany





The French Didactic Tradition (1st hour)

Introduction:

L. Trouche & M. Artigue:
*The French tradition: Roots
and development*



**... with the voices of G. Brousseau, Y. Chevallard and
G. Vergnaud**



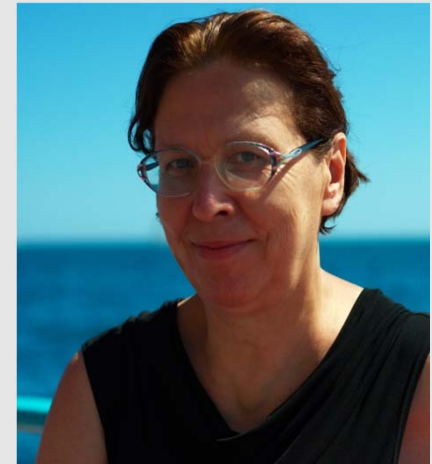
The French Didactic Tradition

(1st hour)

Two illustrative case studies:

V. Durand-Guerrier, A. Chesnais: *Educational research on axial symmetry in the French tradition*

M. Bosch, H. Chaachoua: *Research on school algebra in the French tradition*



Interaction with the audience



The French Didactic Tradition (2nd hour)

Introduction:

The French tradition in the World (animation by P. Gibel)

The vision of a critical friend:

C. Knipping (Germany):

*View of the French tradition,
especially through the lens of
validation and proof*





The French Didactic Tradition (2nd hour)

Examples of interaction:

M. Maschietto (Italy):

Didactic interactions between France and Italy – a personal journey



F. Chellougui (Tunisia):

Didactic interactions between France and African countries - the case of Tunisia





The French Didactic Tradition (2nd hour)

Examples of interaction:

A. Romo Vazquez, F. Pluvinage (Mexico):

Didactic interactions between France and Latin-America - the case of Mexico



Interaction with the audience

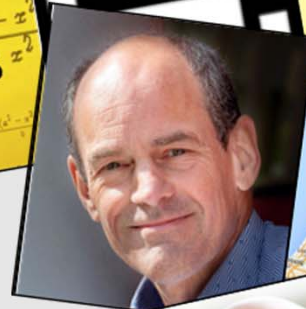
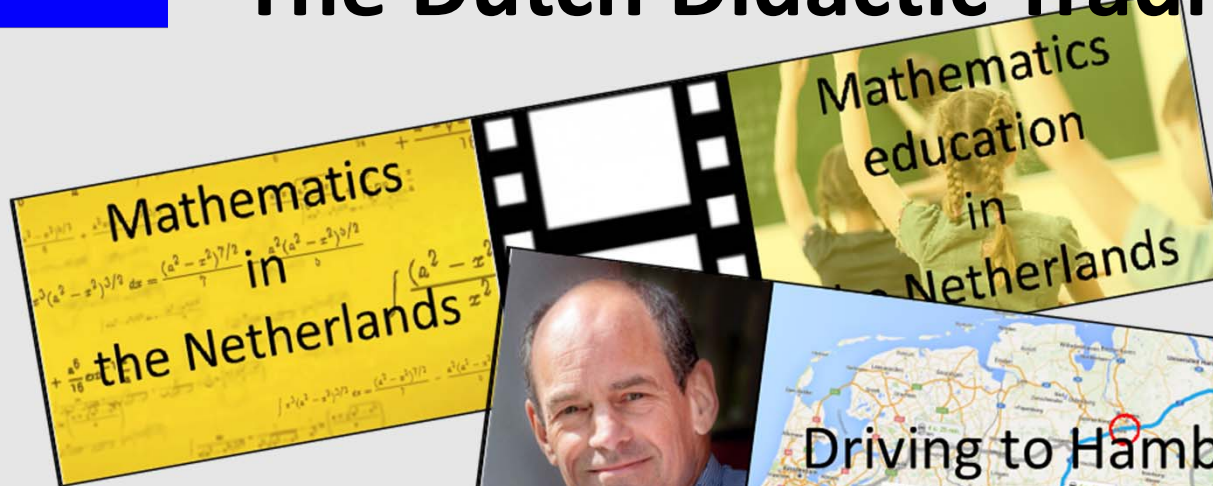
Second hour 16:30 – 18:30:

- the case of France
- **the case of the Netherlands**
- the case of Italy
- the case of Germany

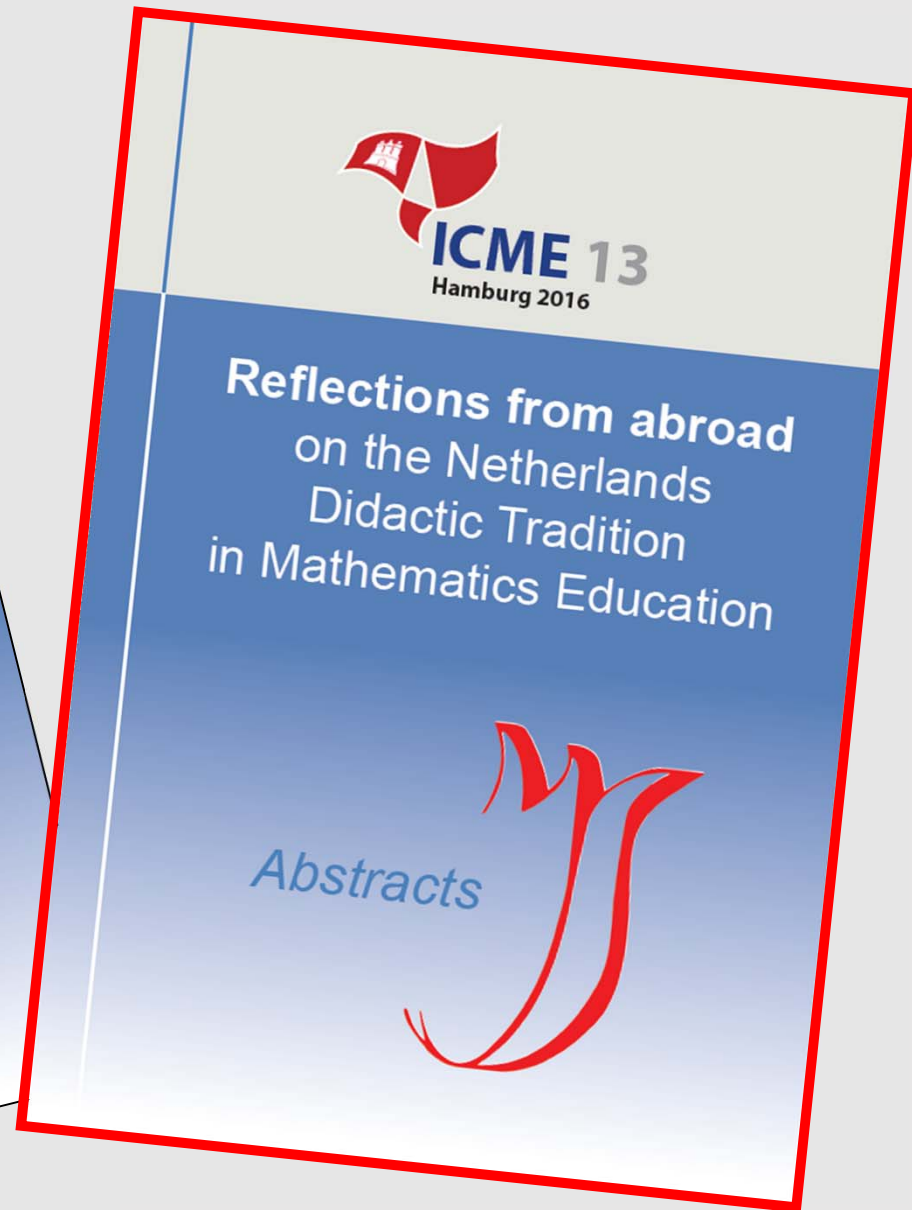




The Dutch Didactic Tradition (1st hour)



RME within the Dutch tradition





The Dutch Didactic Tradition (2nd hour)

Experiences with RME in ...



David Webb
USA



Zulkardi &
Ratu Ilma Indra Putri
Indonesia



Sue Hough
**England &
Cayman Islands**

Critical friends



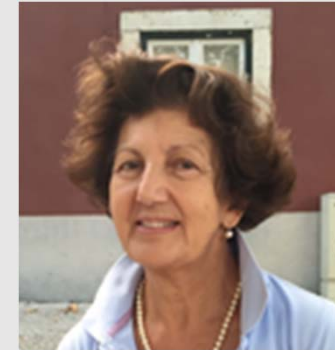
Dirk De Bock
Belgium



Cyril Julie
South Africa

Second hour 16:30 – 18:30:

- the case of France
- the case of the Netherlands
- **the case of Italy**
- the case of Germany





The Italian Didactic Tradition

(1st hour)

Introduction:

M.A: Mariotti: *The emergence and evolution of the Italian Didactic Tradition, an overview*

M. Bartolini Bussi: *Specific features of the Italian situation and salient aspects of Mathematics education in the Italian cultural context*

P. Boero: *To be a researcher in Mathematics Education in the Italian cultural and school context*

Interaction with the audience



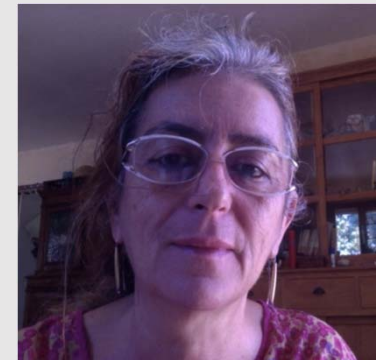


The Italian Didactic Tradition (2nd hour)

**Influences and interactions with other
European traditions**

The vision of a critical friend:

Nadia Douek (France): *SFIDA and other
experiences of collaborations between French
and Italian researchers.*





The Italian Didactic Tradition (2nd hour)

Influences and interactions beyond the European traditions

The vision of a critical friend:

Sun Xuhua (University of Macau, China): *Some impressive points on the Italian tradition of math education from Chinese culture perspective.*



Second hour 16:30 – 18:30:

- the case of France
- the case of the Netherlands
- the case of Italy
- **the case of Germany**





The German Didactic Tradition (1st hour)

A tour of recent German speaking
Didactics of Mathematics
with the help of interview-episodes



E.C. Wittmann



L. Hefendehl-Hebeker



H.-G. Weigand

Clustered by
R. Sträßer



The German Didactic Tradition (1st hour)

Examples of recent German research:

Stephan Hußmann: *Design of learning environments - expanding and enhancing subject matter didactics*

Kerstin Tiedemann: *Helping primary students to learn maths – language and interaction*

Stefan Krauss: *COACTIV- the impact of professional knowledge on student achievement*





The German Didactic Tradition (2nd hour)

Critical friends:



Barbro Grevholm (Norway/Sweden):

Doing empirical research differently:

Nordic countries and Germany



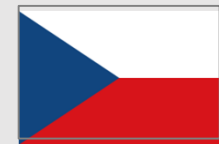
Edyta Nowinska (Poland):

Perspectives on collaborative empirical research in Germany and in Poland



Nada Vondrova (Czech Republic):

Didaktik der Mathematik and didaktika matematiky



Interventions from the floor