

# Didactics of Mathematics in Germany

## A historical sketch

### Introductory Remark :

It is impossible to give adequate credit to all important contributions to Didactics of Mathematics (“DoM”) in the German speaking countries in one hour.

For the sake of a story which can be told :  
One (hopefully informed) perspective on the development of DoM in the last decades  
- with **my sincere apologies** to those whom I have forgotten or misrepresented!

# Starting point in the 1960ies

- ▶ Personal reports from mathematics classrooms (by experienced teachers and education administration) / document analysis for curriculum development
- ▶ Subject Matter Didactics (“Stoffdidaktik”): Mathematical analysis of subject matter to be taught, to find the best (one&only) way to make a mathematics topic accessible, understandable, done by teacher trainers, (mathematicians,) textbook authors and mathematics teachers
  - Two strands: university and “Gymnasium” teachers / teacher trainers for primary and general education
  - Basic methodology: mathematics / anecdotal classroom experience
- ▶ Statistical, mainly comparative studies  
Often done by psychologists / in university departments of psychology

# Institutionalisation of DoM (1960s/70s)

In connection with the 'sputnik crisis' & 'educational catastrophe' / following a social move for more education: expanding educational system in the FRG

=> more universities, more maths teachers, more teacher training for maths, academization of teacher training for primary education teachers

## For Didactics of Mathematics: Creation of

- full professorships in DoM at university
- a scientific society (“**G**esellschaft für **D**idaktik der **M**athematik”)
- a research journal (“**J**ournal für **M**athematik-**D**idaktik”)
- a research institute (“**I**nstitut für **D**idaktik der **M**athematik” at Bielefeld University)

# “Empirical turn to everyday classrooms”

The 1970ies/beginning 1980ies:

Two moves in the developing discipline DoM:

- ▶ **More** detailed empirical research, **less** document analysis and anecdotal reports for developing a description & (causal) explanation of teaching&learning (“**realistic turn**”)
- ▶ **More** qualitative, sometimes linguistic analysis of classroom processes – in Germany initiated by the Bauersfeld group (“**turn to the everyday classroom**”)

# Hans-Georg Weigand on the “real” turn



“There was a change in the last decades: We have had in the 1970ies / 80ies not so much empirical investigations in Germany. We have had smaller, small scale investigations. ... Parallel to this, the qualitative empirical research was developed. ... It was very important to see the disadvantages of quantitative investigations, that you have to look deeper into details, not only seeing the result, seeing why a result is in this way, looking deeper about connections, relationships in this process of learning and teaching.”

“There was an interpretive research method, looking at special transcripts, looking at interviews or at classroom communication, to look with a special theoretical view on these interviews, on this communication. To do an interpretation of this communication, to see in much more detail than you see in a first view. For the teacher and the classroom, for an observer, it might be in the first view, very superficial. If you look deeper, you can discover problems of mathematics education in the communication process. To bring out these problems, to show these problems is a first step of an improvement of this process.”

# **Diversification from 1980ies**

Since the 1980ies, a rather homogeneous field made up of subject matter didactics and classroom studies diversified into a plethora of diversified research on a variety of aspects of the teaching and learning (process) of mathematics, including:

- “empirical research
- subject matter didactics
- applications in mathematics teaching
- historical and philosophical investigations
- methodological aspects of mathematics education-
- principles of mathematics education
- the epistemological dimension of mathematics education
- proving”

(from Burscheid, Struve & Walther 1992)

# **Excursion: German Democratic Republic** **(“Methodik” in the GDR –** **different term for didactics)**

## **4 Characteristics** (following Bruder 2003)

**(1)** policy controlled, uniform planning for a comprehensive school  
(**one** textbook, **one** set of teaching aids)

**(2)** systematic disciplinary (=Mathematics) orientation, trying to cope with the teaching reality  
(research picked up difficulties and experience of teachers and focussed on intervention, minimal role of theory)

# Excursion: German Democratic Republic

**(3)** optimization of instruction and learning environments (periodical repetition, mental training of basics), aiming at a **developed social personality** (socialist idea of man, high esteem of mathematics in the society) – high acceptance of “Methodik” by teachers due to

- high share of didactic-lessons at university
- high share of practical training at school during the study
- no deep gap between pre-service teacher education and in-service teaching experience

**(4)** linear, uniform structure of subjects to be taught, subjects had the priority compared to the individual needs of the students

Consequence: inner differentiation in a uniform educational system with special support of gifted students, e.g. Mathematical Olympiad on different levels, “Spezialschulen”



# **TIMSS and PISA**

## **The (German) “PISA-shock”**

(2<sup>nd</sup> half of the 1990ies / 2000+)

**Hans-Geog Weigand** (in an interview):

“It started with TIMSS in the 1970ies/80ies, when the first results of these international studies came out. Germany was not ranked very well. Many thought in these days, maybe it's because we are not so used to tests and with the special tests, may be we change this and then came PISA with the same results. It was a shock for many people in Germany, for many mathematics educators, also to me. ... We travelled to other countries, of course we've been to Japan and Finland – and we were looking what are they doing differently from Germany. ... We did something. Our first step was to introduce the standards, the German standards and – I think this is a very important point – to do some professional development for teachers – it was the SINUS program.”

**Consequence:** numerous / often policy sponsored efforts to enhance the teaching&learning of mathematics in general education and regional / national evaluation studies

# **The beginning of 21<sup>st</sup> Century / at present : three strands**

## ▶ Stoffdidaktik =>

(11) Stoffdidaktik enlarged

(12) design of learning environments

▶ 'anything' goes as (mostly qualitative) “**case studies**”, especially “classroom studies”, reconstructing aspects of everyday teaching and learning

## ▶ TIMSS&PISA (influenced/sponsored by politics) =>

(31) **quantitative** large scale **evaluation** studies

(32) **qualitative** large scale **development**  
studies

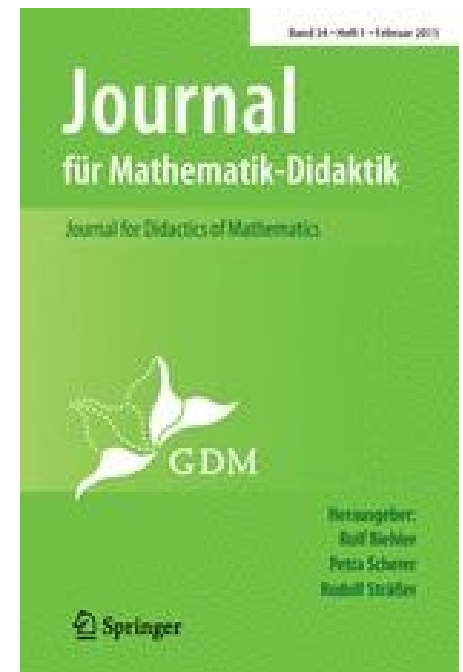
# (11) Stoffdidaktik => Stoffdidaktik enlarged

In addition to traditional Stoffdidaktik:

Take into account

- the history / epistemology of mathematics
- fundamental ideas of mathematics
- the learner and her/his pre-requisites  
(incl. 'basic mental models', beliefs)
- empirical studies on consequences of  
subject matter innovation  
in cooperation with other disciplines

▶▶ **Journal für Mathematikdidaktik (JMD)**  
<http://link.springer.com/journal/13138/37/1/suppl>



## **(12) Stoffdidaktik=>design of learning environments**

Growing out of the search of ***Stoffdidaktik*** for the best way to teach mathematics in an understandable way:

Make the **design of learning environments**  
the defining '**kernel**' of  
**research in mathematics education**  
(didactics of mathematics)

# **Erich C. Wittmann on Didactics of Mathematics as a design science**



“A basic distinction between mathematics and mathematics education occurred to me very early. **I identified mathematics education as a design science.** Of course, mathematics is a related discipline to mathematics education as a design science - in my view the most important one if you look at it carefully and seriously. ... What mathematicians have to understand is that mathematics can be compared to the growth of an organism, a plant. ... This view, this genetic view is the basic difference between the approach of mathematics [education] as compared with the mathematical approach. ... What you also have to become aware of is the growth of mathematics within students. ... (credit to Piaget)... His genetic epistemology was for me an enlightenment. I have made it a habit of looking at mathematics as something evolving.”

## **E.C. Wittmann (continued) on Didactics of Mathematics as a design science**

“There is a wide range of disciplines, which have to contribute to mathematics education, but more in an indirect way. Of course, your knowledge about mathematics is clarified by looking at semiotics, or by looking into developmental psychology, or by looking into the history. But they are not direct information for teaching .... Your mental equipment is very much enriched by taking care of information from other disciplines. So my position is by no means restrictive. I am very open. Why I like to emphasize mathematics [education] as a design science has mainly to do that I strongly believe that you cannot teach these broader information from other disciplines directly to teachers.”

(2) Ongoing diversification =>

**'anything goes'** as (mostly qualitative) **case studies**

Often done in cooperation with

**Psychology, Pedagogy, Educational Sciences, ...**

There is **NO comprehensive overview.**

Typical topics (from the last five years of JMD):

- the use of technology in mathematics teaching and learning
- subject matter analysis
- proof and argumentation
- modelling in mathematics classrooms
- the role of language
- early childhood and primary education
- variables of good class management
- gender and teaching&learning mathematics
- textbook research
- history and epistemology of mathematics
- semiotic and mathematics
- teacher training (preparatory and in-service)
- competencies in mathematics






# Research example 1: a “classroom study”

*Kerstin Tiedemann*: “Helping primary students to learn maths – language and interaction”

- sociological orientation on learning mathematics
- initiated by the works of H. Bauersfeld
- **key assumption**: mathematical knowledge is developed within social interaction
- **research**: reconstructing the social negotiation of meaning

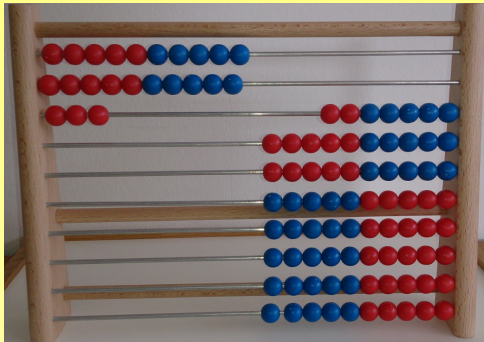
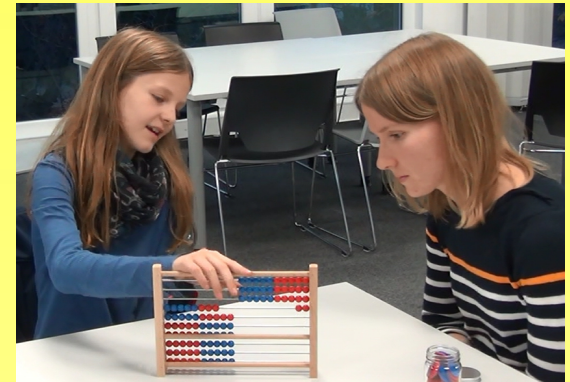


# Classroom Study (2)

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# Classroom Study (3)

Hanna (9.4 years) & Britta



32, ...

*"because here are 3 and here are 2"*

15:

*"1 row of tens and 5 beads"*



53, ...

*"because here are 5 and here are 3"*

25:

*"2 bars of tens and 5 little cubes"*

42:

*"for the 4 tens, you take rows and not bars, okay?"*

**TIMSS&PISA(influenced/sponsored by politics) =>**

**(3) quantitative large scale evaluation studies**

Deceiving results from TIMSS & TIMSS-video =>

Participation in PISA / PISA-E for regional comparisons

Heavy involvement of German Maths Didacticians

from **PISA 2003 onwards**

→ [http://archiv.ipn.uni-kiel.de/PISA/pisa2003/fr\\_reload\\_eng.html?mathematik\\_eng.html](http://archiv.ipn.uni-kiel.de/PISA/pisa2003/fr_reload_eng.html?mathematik_eng.html)

Embedded in PISA 2003: COACTIV-study on  
“teacher competence as a key determinant of  
instructional quality in mathematics”;

From 2007: COACTIV-R on “teacher candidates’  
acquisition of professional competence during  
teaching practice”

-> <https://www.mpib-berlin.mpg.de/coactiv/index.htm>

## Research example 2

# The Impact of Professional Knowledge on Student Achievement

*Stefan Krauss*  
*University of Regensburg*

Compendium by Kunter et al. (2013)

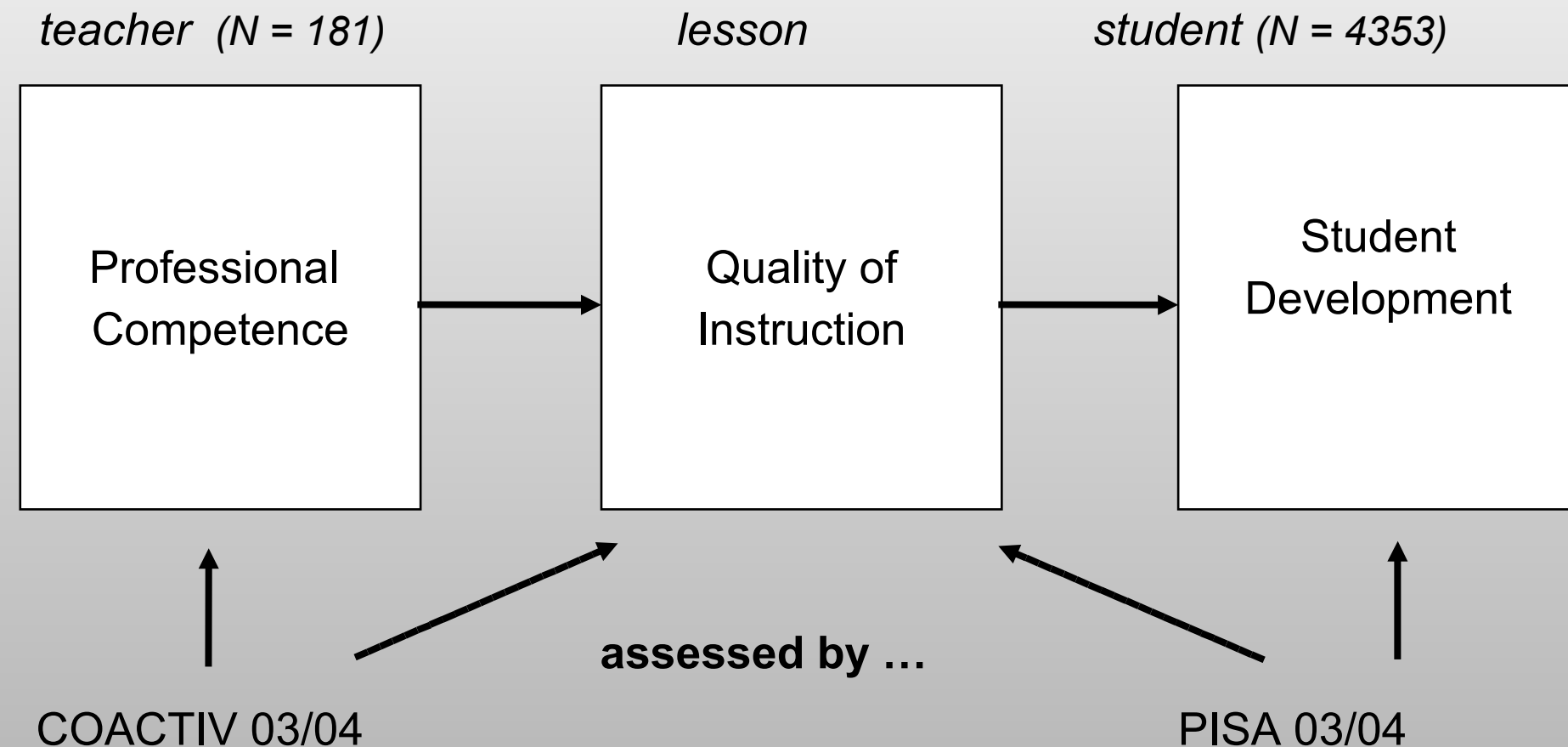
Mathematics Teacher Education 8

Mareike Kunter · Jürgen Baumert  
Werner Blum · Ute Klusmann  
Stefan Krauss · Michael Neubrand  
*Editors*

Cognitive Activation  
in the Mathematics  
Classroom and  
Professional  
Competence  
of Teachers

Results from the COACTIV Project

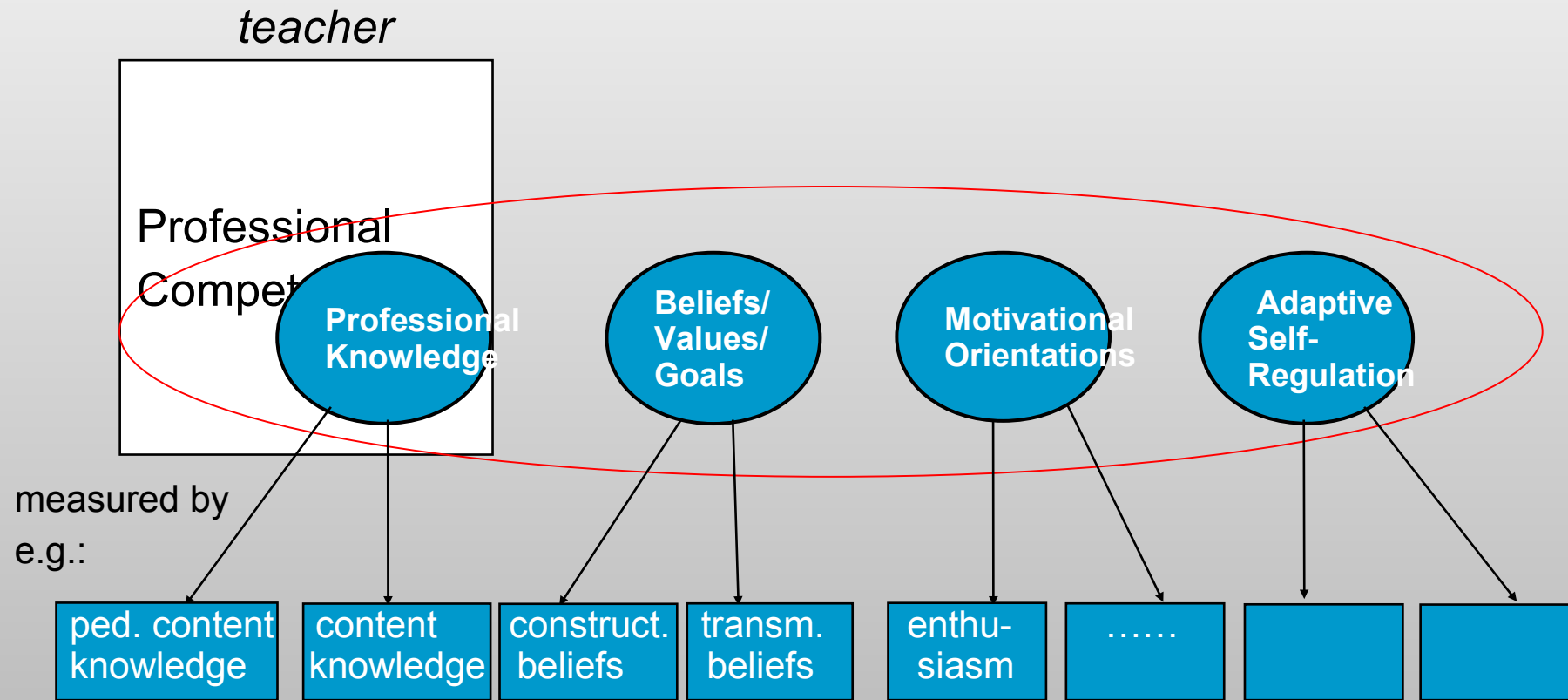
## COACTIV



- - - - - including various perspectives on instructional quality

→ Causality: COACTIV teachers are math teachers of PISA classes

# Model of Professional Competence of Teachers



- Teachers worked about 12 hours on the instruments (between Apr 03 and Apr 04)
- per teacher more than 1000 variables in COACTIV data file

## COACTIV

Quality of  
Instruction

Student  
Development

*Three latent dimensions:*

**cognitive activation** (assessed by tasks implemented by the teachers)

**classroom management**

(e.g., disruption levels, time wasted, both from student and teacher questionnaires)

**individual learning support**

(various scales tapping teacher-student interaction)

*Of course:*

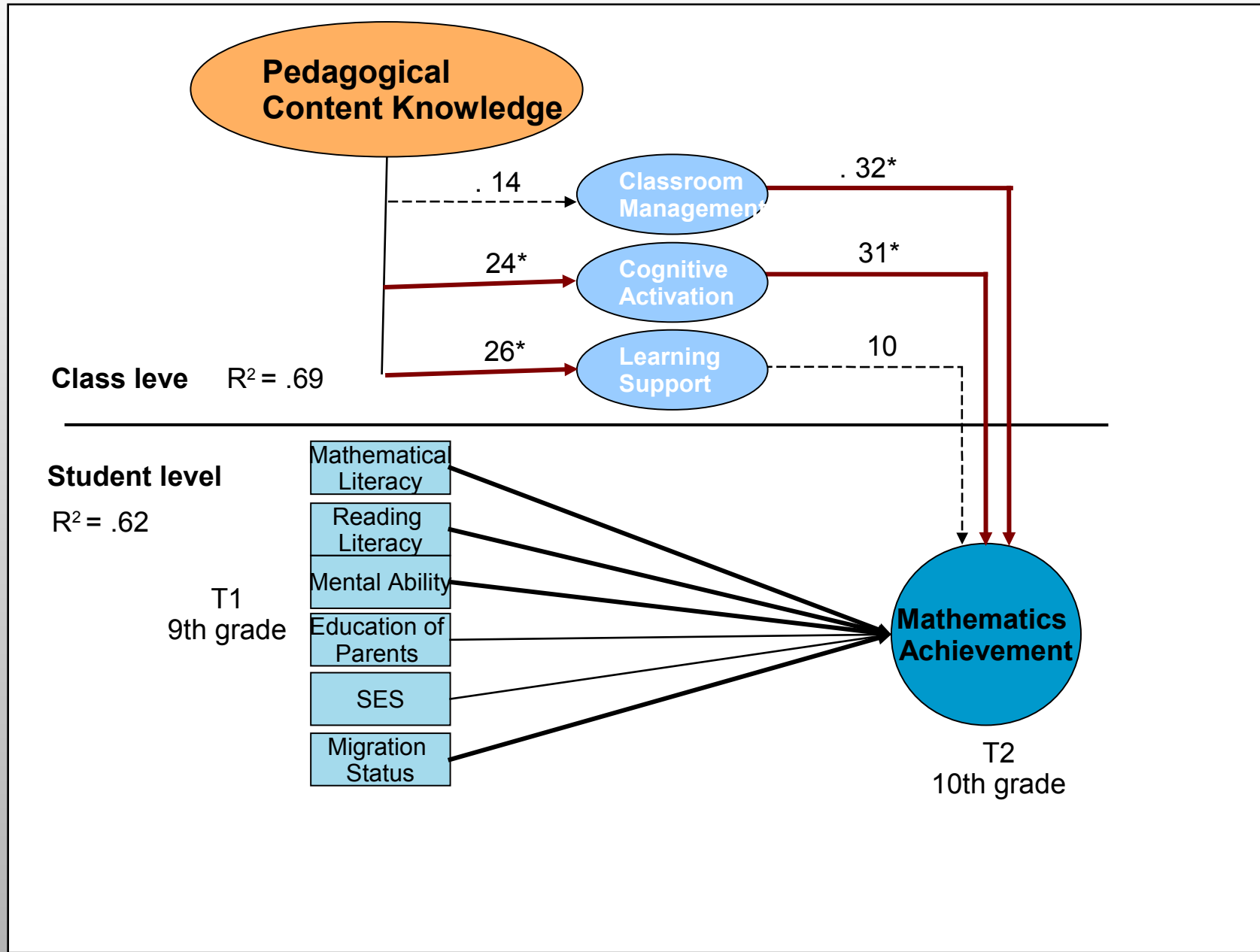
**mathematical literacy** (change from PISA 03 to PISA 04), but also:

**mathematics enjoyment**

**reducing math anxiety**



# Effects of Teacher' Pedagogical Content Knowledge





# In sum



Ped. content knowledge: Effects on cognitive activation and learning support

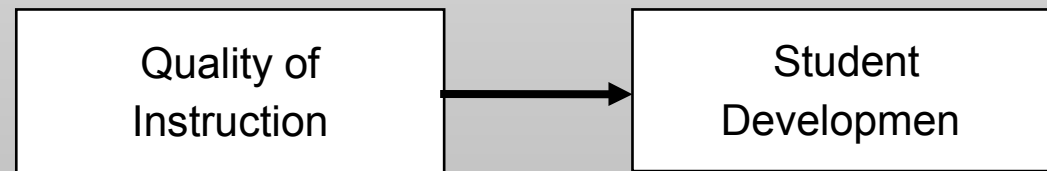
Content knowledge: No effect on instructional quality

*Furthermore:*

Constructivist beliefs: Effect on classroom management

Teacher enthusiasm: Effects on learning support and classroom management

Adaptive self-regulation: Effect on learning support



Classroom management: Effects on achievement and enjoyment

Cognitive activation: Effect on achievement

Learning support: Effects on enjoyment and reducing math anxiety

## **quantitative large scale evaluation studies (2)**

■ Hosted by IEA, with **TEDS-M/+follow-up** in Germany:

Paper&Pencil-Tests of MCK, MPCK and GPK

“The key research questions focused on the relationships between teacher education policies, institutional practices, and future teachers' mathematics and pedagogy knowledge at the end of their pre-service education.” (from the technical report)

■ In most regions (“Länder”): **Assessment studies** (“Schulleistungsstudien”) of teaching&learning mathematics

on various school levels with different methods (tasks, multiple choice, ...)

e.g.:

VERA: comparison of classes / schools in grade 3 /8;

-><https://www.iqb.hu-berlin.de/vera>)

Hamburg: KERMIT -><https://www.lernstand.hamburg.de/>

**TIMSS&PISA(influenced/sponsored by politics) =>**  
**qualitative** large scale development studies

As a consequence of the TIMSS study, in the late 1990s:  
The German government / the “Länder” initiated the  
**SINUS** and **SINUS-transfer study**  
supported by research institutes and didacticians.

**Basic idea:** increase teaching quality.

▶ help the classroom teacher enhance her/his teaching by developing examples of good teaching (11 “modules” on “Developing a Task Culture / Scientific Working / Learning from Mistakes / Gaining Basic Knowledge / Cumulative Learning / Interdisciplinary working / Motivating girls and boys / Cooperative learning / Autonomous learning / Progress of competencies / Quality assurance”)

▶ organise teacher cooperation and dissemination of good teaching units.

-><http://www.sinus-transfer.eu>

# **Lisa Hefendehl-Hebeker on theories of Didactics of Mathematics and topics of research**

There are different initiatives to systematize or to synthesize different theoretical approaches, which can be found all over the world – as Angelika Bikner and Susi Prediger. But the result is not a unifying theory. The result is that they show relations between these different approaches. But we don't have an overarching theory. And I think it is difficult – at least at the moment. We see traces for such a theory. We should know more. The crucial point is still how the human brain works.

If we would know more about this question we could make even a better theory about mathematics learning, about learning mathematics. ... It is difficult because there are so many impacts.: the impact of the personal development of the pupil, the impact of the special situation, how this individual brain works, apart from the question if there are invariant development variables, and ... the setting within the family, where the pupil comes from, the situation in the classroom, the political situation of the school, and the situation in society. There are so many topics which are intertwined in the situation that I think it is difficult to find a theory, which does not simplify these phenomena in an un-useful way.”



# **Lisa Hefendehl-Hebeker on theories of Didactics of Mathematics and topics of research (continued)**

“... Nevertheless I think empirical research is important, but we should take care to keep the balance, because ... it is still necessary also to have other approaches – for example subject oriented analysis and design experiments and ... discussions, philosophical and epistemological discussions, about questions what are good guidelines for mathematics education today. ... Of course, there are also new domains of interest, for example the use of technology in mathematics teaching and learning, the role of language, which has become very important since we have so many pupils with different mother tongues and because we have a new consciousness of the relation between speaking and thinking. ... And then there are completely new branches of research for example to find out about variables of good class management.”

# Hans-Georg Weigand on the future of German speaking Didactics of Mathematics



“In what direction will empirical research go the next years. What is important?

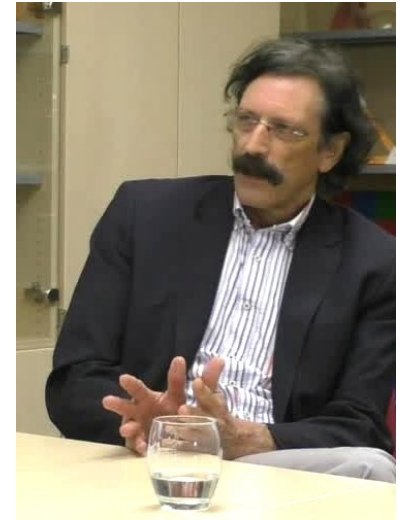
In my opinion, I think we have to get more results, not only qualitative, exploratory studies. ... Getting results means having quantitative investigations! For the future, I think, mixed methods is the right way to see the relationship between qualitative and quantitative research! ... Nowadays, it's not a question between these two big directions. So it is a combination and how to combine these. But again, you have to have a result, you have to have a goal at the end, which you can discuss with with other mathematics educators or with policy. ...

The second (question for the future) is bringing these results of empirical investigations, also of these large scale investigations to the school! Scaling up! ... Not only seeing the results, seeing also how it is brought, transferred to the school, to the learner, to the administration, changing many things. ... ”

Thank you  
for your  
attention!



E.C. Wittmann



H.-G. Weigand



L. Hefendehl-Hebeker



Kerstin Tiedemann



Stefan Krauss

**Compiled by  
R. Sträßer**