

European Didactic Traditions

Critical friends

Germany

(3rd hour)

Barbro Grevholm:

Doing empirical research differently: Nordic countries and Germany



Edyta Nowinska:

Perspectives on collaborative empirical research in Germany and in Poland



Nada Vondrova:

Didaktik der Mathematik and didaktika matematiky



Doing empirical research differently: The Nordic Countries and Germany

A view from the Nordic countries

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Reasons for Nordic connections to Germany

Germany is a neighbour to the Nordic area and direct neighbour via land to Denmark, or sea to Sweden, Norway, Finland, and Iceland

German language is often the second foreign language learnt in the Nordic area and it used to be the first foreign language . The linguistic connections are many, in historical times German was the spoken language in Stockholm, for example.

The classical Bildningsresorna (Bildungsreisen) went to Germany for many young persons in the Nordic countries. One famous example is the young Norwegian mathematician Abel.

Some early examples of German influence

Salomon Eberhard Henschen 1920, Klinische und Anatomische Beiträge zur pathologie des Gehirns. 5 Teil. Uber Aphasie und Akalkulie. He was medical professor in Stockholm.

It can be considered a very early contribution to studies on dyskalkulie.

The Kassel-Exeter study – had links to Norway and the KIM-study and there are also later master studies where the Kassel-Exeter tasks were used again in Norway.

Hanseatic traditions – being in Hamburg we are reminded of all the Hanseatic cities in the Nordic and Baltic countries that were linked to German centers of commerce and communication.

German textbooks used in Swedish University Studies

In the 60ies the Knopp Funktionen Theorie and Non-Euklidian Geometrie were regularly studied

Van der Waerden Algebra was also part of the doctoral study curriculum

Visits from German researchers, example Professor Doktor Emil Artin und Frau Braun (the very first and only lady in mathematics that I as a young doctoral student had the opportunity to meet)

How and why are we doing empirical research differently?

Some important differences in the conditions for research in didactics of mathematics (DoM):

The first professorships in DoM were created 1992-1993 in Denmark, Finland and Norway and in Sweden 2001 at Luleå University of Technology, in contrast to the 60 professorships created in Germany in the 1960ies

The academization of teacher education took place around 1960 but not until in the 1980ies was it explicit that teacher education should be research based.

A scientific society of DoM was created in Sweden in 1998 and some years earlier in Denmark and Finland. Norway and Iceland still do not have such societies.

Conclusion: The Nordic countries were about 20-30 years behind in the development compared to Germany

What triggered the development in the Nordic countries?

The First Mathematics and Science Study, FIMSS, created a huge debate in Sweden and a governmental committee was set up. They published their report in 1986, called mathematics in School, and it suggested academic courses in DoM, positions and revision of teacher education. A new teacher education started in 1988, where DoM was introduced and the education clearly research based. The student was supposed to carry out a small research study and write a scientifically oriented paper on it, the so called examensarbete.

The International trends also swept over the Nordic countries First in the 1960ies the modern mathematics was in focus, then back to basics, use of ICT and problemsolving.

Results from TIMSS and PISA are influencing the politicians much and creating debate in society about school mathematics.

Nordic teachers were taught methods of teaching which links to Eric Wittman's view of DoM as a design science. It was not seen as research.

National centers for teaching of mathematics

In Sweden NCM, National Center for Mathematics Teaching was created in the end of the 1990ies and following that came a similar center in Trondheim, NSMO. Again politicians emphasised the teaching and research was not included.

But in Norway a great effort was also given to creating research in DoM. At University of Agder (UiA) a Masters education started in DoM in 1994 followed by a doctoral education in 2002. Four professors of DoM were hired (all women!!) and asked to build up a research environment and stable doctoral education. This was the first time in the Nordic countries when a group could work together in DoM at the same university.

Nordic Graduate School in Mathematics Education

A huge five year grant was given to UiA in order to set up a Nordic Graduate School. Most of the Nordic universities with DoM-students were linked to this Graduate school and it held about 90 doctoral students and 100 supervisors, also from the Baltic countries. To this Graduate school several German scholars were invited to lecture, participate in summer schools or hold seminars for supervisors and thus the link to Germany was kept alive.

The Graduate School was alive between 2004 and 2010 and was followed by a fruitful networking collaboration between the countries in the form of common conferences, the so called NORMA-conferences, a common scientific journal, NOMAD (started in 1993 and revived in 2004), and joint activities in doctoral education in the form of courses, summer schools and the like. Also here German scholars took part.

Examples of collaboration with German-speaking colleagues

Erkki Pehkonen collaborated with Gunter Törner and created a series of conferences on Problemsolving in mathematics, the ProMath-conferences.

Markku Hannula has done joint studies with Bettina Rösken on attitudes and beliefs and held conferences upon this theme.

Heinz Steinbring has contributed to Madif-conferences as invited main speaker and published in the proceedings.

Peter Berger contributed to one of the early Norma-conferences.

Williblad Dörfler was group leader during one of the summer Schools of NoGSME, and Gabriele Kaiser was presenting about ZDM in a supervisors seminar.

Many, many others could be mentioned here.

- In the Nordic countries empirical research was most often carried out by single researchers on their own and it resulted in a fragmented picture of results from the research. The only place where we find a larger group of researchers is at university of Agder in Norway. The only school of study where many researchers chose the same framework is the phenomenography group around Ferenc Marton in Gothenburg. These studies are in pedagogy and has not mathematics as a strong component in the work. In Sweden in the beginning these studies were much questioned by people working in DoM.
- A wish for the future is to forge stronger bonds between Germany and the Nordic countries in DoM and to develop collaboration in many ways. Doctoral student exchange could for example be one excellent way to do this. Another could be academic teacher exchange.
- German researchers are welcomed to continue to participate in the Nordic activities like conferences and courses and to publish in Nomad and create joint research studies with colleagues from the Nordic countries. Such a collaboration is fruitful for all who take part.

Perspectives on collaborative empirical research in Germany and in Poland

Edyta Nowinska

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Cultural contexts influencing
the work of researchers
in **Didactics of Mathematics (DoM)** in Poland

- Research in DoM in Poland has its roots in the movement initiated by Professor Anna Zofia Krygowska in the **1960ties** and called “**Cracow school of mathematics education**”.
- 1982 Anna Zofia Krygowska succeeded in establishing the first **Polish** journal for publishing research work dealing with didactics of mathematics. It was also used to publish translations of papers written at this time in English or German and related to DoM.

- research work in the tradition of Anna Zofia Krygowska
 - strong epistemic component, attention to mathematical meanings and mathematical understanding specific to particular concepts
 - mostly didactics of **MATHEMATICS** ... and less **PSYCHOLOGY** or **SOCIOLOGY** of learning mathematics (quite homogeneous field)

- research work in the tradition of Anna Zofia Krygowska

- “Typical” research themes are related to particular mathematical concepts

(functions, limits, proofs, algebraic thinking, generalization, elementary geometry, fractions)

and presented as **case studies**.

- **institutional contexts** of the work of researchers in DoM in Poland
 - institutes for mathematics (evaluation of the quality of research from the perspective of mathematicians)
 - “Didactic of mathematics” does not have the status of scientific discipline in Poland.

- **institutional contexts** of the work of researchers in DoM in Poland
 - no tradition of research projects supported by funds and research grants to enhance the quality of teaching and learning of mathematics
 - no tradition of research on in-service teacher professional development programmes
 - no tradition in video-based classroom research

Consequences for / perspectives on collaborative empirical research in Germany and in Poland

- German DoM as a source of “inspiration” for theoretical considerations and methodological work of *individual* Polish researchers, and as a source of “inspiration” for empirical work on problems related to teaching and learning situations and in-service teachers’ professional development.

Edyta Jagoda - Substantial Learning Environments - "Tiles", Didactica Mathematicae no32, (2009) (DOI): 10.14708/dm.v32i0.25

Jagoda, E., Swoboda, E. (2010). Pomysły na naturalne różnicowanie w nauczaniu szkolnej matematyki. Środowisko geometryczne „Kafelki” , ISBN: 978-83-7338-581-8, Wydawnictwo Uniwersytetu Rzeszowskiego, Rzeszów. S. 78.

Hospesova, A., Jagoda, E., Roubicek, F., Swoboda, E. (2010). Ideas for Natural Differentiation in Primary mathematics classrooms. Geometrical Environments. ISBN: 978-83-7338-582-5 Wydawnictwo Uniwersytetu Rzeszowskiego, Rzeszów, s. 164.

Jagoda, E., Swoboda, E. (2009) Środowisko edukacyjne "Kafelki". Współczesne Problemy Nauczania Matematyki, Prace amonograficzne z dydaktyki Matematyki, Koło SNY, Forum Dydaktyków Matematyki, Bielsko- Biała .

- no tradition of collaborative empirical research work in Poland and in Germany
- no tradition of collaboration to design actions aimed at a better understanding and improvement of similar educational problems in different cultural contexts
- Researchers aiming at collaborative empirical work in Poland and in Germany must take into consideration the cultural contexts influencing their work.

*Coming together is a beginning;
keeping together is progress;
working together is success.*

Henry Ford

Didaktik der Mathematik and didaktika matematiky

**Nad'a Vondrová
Charles University Prague
Faculty of Education**



Prior to 1989 (Velvet Revolution)

- Czech mathematics education similar to that of German Democratic Republic (see the Excursion in the main presentation)
- Little connection to German mathematics education (few opportunities to travel or read international journals)
- YET, some links established: mutual visits at universities (Jaroslav Šedivý, Oldřich Odvárko, Leoš Boček, Jiří Mikulčák and others, often from the Faculty of Mathematics and Physics, Charles University)
- Jan Vyšín - member of Wissenschaftlicher Beirat ZDM
- Jan Vyšín and Oldřich Odvárko - Members of the editorial board of Zentralblatt für Didaktik der Mathematik
- Czech mathematicians and mathematics educator followed New Math movement abroad, mostly in East Germany but in Western countries, too
- Articles published in Czech journals by Miloš Jelínek (1962-63) about New Math which influenced the movement in Czechoslovakia



Prior to 1989 (Velvet Revolution)

- Hans Georg Steiner had lectures at the Faculty of Mathematics and Physics in Prague repeatedly, similarly Roland Stowasser and probably others
- Hans Georg Steiner „pulled strings“ to get authorities to allow the organisation of the *International Symposium on Research and Development in Mathematics Education* (and edited proceedings with Milan Hejný) in Bratislava in 1988: a very important event for Czech and Slovak mathematics education



Jaroslav Šedivý

Institut pro didaktiku matematiky v Bielefeldu

An example of an article about German mathematics education prior to 1989

Pokroky matematiky, fyziky a astronomie, Vol. 29 (1984), No. 2, 112-117

Na počátku 70. let se naopak začala rozvíjet činnost výzkumného ústavu v Bielefeldu (NSF) a nyní bychom chtěli stručně charakterizovat v tomto příspěvku.

Mladé univerzitní město

Bielefeld leží v průsmyku Teutoburského lesa, horského hřebene známého z dějin bojů Římanů s Germány, později též Franků se Sasy. Od svého založení ve 13. století prošlo vývojem hanzovního města, stalo se „městem lnu“ s rozvinutým tkalcovstvím, ale průmyslová revoluce 19. století přinesla i další odvětví průmyslu. V sousedství ruhrské pánve, na trati spojující ji s Berlínem, byly dobré podmínky pro hospodářský vzestup města, které má nyní přes 320 000 obyvatel, z toho 65 000 dělníků.

...ové centrum pro vysokoškolskou didaktiku (IZH) aj. V době největšího rozmachu vědecké činnosti (za ještě dobré finanční situace) řešili pracovníci univerzity v r. 1977 přes 350 vědeckých úkolů, v posledních letech však úsporná opatření vedou ke snižování fondů, počtu pracovníků i řešených úkolů.

Vývoj Ústavu pro didaktiku matematiky (IDM)

V r. 1969 vytvořil koncern Volkswagen nadaci určenou k podpoře oborových didaktik matematiky a přírodních věd; prvním projektem, který byl v zakládací listině uveden, bylo založení nadregionálního (= überregionalen) Ústavu pro didaktiku matematiky. [Většina čtenářů jistě ví, že kapitalistické koncerny vytvá-



An example of influence of Wittmann's work on Czech mathematics education: an article about Strategies in Heuristics

Strategie v heuristice

Jan Kopka, Ústí nad Labem

V roce 1975 vyšlo v *Int. J. Educ. Sci. Technol.* obsáhlé pojednání profesora E. WITTMANNA s názvem *Matrix strategies in heuristics*. Wittmann zde podává stručnou charakteristiku strukturální matematiky (podle N. BOURBAKIHO), dále popisuje elementární struktury v genetické psychologii (podle J. PIAGETA) a základní strategie v heuristice (podle G. POLYI). Potom ukazuje úzké vztahy, které mezi těmito oblastmi existují. Dospívá k závěru, že se ve vyučování matematice musí odrazit nejen samotná matematika, ale i psychologie a heuristika. Cílem našeho článku je seznámit čtenáře s některými myšlenkami Wittmannova pojednání a připojit k nim některé poznámky a kritická zhodnocení.

právě v zanedbávání heuristických strategií. Tento příklad nám bude sloužit jako motivace další teoretické části. Bude však dobré, když se čtenář po přečtení článku opět k tomuto příkladu vrátí a znovu si promyslí jednotlivé heuristické postupy jeho řešení.

P: Rozdělte rovnoběžník na tři části stejného obsahu a použijte k tomu přímeek procházejících jedním vrcholem¹).

Jedno možné řešení *P*.

*P*₁: Rozdělte rovnoběžník na dvě části stejného obsahu a použijte k tomu přímeek procházející jedním vrcholem.

*P*₂: Rozdělte rovnoběžník na čtyři části stejného obsahu a použijte k tomu přímeek procházejících jedním vrcholem.

*P*₃: Rozdělte trojúhelník na tři (dvě, čtyři) části stejného obsahu a použijte k tomu přímeek procházejících jedním



After 1989

- New opportunities offered by conferences: *Tagung für Didaktik der Mathematik* (Czech mathematics educators present: František Kuřina, Milan Koman, Leoš Boček, Oldřich Odvárko, Marie Tichá, Alena Hošpesová and others)
- German university researchers inviting Czech researchers to give lectures at their universities (often providing financial support, too)
- „Empirical turn to everyday classrooms“ in Czech mathematics education under the influence of Western (also German) research
- *Stoffdidaktik* always close to the Czech conception of mathematics education (where ‚mathematics‘ never disappeared)
- *Stoffdidaktik enlarged* taking into account the history and epistemology of mathematics and fundamental ideas of mathematics (an example is the cooperation of Ladislav Kvasz, Rainer Kaenders and Ysette Weiss-Pidstrygach)



German researchers

- **Erich Wittmann: well-known to Czech researchers**
 - Project Mathe 2000
 - His and his colleagues' view of mathematics education as a design science (often cited by F. Kuřina)
 - The idea of substantial learning environments - very well known in CZ (often cited by M. Tichá, N. Vondrová, and mainly M. Hejný for whom „the design of learning environments indeed is the defining ‚kernel‘ of research in mathematics education“)
- Other researchers whose work inspired Czech researchers: H. Steinbring (epistemological investigation of classroom interaction), E. Cohors-Fresenborg, K. Hasemann, H. Meissner, G. Muller, P. Scherer, B. Wollring, E. Glasserfeld (constructivism), H. Freudenthal (had German roots)



Common Socrates Comenius projects

- *Understanding of mathematics classroom culture in different countries (2000-2004)* (Czech Republic, Germany, Italy; M. Tichá, A. Hošpesová, P. Scherer, H. Steinbring and others - German inspiration in terms of the concept of reflection)
- *Motivation via Natural Differentiation in Mathematics (2008-2010)* (Czech Republic, Germany, Netherlands, Poland; G. Krauthausen, P. Scherer, M. Tichá, A. Hošpesová and others - inspiration by substantial learning environments as a connecting element in the project; the German team „had the biggest influence on the implementation of the theory“ - final report)
- *Implementation of Innovative Approaches to the Teaching of Mathematics (2003-2006)* (Czech Republic, United Kingdom, Germany, Greece; M. Hejný, D. Jirotková, N. Vondrová, B. Wollring, B. Spindeler and others - German inspiration by hand-on activities)
- *Communicating Own Strategies In Primary Mathematics Education (2006-2006)* (Czech Republic, Germany, the United Kingdom; M. Hejný, J. Kratochvílová, B. Wollring, A. Peter-Koop)

