



*Thematic Afternoon at ICME-13. The French Didactic Tradition*

# Research on school algebra. From didactic transposition to instrumental genesis

---

MARIANNA BOSCH (Univ. Ramon Llull, Barcelona, Spain)

HAMID CHAACHOUA (Univ. Grenoble Alpes – ESPE, France)

# Research on school algebra: 1979-2016

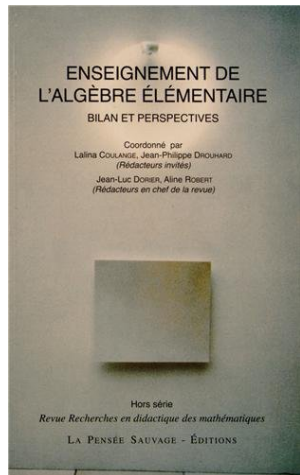
---

**First studies in the 1980s**, by Y. Chevallard, in an attempt to approach secondary school mathematics from the perspective opened by the Theory of Didactic Situations

This gave rise to the first analysis in terms of *didactic transposition* that later became the Anthropological Theory of the Didactic

Other frameworks have also been developed in a close relationship to research on school algebra, like the *semio-linguistic approach* (J.P. Drouhard et al, F. Arzarello et al) and the *Theory of Instrumentation* (L. Trouche et al)

# Recent surveys: 2012 and 2015



Coulange L., Drouhard J.-P., Dorier J.-L., Robert A. (eds) (2012), *Enseignement de l'algèbre élémentaire: bilan et perspectives*. Recherches en Didactique des Mathématiques (special issue).

- 16 papers
- Part 1: Teaching algebra practices
- Part 2: Cross-perspectives on the algebra to be taught



Butlen D., Bloch I., Bosch M., Chambris C, Cirade G., Clivaz G., Gobert S., Hache C., Hersant M., Mangiante M. (2015, eds.), *Rôles et places de la didactique et des didacticiens des mathématiques dans la société et dans le système éducatif*. Grenoble : La Pensée sauvage.

# Research on school algebra: 1979-2016

Alves et al (2013), Artigue (1997), Artigue et al (2001), Assude (1993, 2002), Assude et al (2012), Barallobres (2004), Bardini (2003, 2012), Ben Nejma (2009), Bolea (2003), Bolea et al (2001, 2004), Bosch (2015), Bouhineau et al (2001), Briant (2013), Broin (2002), Capponi et al (1989, 1990) Capponie & Balacheff (1999), Chaachoua (2015), Chaachoua et al (2007, 2012), Chevallard (1984, 1988, 1989, 1990, 1992, 1994), Chevallard & Bosch (2012), Chevallard & Conne (1984), Cid & Bolea (2010), Cid et al (2013), Constantin (2014), Coppé et al (2015), Coulange (2001), Coulange et al (2012), Coulange & Grugeon-Allys (2008), Croset (2006, 2009), Croset & Chaachoua (2010), Drouhard (1992, 2012), Drouhard & Panizza (2012), El Mouhayar (2007), Erdogan (2006), Gascón (1993, 1994, 1999, 2011), Grugeon-Allys (1995, 1997, 2008), Grugeon-Allys et al (2012), Haspekian (2005, 2012), Lagrange & Gelis (2008), Lenfant (2002), Mercier (1992, 2012), Pilet (2012, 2015), Ruiz-Munzón (2010), Ruiz-Munzón et al (2007, 2011, 2012, 2013), Tonnelle (1979), Trgalová & Chaachoua (2009), Vergnaud et al (1988), ...

	Publications
Before 1990	9
1991-2000	12
2001-2010	25
After 2010	23

# Research questions and approaches (1)

What is taught as “school algebra”? What is (not) taught?  
What was taught/considered as “school algebra” before?

## → DIDACTIC TRANSPOSITION

Vanishing of algebra in current curricula: not systematic work on algebraic expressions with parameters and disconnection with quantities and functional modelling

## → INSTITUTIONAL CONSTRAINTS (ECOLOGY OF KNOWLEDGE)

Lack of sense/rationale of many algebraic handlings  
Pejoration of algebra (writing) in Western cultures  
Need for a debate about (and renew of) what mathematics should be taught at secondary school

Sunday Review | OPINION

The New York Times

Is Algebra Necessary?

By ANDREW HACKER JULY 28, 2012



# Research questions and approaches (2)

**What algebra could be taught? Under what conditions?  
How to implement them? What constraints are found?**

→ **REFERENCE EPISTEMOLOGICAL MODELS (REM)**

“*Computation programmes*” as a basic notion to rebuild and give meaning to algebraic notions and activities  
New interactions between the algebraic and numerical work (e.g. negative numbers) and functions

→ **DIDACTIC ENGINEERING (NEW ECOLOGIES)**

Experimentation of teaching sequences using ICT  
Conception of computing environments in collaboration with computer experts.  
Potential errors and students’ trajectories using REM



$$\left( \frac{120+x+30-x}{4} \right) + \left( \frac{25 \cdot x - 0}{x} \right) \cdot 2 = 50$$

$$\left( \frac{120+x+30-x}{4} \right) + \left( \frac{25 \cdot x - 0}{x} \right) / 2 \rightarrow 50$$

# Research questions and approaches (3)

**How do ICT modify the nature and the way of using, teaching and learning algebra?**

→ CONCEPTION OF ICT AS PART OF THE ADIDACTIC MILIEU

ICT environments are integrated to algebraic activities  
Importance and nature of their feed-back

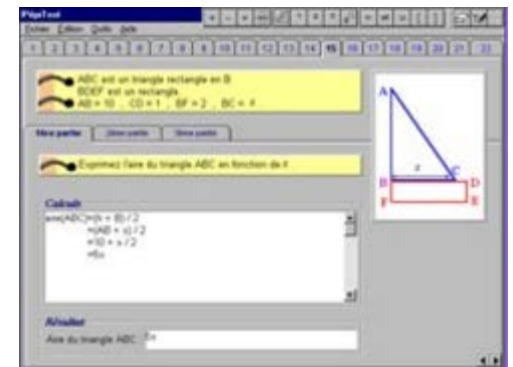
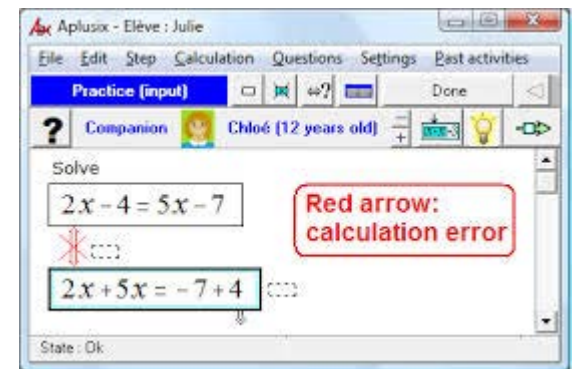
→ TAKING INTO ACCOUNT THE INSTRUMENTAL GENESIS

Problems of integration and teachers' legitimacy

→ DIDACTIC AND COMPUTER TRANSPOSITION

Design of new ICT environments (APlusix and Pépité)  
based on explicit epistemological models of algebra

	A	B	C	D
1	PRIX (euros)	TAUX (%)	TVA (euros)	
2	1000	19,60	196	
3	60	19,60		
4				



# Research questions and approaches

**How to teach algebra?**

**How to overcome students' difficulties with algebra?**

**What is taught as “school algebra”? What is not taught?  
What was taught/considered as “school algebra” before?**

**What algebra could be taught? Under what conditions?  
How to implement them? What constraints are found?**

**How do ICT modify the nature and the way of using,  
teaching and learning algebra?**

DIDACTIC TRANSPOSITION  
ECOLOGY OF KNOWLEDGE  
REFERENCE EPISTEMOLOGICAL  
MODELS (PRAXEOLOGIES) ...  
DIDACTIC ENGINEERING  
INSTRUMENTAL  
GENESIS DIDACTIC MILIEU  
INSTITUTIONAL LEGITIMACIES



# THANK YOU VERY MUCH

## In Memoriam: Jean-Philippe Drouhard

---



A research life in didactics devoted to the study of the semio-linguistic complexity of elementary algebra

Creator of SFIDA – Franco-Italian Seminar in Didactics of Algebra (1992 -2012)

Coulange L., Drouhard J.-P., Dorier J.-L., Robert A. (eds) (2012), *Enseignement de l'algèbre élémentaire: bilan et perspectives*. Grenoble: La Pensée sauvage.