Educational research on axial symmetry in the French tradition

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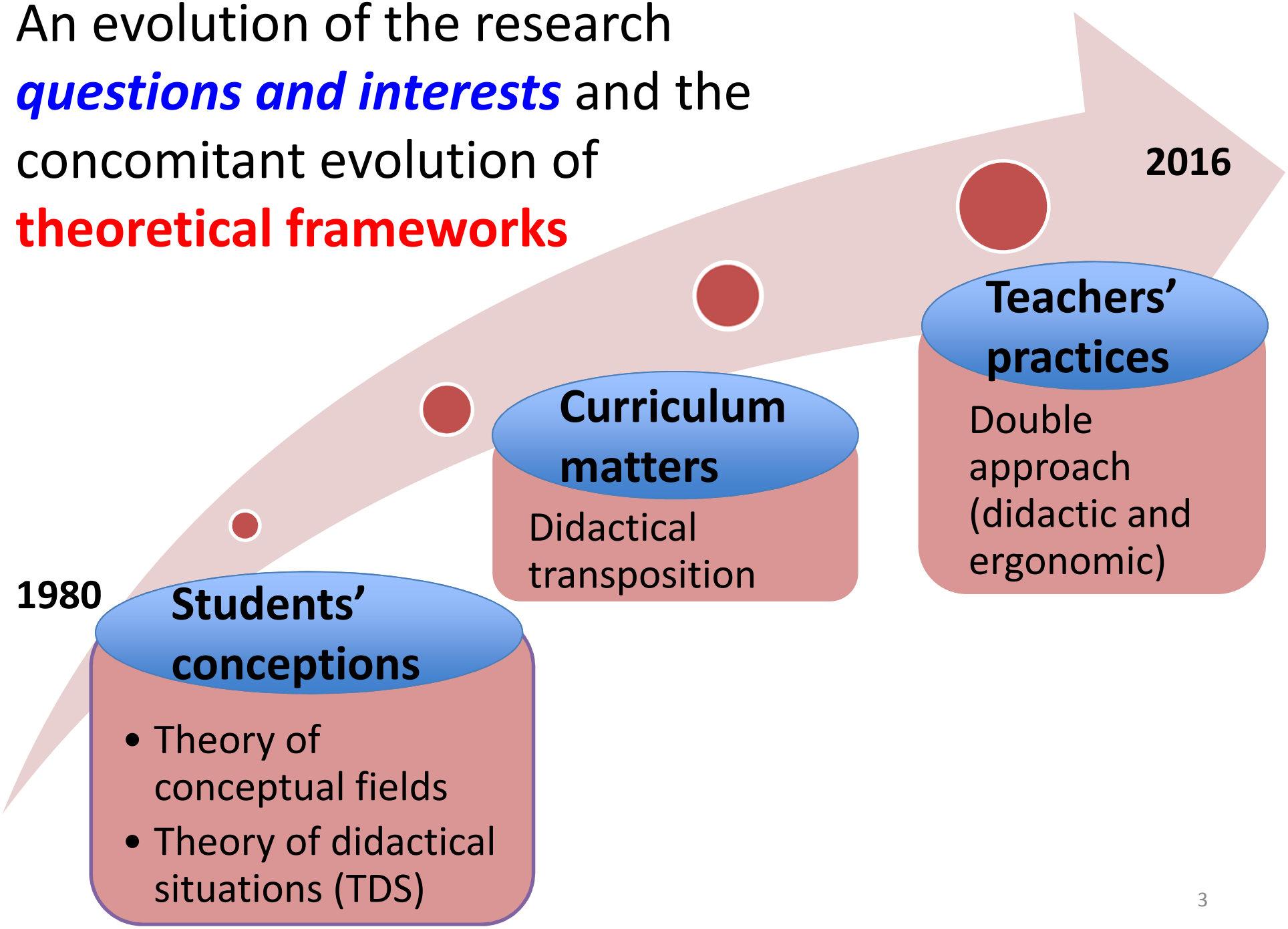
In France, Symmetry has been an object of research in Didactic of Mathematics from the eighties till nowadays, mainly because

- ***It is an everyday notion, familiar to students***;
- ***It is taught from primary school to university***;
- ***It is a central notion among the geometrical transformations;***
- ***It is a central notion in the curriculum since the new maths reform;***
- ***It is involved in many professional activities***;
- ***It is emblematic of the gap between pragmatic validation and theoretical proofs in Geometry.***

➢ A good subject to present an evolution of research questions and frameworks in French research in didactic of mathematics

An evolution of the research ***questions and interests*** and the concomitant evolution of **theoretical frameworks**



Students' conceptions
Including
proof and proving
and
designing of activities

Four PhD thesis have been defended in Grenoble from 1988 to 2006.

These studies articulated

the theory of conceptual fields

the theory of didactical situations (TDS)

and since 1993, they took into account the ***contribution of the ICT***, with the development by Balacheff (1995) of **the cK¢ model for knowledge**.

A new thesis articulating the **theory of conceptual fields** and **the geometrical working spaces** approach (Kuzniak 2003) was defended in 2008 in Paris.

The **cK¢ model** (Balacheff, 1995) is an enrichment of the model of concept of Vergnaud, permitting to bridge a link between *knowing* and *proving* by constructing links between controls and proofs (Balacheff 2013).

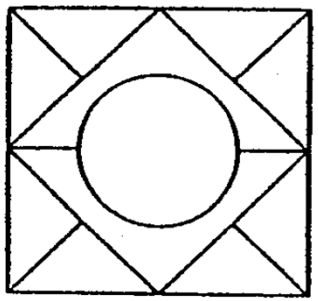
A conception is characterised by :

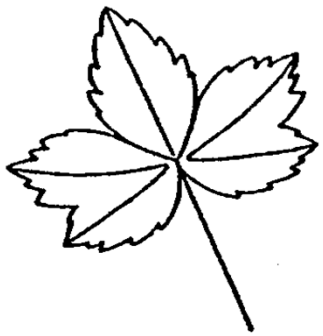
- a set of *problems (*P)
- a set of *operators (R)*
- a *representation system (L)*
- a *control structure (* Σ)

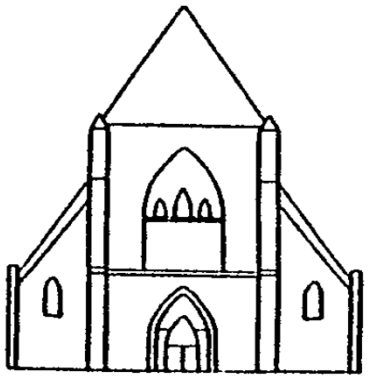
Denise Grenier (1988)

- studied very precisely ***students' conceptions*** on axial symmetry
- elaborated ***a didactical engineering for the teaching of axial symmetry*** experimented in middle school (Grade 6).

what do these figures have in common ?







An important result is that to communicate a teaching process to teachers, it is necessary to consider not only students' conceptions, but also teachers' representations about

- the mathematic knowledge at stake,
- the previous knowledge of the students,
- and the mode of elaboration of their knowledge.

Takeshi MIYAKAWA (2005) studied **the relationships between mathematical knowledge and proof** in the case of axial symmetry, considering with Balacheff that « learning proof is learning mathematics ».

The main question concerns **the gap between pragmatic validation and theoretical validation**.

He concluded that although the constructions are playing an important role to overcome this gap, it appears to be non-sufficient.

SalimaTAHRI (1993) proposed a ***modelling of didactical interaction in a hybrid tutorial*** on the micro-world CABRIGEOMETRE, ***in order to analyse didactical decisions***.

Iranette LIMA (2006) proposed ***a modelling of conceptions by identifying their structure of control***, and used it to elaborate an experimentation aiming ***at studying the teachers' didactical decisions***.

In both cases, the mathematical content at stake was ***axial symmetry.***

Caroline Bulf (2008) studied the effects of axial symmetry on the other isometries' conceptualization and on the nature of geometrical work at secondary school.

As part of her research, she paid interest to ***the role of symmetry in the work of stone carvers and carpenters***.

Through observations and interviews, she tried to identify invariants, considering, in line with G. Vergnaud, the role of action in conceptualization.

She concluded that ***the concept of symmetry is organizing their action*****.**

Didactical transposition and curriculum perspectives

The symmetry appears as a relevant subject to study ***curriculum matters*** in geometry

- the phenomenon of ***didactical transposition*** has been studied in the case of axial symmetry in the thesis of Patricia Tavignot (1991).
- recently, symmetry has been used as a key example to illustrate ***progressive curriculum development*** in Geometry by Marie-Jeanne Perrin-Glorian (2012)

The study of teachers' practices
and of their cognitive effects using
the double approach (didactic and ergonomic)

- Teacher considered as an element of the didactical system, means of the transmission of didactical engineering
- Difficulties of this transmission appeared to result from decisions that teachers had to make in order to react to the reality of students / classes
- Some decisions of teachers don't seem to be only determined by didactical considerations

-> ***necessity to study teachers' practices***

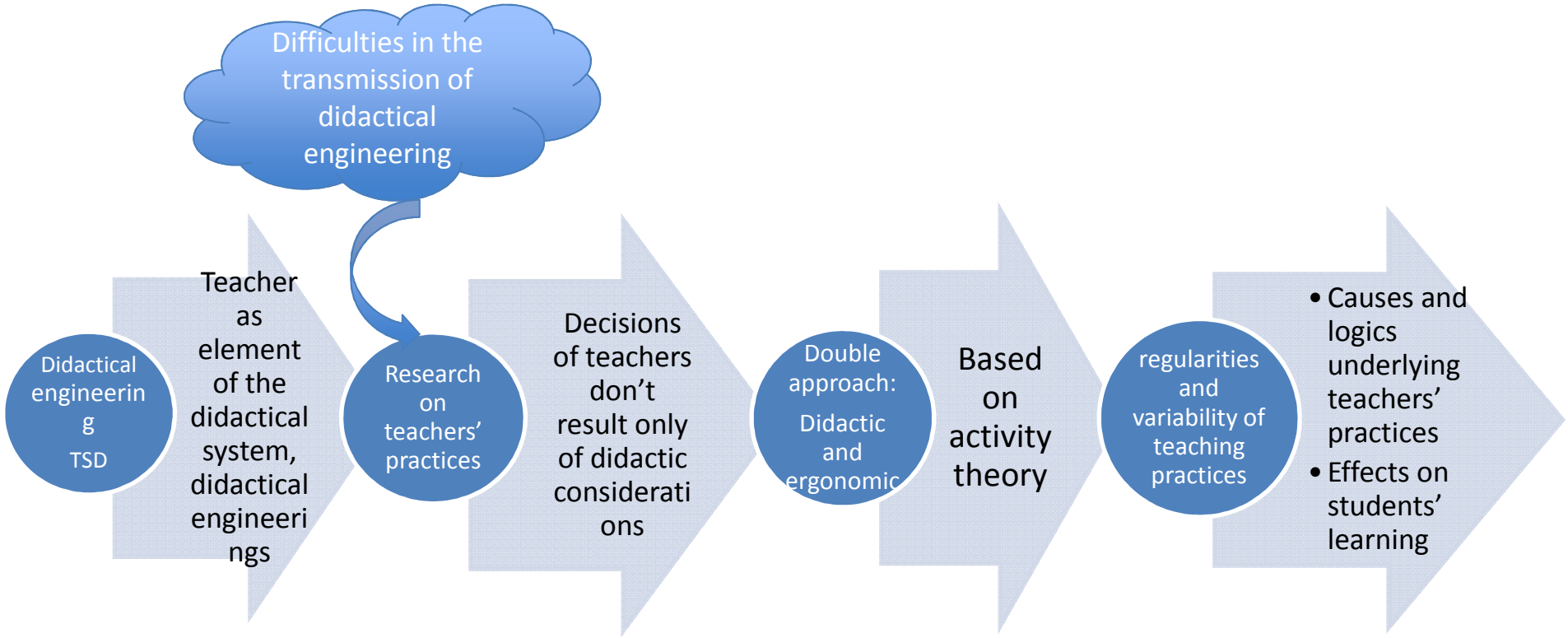
In the 80's, development of research questions on teachers' practices in several frameworks and a new approach emerged

-> Robert and Rogalski (2002, 2008, 2013) developed the ***« double approach » of teachers' practices***:

articulating ***a didactic and an ergonomic points of view*** on teachers' practices,

based on ***activity theory***

Emergence of a new framework



- Research on ***regularities and variability of teaching practices*** according to contexts, mathematical subjects, grades, teachers etc.
- Questioning
  - Differences and similarities,
  - rationales underlying teachers' practices,
  - causes,
  - effects on students' learning

Aurélie Chesnais (2009): ***comparison of the teaching and learning of axial symmetry between a socially disadvantaged area school and an « ordinary » one***

The research was based on both ***« natural observation »*** and an ***experiment*** consisting in ***the transmission of a teaching scenario*** about axial symmetry elaborated by one teacher in an ordinary context to another one teaching in a socially disadvantaged area.

Results:

socially disadvantaged students can do as well as « ordinary » ones provided that certain conditions are fulfilled

reasons for teachers' choices are multiple

collaborative work between teachers might be a good lever for professional development under certain conditions

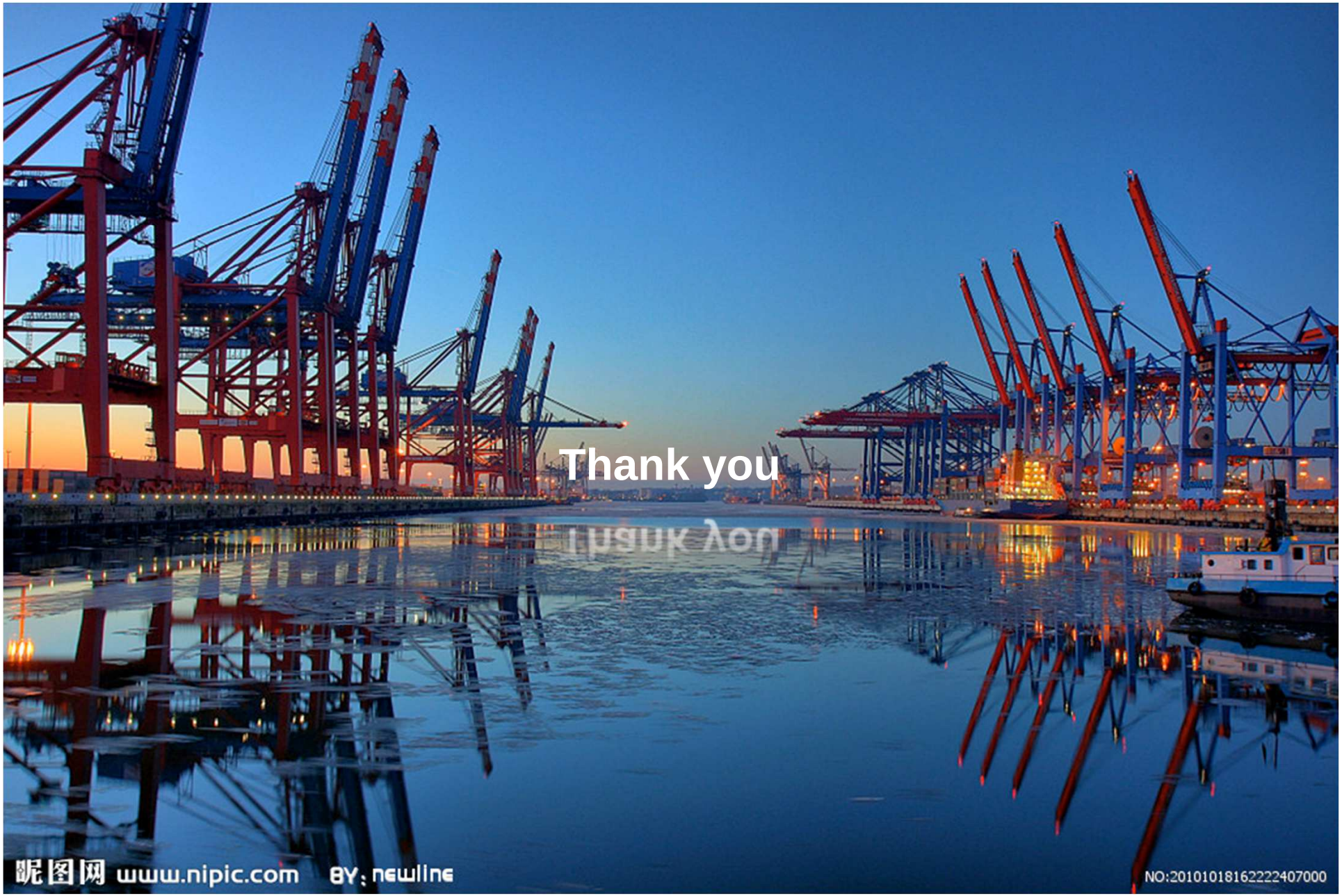
Current research on the role of language in the learning and teaching of mathematics involving symmetry

Language as

- an ***object of learning***,
- a ***media for learning and for teaching***
- a ***methodological object for researchers*** to get access to students' and teachers' activity

- Symmetry as a ***property*** of a given figure.
- Axial symmetry as a ***ternary relationship*** involving two figures and an axis *versus* a ***geometrical transformation*** involving points

➢ Good subject to study the ***relationships between action and language in mathematical activities*** involving symmetry.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We need to ground mathematical proofs also on geometric judgments which are no less solid than logical ones: ***"symmetry", for example, is at least as fundamental as the logical "modus ponens"***; it features heavily in mathematical constructions and proofs. (Longo, 2012)

Second generation didactical engineering

- Development of teaching ressources
- Collaborative research
  - means of professional development
  - research on teaching practices