

What number does the orange arrow point to?
Explain how you figured it out.



UNCOVERING THE SPECIAL MATHEMATICAL WORK OF TEACHING*

Deborah Löwenberg Ball
University of Michigan USA

dball@umich.edu • @deborah_ball

30 Juli 2016 • Hamburg Deutschland
13th International Congress of Mathematics Education (ICME)

*My slides will be posted on my website.



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

ACKNOWLEDGMENTS

- Hyman Bass and Mark Hoover
- Imani Goffney
- My students: Esther Enright, Susanna Farmer, Rebecca Gadd, Lauren Hickman, Lindsey Mann, Blake Noel, Sabrina Salazar, Rachel Snider, Charles Wilkes, Amber Willis
- Meghan Shaughnessy



30.07.2016

2



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

A SORT-OF-SCARY MYSTERY, IN THREE PARTS

1. *A Story of a Journey*
2. *Getting a Little Bit Lost*
3. *Finding Our Way, Using Tools and Maps*



30.07.2016

3



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

Chapter 1: *A Story of a Journey*



30.07.2016

4



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

IN SEARCH OF TEACHERS' MATHEMATICAL KNOWLEDGE

*ONCE UPON A TIME, long, long ago,
mathematics educators, mathematicians, and
policymakers were discussing how to make
mathematics teaching better.*



30.07.2016

5



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

IN SEARCH OF TEACHERS' MATHEMATICAL KNOWLEDGE

Teachers must know mathematics in order to teach it.

What mathematics do teachers **NEED** to know?

What mathematics **DO** teachers know?

Teachers don't know enough mathematics..

Teachers should major in mathematics/take more courses.

A RELUCTANT DISCOVERY, AND A NEW QUESTION

The amount of mathematics
a teacher studies does not
ensure good mathematics
teaching!

What mathematical
skill and insight DOES
teaching require?



30.07.2016

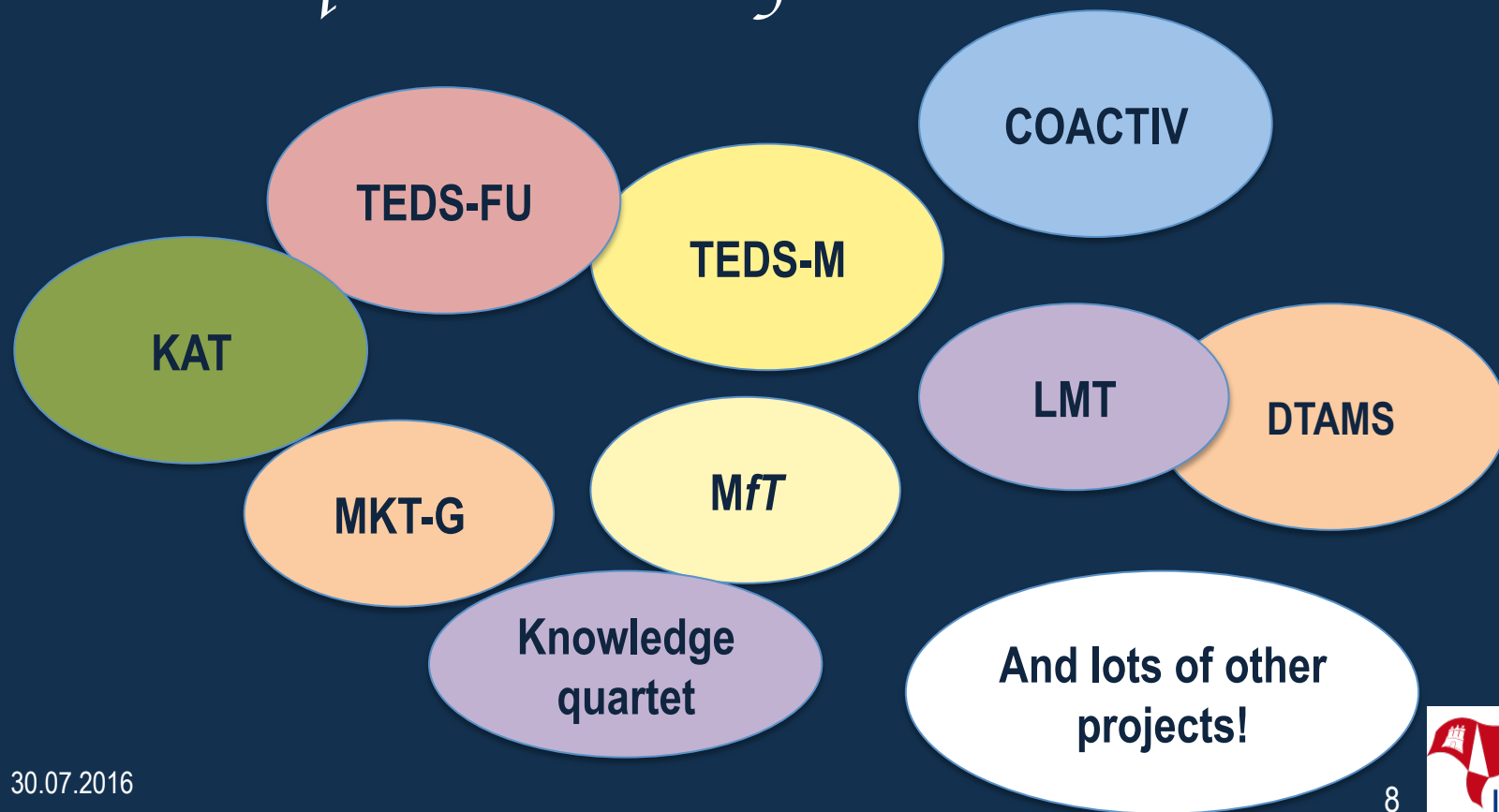
7



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

Many new efforts began around the world, to ask better questions and find better answers.



FROM NEED—TO USE

How is mathematics
USED in teaching?

Knowledge quartet
(Rowland)

Noticing learners
(Phillips, Sherin, Jacobs)

Responding to
instructional
situations
(Herbst and Chazan)

Mathematical
perspective
(Klein project)

Situated reaction
competency
(COACTIV)

Tasks of
teaching
mathematics
(LMT)

Equitable teaching
(Goffney)

Mathematical
discourse in
instruction (Adler)

Commognition
(Sfard)

Chapter 2: Getting a Little Bit Lost



30.07.2016

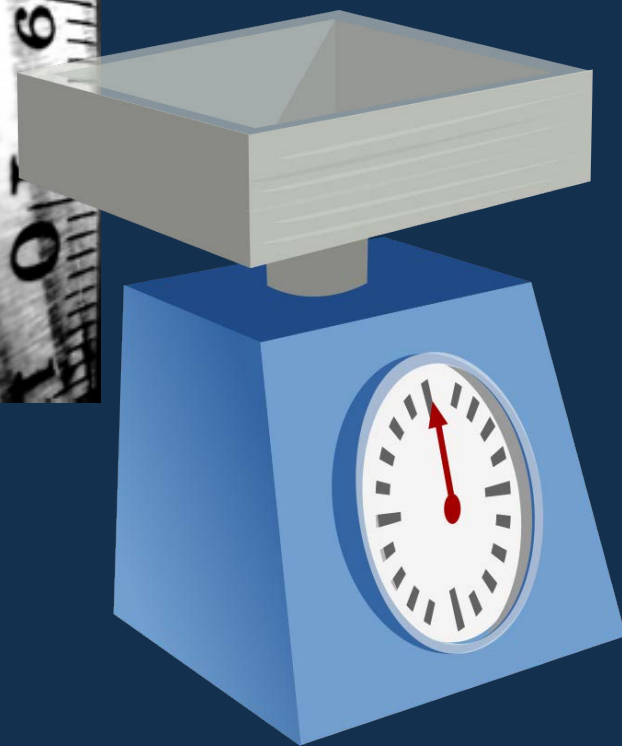
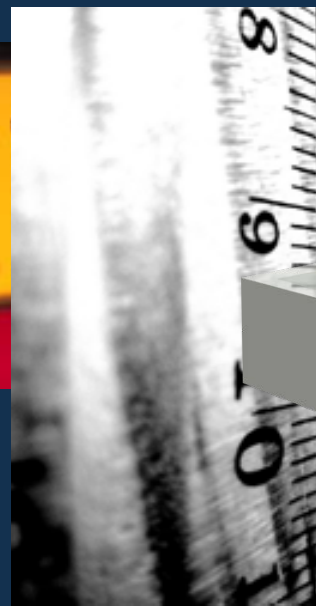
10



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

BUILDING TOOLS TO “MEASURE”



Suddenly it was really valued to be measuring teacher knowledge. Many of the projects developed items and tasks to measure teachers' mathematical knowledge.

HOW DID THE MEASUREMENT WORK AFFECT OUR JOURNEY?

How is mathematics
USED in teaching?

How does
mathematics help
make teaching
GOOD for learners?



30.07.2016

12



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

HOW DID THE MEASUREMENT WORK AFFECT OUR JOURNEY?

ADVANCES

- Established that there are special kinds of knowing for teaching
- Developed ways to study outcomes of teacher education and professional development

IMPEDIMENTS

- Fell back from practice to knowledge (from sociocultural view to cognitivist view)
- Not fully dynamic, about what teachers actually have to DO mathematically
- Compartmentalized teaching— e.g., attention to equity



30.07.2016

13



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

Teachers' competencies

Teachers' knowledge of some mathematics

Teachers' pedagogical content knowledge

Teachers' professional knowledge

Teachers' values and beliefs

Teachers' reasoning



mathematical
knowledge
values, beliefs, PCK, etc.



mathematical
work of teaching

teaching and
learning



WHAT IS THE “WORK” OF MATHEMATICS TEACHING?



NEXT CHAPTER IN OUR STORY

*How to calibrate our perspectives to try to
see, name, and understand
the actual
mathematical work of teaching?*



30.07.2016

17



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

Chapter 3: Finding Our Way, with Some Tools and Maps



30.07.2016

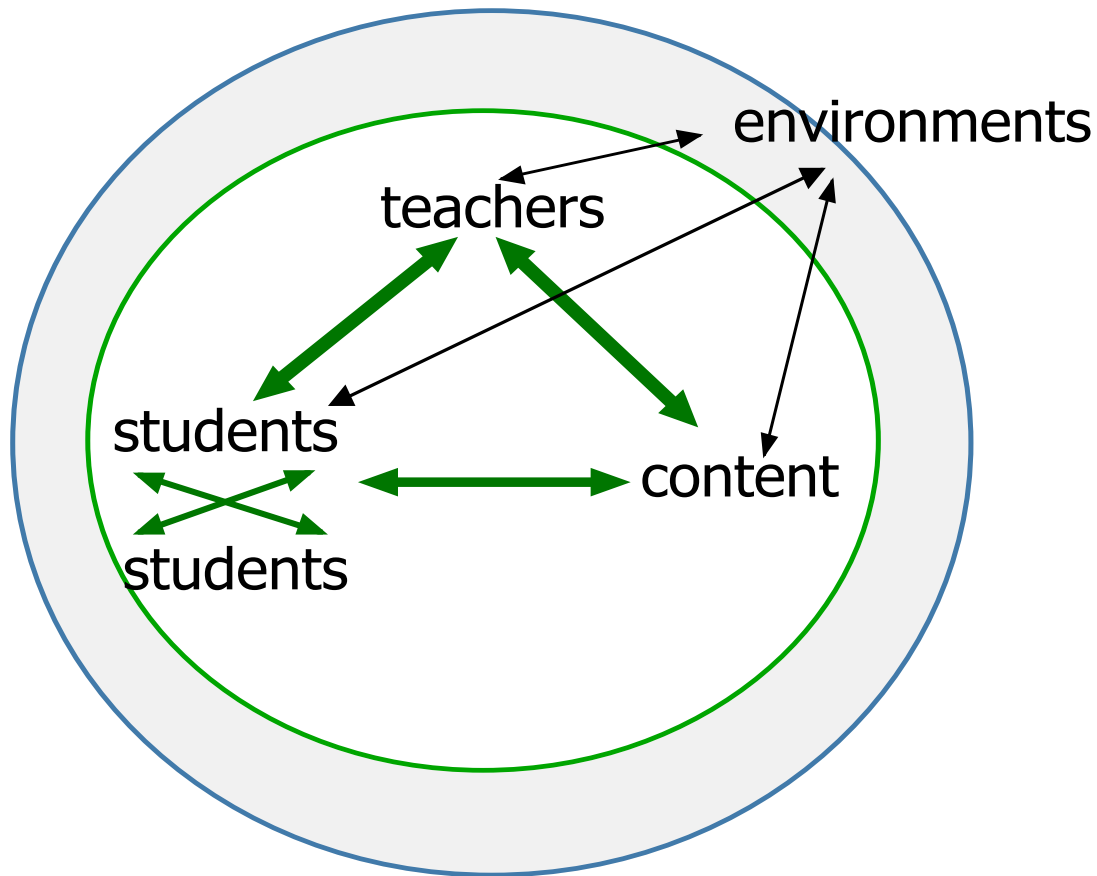
18



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

WHAT IS TEACHING?

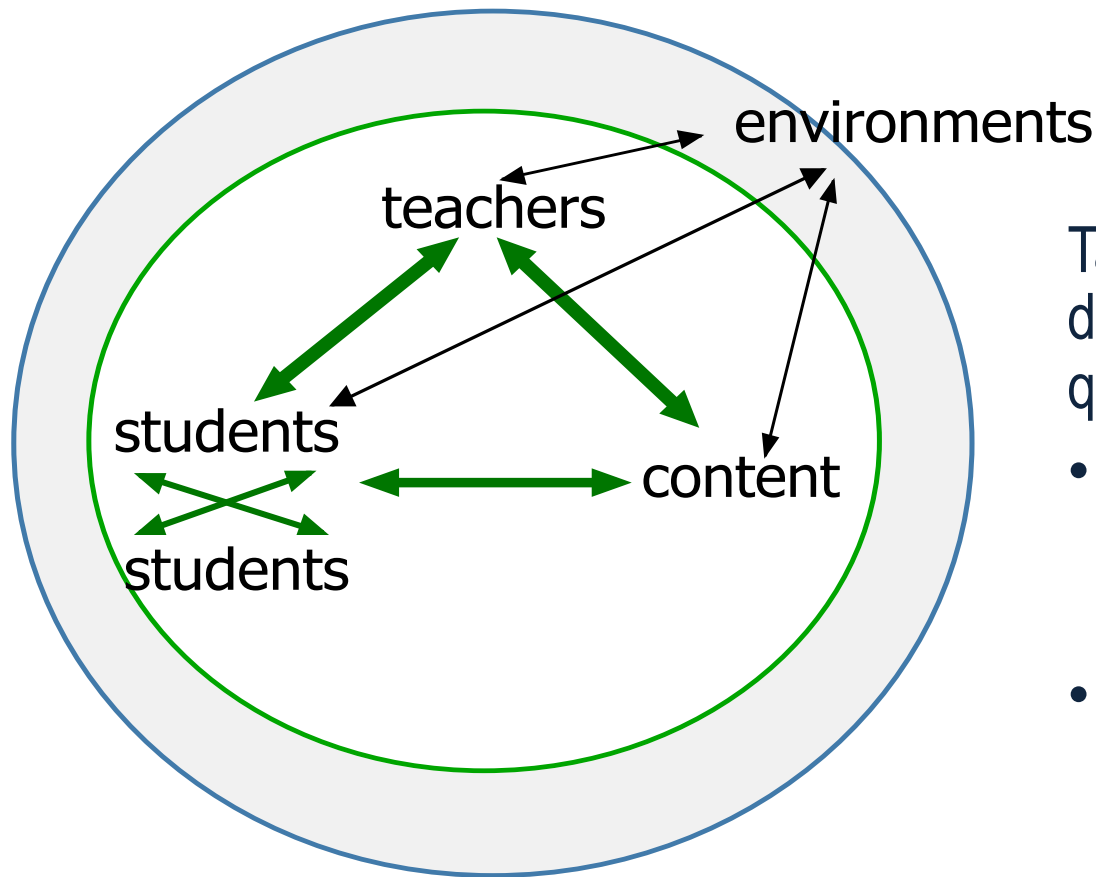


Teaching is co-constructed

- . . . in broad socio-political, historical, economic, cultural, community, family environments
- . . . through the interpretations and interactions of teachers, students, and “content”

Cohen, Raudenbush, and Ball (2003)

WHAT IS THE WORK OF TEACHING?



Taking responsibility for deliberately maximizing the quality of these interactions . . .

- . . . in ways that maximize the probability that students learn
- . . . worthwhile content and skills

WHY “**WORK**” OF TEACHING”?

1. To focus our attention on what teachers DO and to distinguish this from other features of classrooms, such as instructional formats, classroom culture and norms, what students are doing, how the curriculum is designed

But what about small group work, or problem solving with open-ended problems, or seatwork? Aren't those what teachers DO?

WHY “**WORK**” OF TEACHING”?

1. To focus our attention on what teachers DO and to distinguish this from other features of classrooms, such as instructional formats, classroom culture and norms, what students are doing, how the curriculum is designed
2. To honor the effortful and deliberate nature of teaching and not to leave it invisible, implicit, and taken for granted

TRYING TO SEE THE WORK OF TEACHING

Taking a socio-cultural perspective on teaching and learning, and drawing on many others' work:

- Discursive nature of teaching and learning (e.g., Sfard, Adler, many others)
- Diversity: language, identities, race and ethnicity, class, gender

means that there is something to the **mathematically interactive, discursive, and performative work of mathematics teaching** that is important to understand.

WHY “**MATHEMATICAL**” WORK OF TEACHING”?

To look at how **mathematical** listening, speaking, interacting, acting, fluency, and doing **are part of** the work of teaching, not just resources for it

This is what I mean by “**the special mathematical work of teaching.**”

Let's look!



30.07.2016

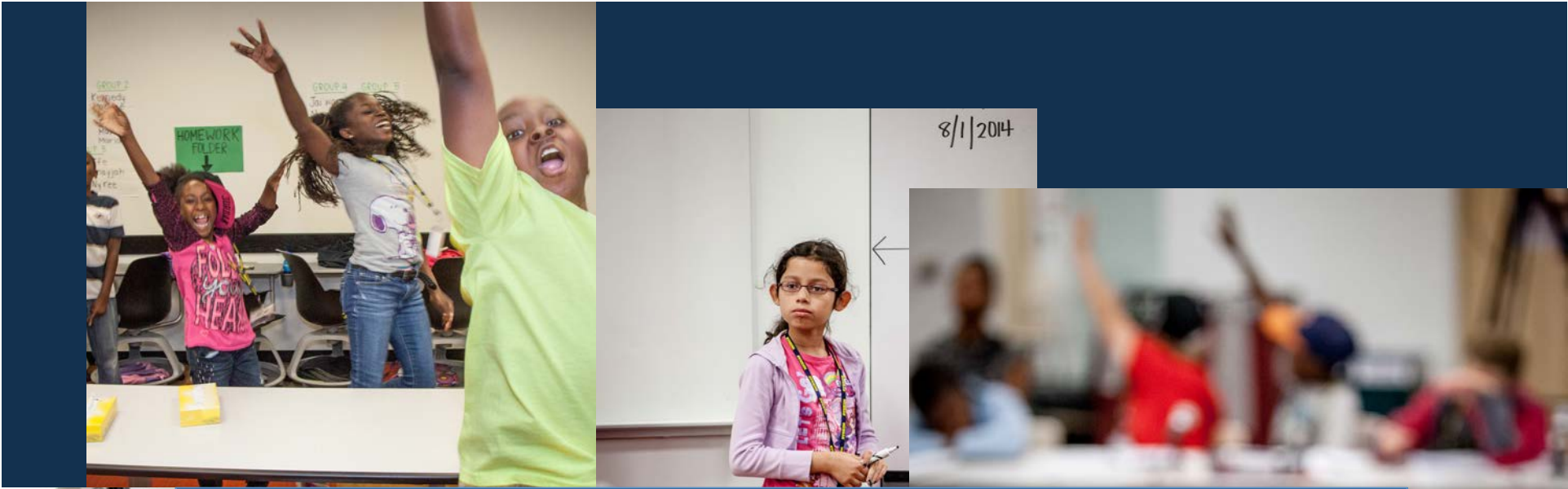


This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

24





Grade 5 learners, U.S.
29 pupils
21 African American, 4 Latin@, 4 White
Low-income community
Most children have been unsuccessful in school mathematics



AN INHERENT FACT OF TEACHING

Is that we are always communicating, relating, and making sense across difference, including:

- Age
- Gender
- Race, ethnicity, culture, religion
- Identities
- Language
- Experience

So this means that fundamental to the work of teaching is attuning to other people, and being oriented to others' ideas and ways of thinking and being



30.07.2016

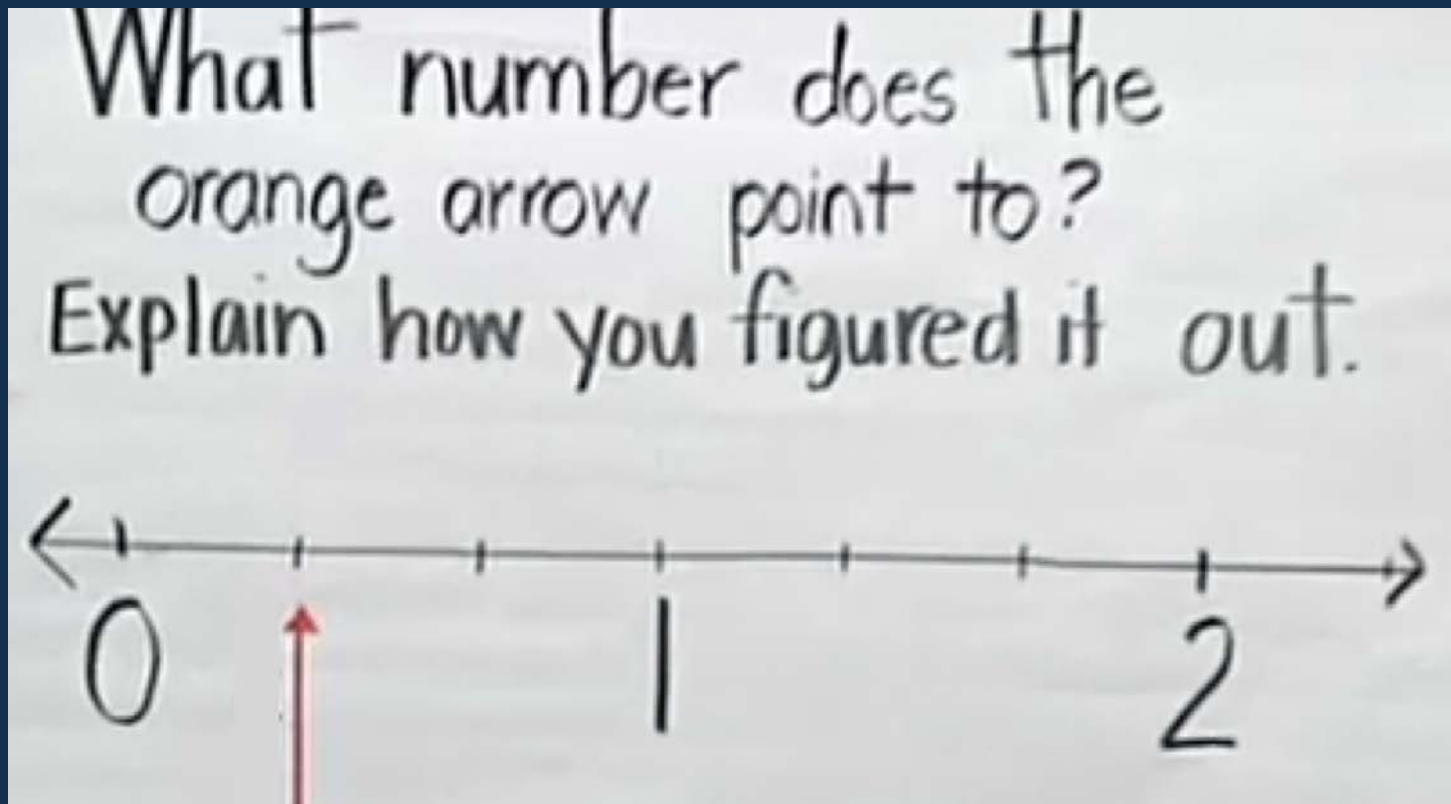
26



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

THE WORK OF USING A MATHS PROBLEM



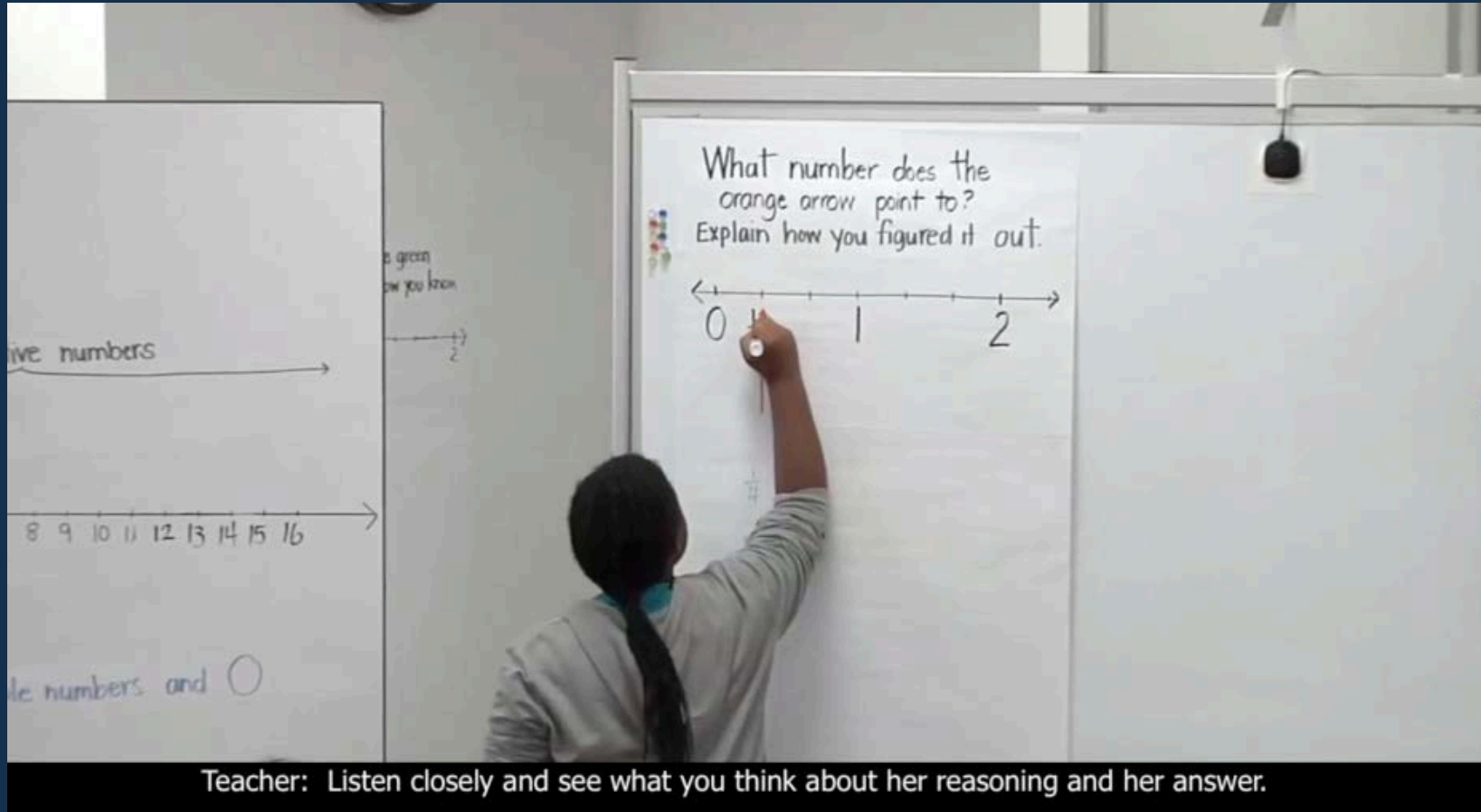


ANIYAH



TONI

VIDEO: WHAT MATHEMATICAL WORK OF TEACHING CAN YOU IDENTIFY?



WE WILL WATCH ANIYAH AND TONI AGAIN, AND A LITTLE MORE, BUT FIRST . . .

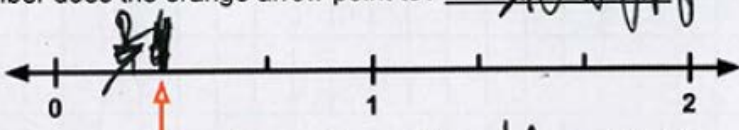
Try a little mathematical work of teaching.

- While all the children are working on this task, before Aniyah goes to the board, the teacher circulates around the room.
- The teacher scans what children are writing to decide how to discuss solutions.

READING STUDENTS' WORK

2/4

What number does the orange arrow point to? $\frac{2}{4}$



Explain how you know: because there
four equal parts and
you are pointing to the
second one so its $\frac{2}{4}$

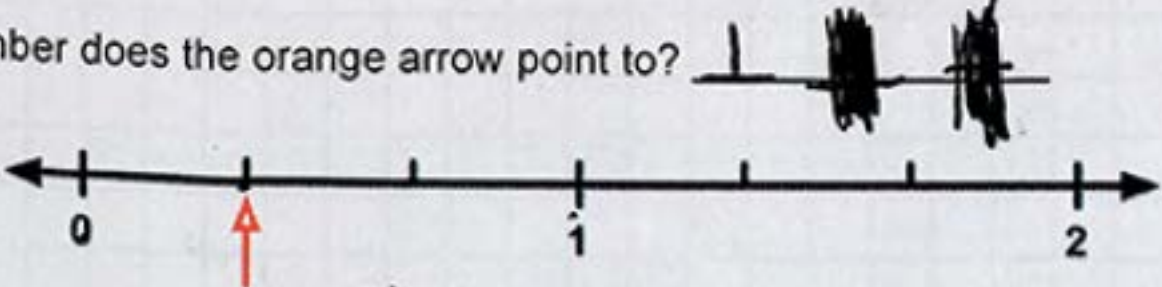
Write a complete sentence with one goal for yourself for our math class
Give an example of what it looks like to do this really well.

to listen to other people's
ideas. Like just because
I know the answer that
I will still listen to others.

READING STUDENTS' WORK

1

What number does the orange arrow point to?

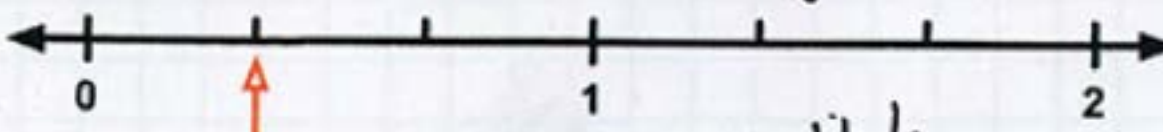


Explain how you know: There's 0 2 spaces
1 2 spaces then 2 2 spaces
in those 2 spaces
are fractions

READING STUDENTS' WORK

1/4

What number does the orange arrow point to? ~~1/4~~ $\frac{1}{4}$

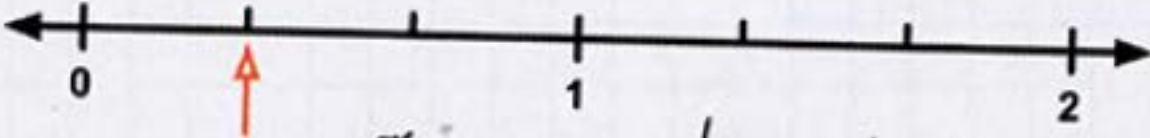


Explain how you know: because it's
divided in ~~4~~ 4 units and
it's 1 of of the parts.

READING STUDENTS' WORK

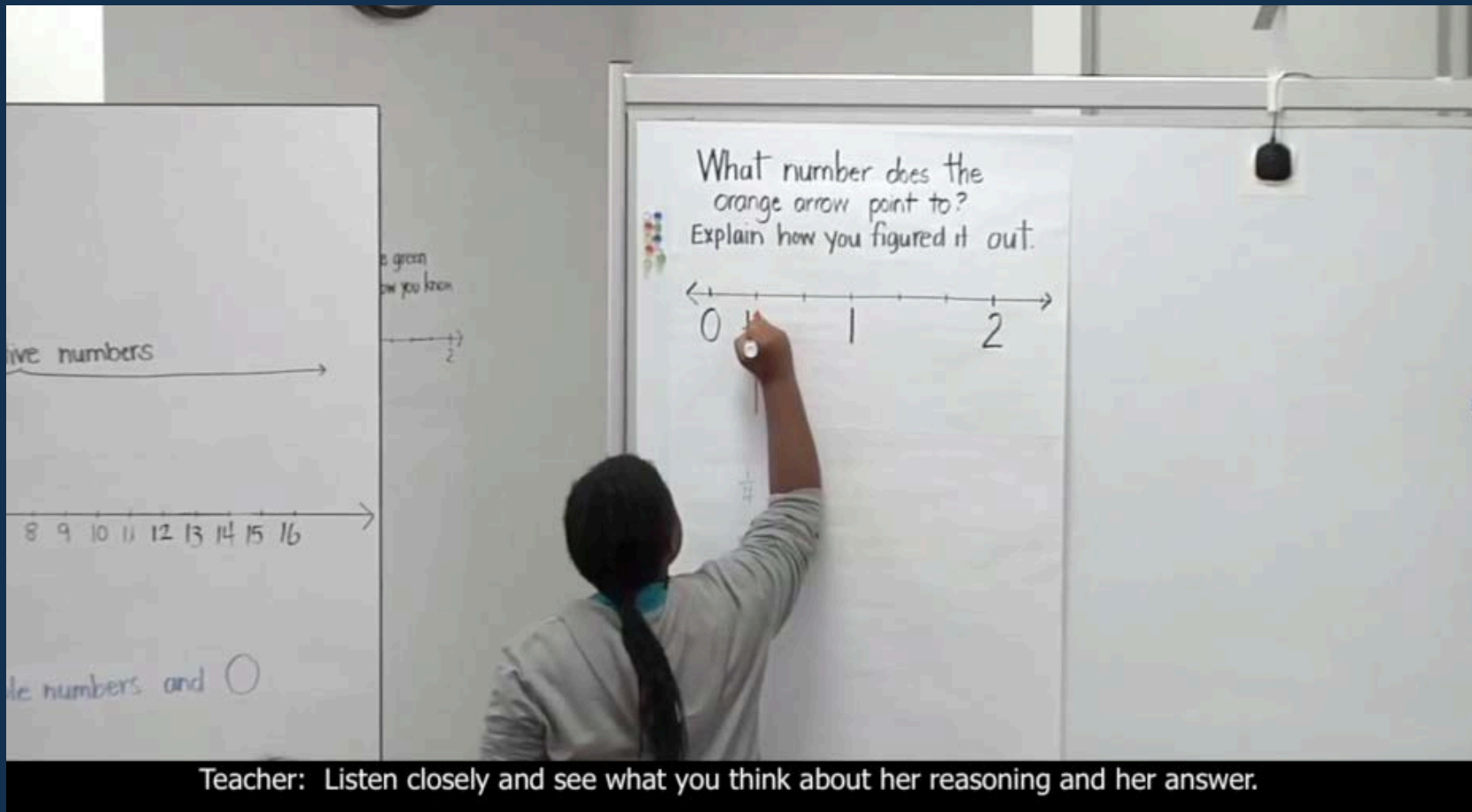
$\frac{1}{3}$ without mathematical explanation

What number does the orange arrow point to? $\frac{1}{3}$



Explain how you know: There is two
lines instead of
3 plus the 1 equal
3.

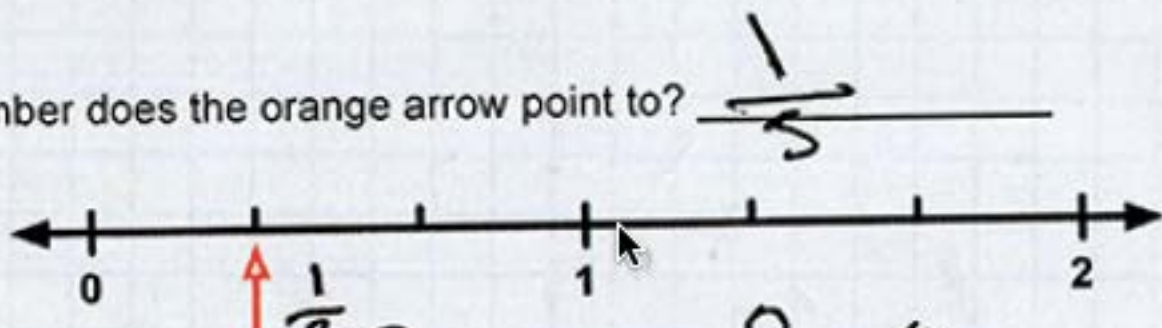
VIDEO: WHAT IS THE MATHEMATICAL WORK OF TEACHING IN THESE THREE MINUTES?



TONI

What is the mathematical work of teaching in these three minutes?

What number does the orange arrow point to? $\frac{1}{3}$

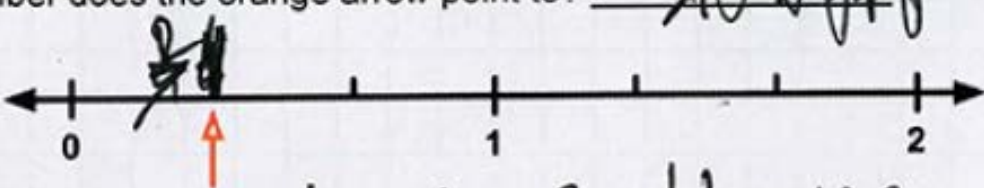


Explain how you know: Because it's in
3 parts

LAKEYA

What is the mathematical work of teaching in these three minutes?

What number does the orange arrow point to? ~~2~~ ~~1~~ ~~1~~ ~~2~~ ~~2~~



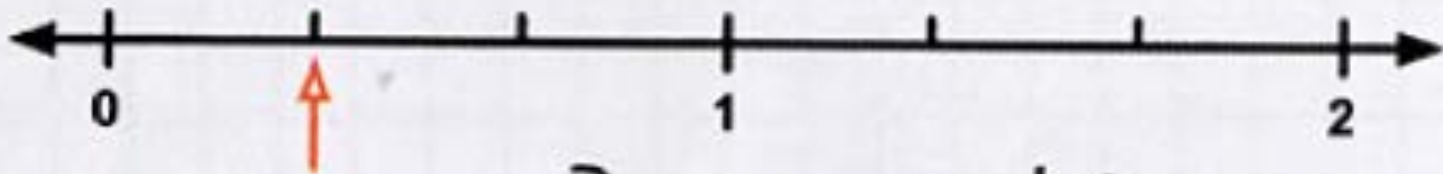
Explain how you know: because there
^{seven} four equal parts and
you're pointing to the
second one so it's $\frac{2}{4}$

DANTE

What is the mathematical work of teaching in these three minutes?

What number does the orange arrow point to? _____

$\frac{1}{2}$



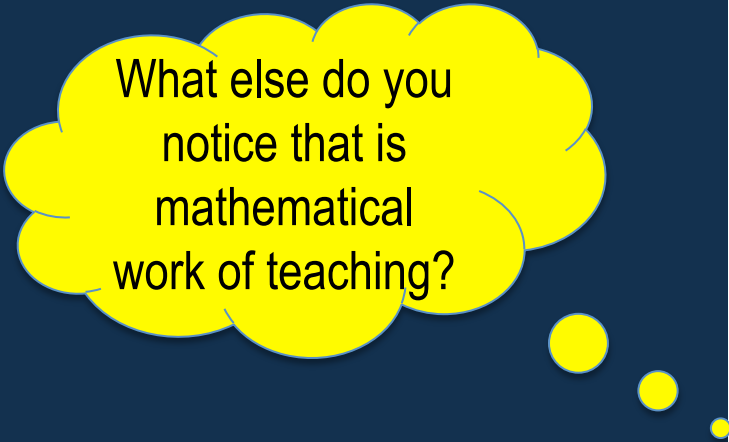
Explain how you know: _____

Because if you look at it and count.

THE MATHEMATICAL WORK OF TEACHING

First example:

- Hearing, seeing, and reading students, in “real time”
 - During a class discussion
 - While circulating in the classroom
 - When reading students’ writing



What else do you notice that is mathematical work of teaching?

Second example:

- Assigning competence

A SECOND EXAMPLE: ASSIGNING MATHEMATICAL COMPETENCE

1. Broaden and name what being competent in mathematics means
2. Intervene to position who (and what) is seen as competent in maths class
3. Support individual children to develop their mathematical and academic identities and competence

Sources: E. Cohen and R. Lotan, complex instruction; J. Boaler's work; *Smarter Together: Collaboration and Equity in the Elementary Mathematics Classroom* (Featherstone, Crespo, et al., 2011);



30.07.2016

40



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu



ANIYAH



LAKEYA



TONI



DANTE



What does **Aniyah** know
and what can she do?
What does **Toni** know
and what is she able to
do?



WHAT DO MANY “HEAR” IN ANIYAH AND TONI?

ANIYAH

- She has the wrong answer: 1/7

TONI

- She is playing with her hair and trying to get attention
- She is trying to embarrass Aniyah



30.07.2016

43



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

WHAT DO ANIYAH AND TONI KNOW AND WHAT CAN EACH DO?

ANIYAH

- Uses the definition for a fraction to explain
 - She identifies the “whole”
 - She makes sure the intervals are equal
 - She counts intervals and not tick marks
 - She knows how to write “one-seventh”
- Produces a mathematically well-structured explanation
- Presents her ideas clearly

TONI

- Listens closely to a classmate’s presentation
- Uses the definition for a fraction to ask
 - How Aniyah decided on 7 parts
- Asks a pointed mathematical question

ANOTHER EXAMPLE: TRANSLATING INTO LEARNER LANGUAGE

3.NFA.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NFA.1 and 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

MP.1. Make sense of problems and persevere in solving them.

MP.3. Make and critique mathematical arguments.



But how can I
translate this in ways
that make sense for
my learners?



30.07.2016

45



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

TRANSLATING: DEFINING A FRACTION

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts.

- Figure out what the whole is.
- Figure out if the whole is divided into equal parts.
- If not, make equal parts.
- Count how many equal parts there are.
- Write $1/d$ to show one of the equal parts. This is a unit fraction.

Understand a fraction a/b as the quantity formed by a parts of size $1/b$.

- If more than one of those parts is shaded, count them (n) and write n/d .

Steps for Naming a Fraction Correctly

1. Figure out what the **whole** is.
2. Figure out if the whole is divided into **equal parts**. If not, make equal parts.
3. Count how many equal parts there are.
4. Write $\frac{1}{d}$ to show one of the equal parts. This is a **unit fraction**.
5. If more than one of those parts is shaded, count them (n) and write $\frac{n}{d}$.

6. $d \neq 0$ 7. d is a whole number now.

TRANSLATING: DEFINING A FRACTION

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts.

- Figure out what the whole is.
- Figure out if the whole is divided into equal parts.
- If not, make equal parts.
- Count how many equal parts there are.
- Write $1/d$ to show one of those parts. This is a unit fraction.

Understand a fraction n/d as the quantity formed by a part of a partitioned whole.

- If more than one part is needed, count them (n) and write n/d .

I need to “talk” the idea of $1/d$ in ways that make sense for my learners.

Steps for Naming a Fraction Correctly

1. Figure out what the **whole** is.
2. Figure out if the whole is divided into **equal parts**. If not, make equal parts.
3. Count how many equal parts there are.
4. Write $\frac{1}{d}$ to show one of the equal parts. This is a **unit fraction**.
5. If more than one of those parts are needed, count them (n) and write $\frac{n}{d}$.

7. d is a whole number now.

MATHEMATICAL WORK OF TEACHING

Some examples:

- Hearing students, reading students
- Translating across many differences
- Speaking mathematically fluently and across differences
- Building students' mathematical identities
- Using mathematical tasks as tools for students' learning



30.07.2016

48



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

WHAT MAKES THIS MATHEMATICAL WORK?

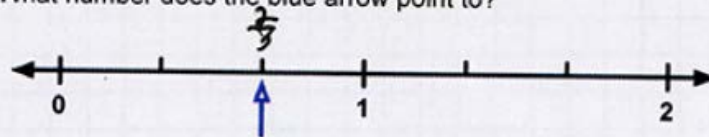


Steps for Naming a Fraction Correctly

1. Figure out what the **whole** is.
2. Figure out if the whole is divided into **equal parts**. If not, make equal parts.
3. Count how many equal parts there are.
4. Write $\frac{1}{d}$ to show one of the equal parts. This is a **unit fraction**.
5. If more than one of those parts is shaded, count them (n) and write $\frac{n}{d}$.

6. $d \neq 0$ 7. d is a whole number now.

1. What number does the blue arrow point to?



Explain how you figured this out:

Because you half to count by the equal measurements as the other ones.

LEARNING TO SEE THE MATHEMATICAL WORK OF TEACHING IS A COLLECTIVE AGENDA, NOT ANY ONE PROJECT

New scholars, many of you doing dissertations to advance understanding and improvement of teaching and learning

FOR EXAMPLE:

- Developing common language
- Studies of classroom activities and norms
- Investigating the enacted curriculum
- Theorizing about and studying mathematical discourse
- Professional knowledge and competencies
- International comparisons of classrooms
- Equitable, anti-racist, liberatory pedagogy

TO TRY TO DEVELOP BETTER UNDERSTANDING OF THE WORK OF TEACHING IN AND ACROSS CONTEXTS



30.07.2016

50



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

OUR WORLD, AND OUR FUTURE

- There are 2,600,000,000 people under the age of 18 in our world. 1,900,000,000 are in schools.
- Almost 40% of the world's population.

How can we teach mathematics so that people stop killing each other?

- Maisha Winn, inspired by Ihab Hassan



30.07.2016

51



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

Skillful and caring teaching
is a resource too powerful
to leave to chance.

Not The End



30.07.2016

52



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

VIELEN DANK

- The ICME-13 organizers for this honor
- My doctoral students, present and past
- My young students, the children I have taught for more than four decades
- My colleagues at the University of Michigan
- My U.S. and international professional colleagues

Email: dball@umich.edu



30.07.2016

53



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License: <https://creativecommons.org/licenses/by-nc-nd/4.0/>

© 2016 Deborah Loewenberg Ball • School of Education • University of Michigan • Ann Arbor, MI 48109 • dball@umich.edu

CREDITS



Image on slide 11 (cropped and rotated):

“Ruler” by Flickr user Scott Akerman

Licensed under a Creative Commons Attribution 2.0 Generic License

<https://creativecommons.org/licenses/by/2.0/>

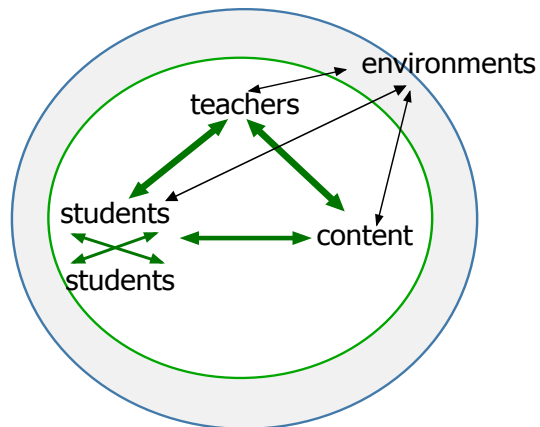


Image on slides 14 and 15:

“LOMO MBR-3 Microscope” by Flickr user Ian Sutton

Licensed under a Creative Commons Attribution 2.0 Generic License

<https://creativecommons.org/licenses/by/2.0/>



Graphic on slides 19 and 20:

Cohen, D. K., Raudenbusch, S., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, 25 (2), 119-142.