Mathematics, Education, & Culture: A contemporary moral imperative

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The Pleasure Principle

Let me begin with the pleasure we all get from mathematics.

The pleasure when we see the uncomplicated equation

\[ z_{n+1} = z_n^2 + c \]

transform into the Mandelbrot Set:
The Pleasure Principle

The pleasure when we first see a visual proof e.g. that the sum of consecutive triangular numbers will be a square number:

\[
\begin{align*}
(1 + 3) + (3 + 6) &= 4 + 9 \\
&= 16
\end{align*}
\]
The Pleasure Principle

The pleasure of realising that the sum of sines formulae we learned so arduously in trigonometry at school explains what surfers know from experience: big waves come in threes.

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$
$y = \sin(7x) + \sin(6x)$
The Pleasure Principle

And you will have your own examples.
If you would like to see some more, see the Klein Project—an ICMI/IMU project to make contemporary mathematics accessible to teachers for their pleasure and inspiration.

Google “klein project blog”
Or go to < http://blog.kleinproject.org >
Feel free to do this now......
The Pleasure Principle

Wouldn’t it be nice if the pleasure we get from mathematics imbued mathematics education at every level at all times.

xxxxxx

I will come back to this, .....
I have been invited to follow-up Ubiratan D’Ambrosio’s ICME-5 plenary address in Adelaide in 1984, in which he introduced the concept of ethnomathematics.
An Agenda

Ubiratan outlined for us what we must do in order to properly address inequity, racism, and Eurocentrism in mathematics education. Many have worked on this agenda, critiquing and building on his theoretical constructs.
“Mathematics is regarded as the dorsal spine of Modern Civilization. It is the basis upon which Science, Technology and Human Behavior relies. On the other hand, no one disagrees that Science and Technology have controversial effects on the quality of life, threatening the basic conditions for sustainable life. And that Human Behavior is, frequently, despicable.”
Goals

Ubiratan suggests that our essential goals are: **responsible creativity** and **ethical citizenship**, and highlights the role of mathematics in both.

But how?

How are we to build a beautiful creature on our dorsal spine, and not a destructive monster?
But how?

Ubiratan suggests mathematics education must include its cultural roots: reinstating the essential cultural processes of techniques of doing, explaining, and knowing about our natural and social environment. He emphasises mathematical experimentation, modelling and ethnomathematical research.
But how? (2)

Today I want to give more detail and indicate practical strategies on how we can achieve D’Ambrosio’s goals. To help, I wish to invoke Ecological Systems Theory.

D’Ambrosio introduced the term *ethnomathematics* in 1977, two years before the development of *Ecological Systems Theory* by Urie Bronfenbrenner to describe child development. They have overlapping principles.
Ecology

Ecology is the study of living things in their environment. So ecological mathematics (education) is the study of mathematics (education) within its wider social, political, historical, cultural, physical, .... environment.
Ecological Systems Theory

Bronfenbrenner identified five environmental systems:

1. Microsystem: institutions and groups most directly involved.

2. Mesosystem: interactions within the microsystem and between it and the organism.

3. Exosystem: links between the organism and the wider social setting not directly related to it.

4. Macrosystem: the cultural contexts including regional or national features, socioeconomic status, and ethnicity.

5. Chronosystem: the pattern of environmental events and transitions, as well as socio-historical circumstances.
Microsystem:
institutions and groups most directly involved
Mesosystem:
interactions within the microsystem and between it and the organism
Exosystem:

links between the organism and the wider social setting not directly related to it
Macrosystem: the cultural contexts including regional or national features, socioeconomic status, and ethnicity
Chronosystem:

The pattern of environmental events and transitions, as well as socio-historical circumstances
An Expanded Research Field

So Bronfenbrenner’s environmental systems expand the research field of the sociology of mathematics education, and heighten the imperative for ethnomathematical understanding—but we still do not know how to help students achieve responsible creativity and ethical citizenship.

A little more theoretical help......
Ecological Humanity

This seeks to bridge the divide between science and the humanities. (See also C.P. Snow’s *The Two Cultures*).

*Its assumes that the organic and inorganic worlds are a single linked system. In this system justice (and, for example, education) are enlarged into the wider environment.*

A consequence is a requirement to take account of “plural and diverse knowledges”.
Ecological Humanity

Connections between organisms (humans and other living things) define how the whole system works, and thus constitute laws of existence and guide behaviour.

In other words, to the extent that we free mathematics (education) from culture, we are freeing ourselves of social and cultural responsibility.

That makes us amoral.
Deep Ecology

An ideology developed by the Norwegian Arne Naess, who argues that the prevailing approach to environmental management is anthropocentric, and our environment is not only “more complex than we imagine, it is more complex than we can imagine”.

I argue that mathematics, and mathematics education, are also like this.
Deep Ecology

Deep ecology reinforces the idea that the living system encompasses the individual, so what we do must be guided not just by altruism to each other, but to “biospherical equality”.
Out of this theory I distill three principles for carrying forward D’Ambrosio’s agenda in both mathematics and mathematics education.

The Perspective Principle

The Reflexive Principle

The Pleasure Principle
The Perspective Principle
Be aware of other ways of understanding

---x---

The Reflexive Principle
Do unto others as you would have them do unto you

---x---

The Pleasure Principle
Act so as to increase pleasure

---x---

These operate on a system level, as well as on an individual level.

Let me illustrate with examples...
The Perspective Principle
Mathematics

Mathematics illustrates, par excellence, Naess’ comment “not only more complex than we imagine, it is more complex than we can imagine.” As are the consequences of developing any particular mathematical idea. This highlights the question of responsibility for what it may lead to. IMU is the body which can take the lead by developing an interest in their relationships with the exo-, macro- and chrono-systems.
Was the mathematics community responsible for the Global Financial Crisis?

Yes ... or maybe no. The argument still rages.

“Banks did not listen to us enough”
“Our models worked throughout the crisis”
“It was greed, not models that caused the crash”
“Everyone knows risk cannot be 100% calculated”

In 2009 Wilmott developed an Ethical Manifesto for inventors of financial models.
The Reflexive Principle
Mathematics

“Do unto others only what you would be happy to have done unto you”

Mathematicians rightly expect teachers to continue to love mathematics; do they imbue mathematics education with love?

Do mathematicians evaluate themselves and colleagues, and dismiss annually the 20-30% who do most poorly?

If not why do they evaluate students in this way?
But let us turn these questions on ourselves, the members of the mathematics education community. I know that mathematicians nurture their love for mathematics. Do we as teachers of mathematics similarly nurture our love for the subject? If not why not?

What have been the exo-, macro-, and chrono-system characteristics that have led to this situation in your environment?
The Perspective Principle
Mathematics Education

This is the Ethnomathematical Agenda: understanding other points of view, assuming there are always more ways of understanding a situation.

In mathematics education a universal activity is to allow all students to begin with Ubiratan’s “essential cultural processes of techniques of doing, explaining, and knowing about our natural and social environment.”

An full understanding of the nature of conventional mathematics must start from there.
The Perspective Principle
Mathematics Education

We all need to make efforts to ensure mathematics education benefits from multiple points of view. Mea culpa: where are the translations for my talk? I believe that ICMI could do more in this respect, making available more funding for conferences and meetings to be more “language friendly”. The Klein Project is one model of how this could work.
The Reflexive Principle
Mathematics Education

“Do unto others only what you would be happy to have done unto you”

This principle comes from the ecological assumption that we are all in this together. It can be a basis for thinking about (acting upon) mathematics education for immigrants, refugees, cultural or social groups. We are the same, we have the same rights.

For example, human rights are not suspended if you cannot speak a particular language. So offering fewer mathematical opportunities in any way because of language is unacceptable.

Let me further illustrate with streaming or setting or banding or other identity ranking systems.
First: what does research say?

• No significant differences on student achievement, regardless of level or ability….. Or…..
  Slight gains for higher placed students and losses for middle and lower-placed students.
• A detrimental effect on the attitudes and self-esteem of average and below average students.
• Grouping reinforces social segregation, and accentuates socio-economic differences.
• Neither setting nor mixed ability provides an appropriate learning environment for all pupils to reach their full potential.
Streaming seen reflexively ....

Imagine that who you are allowed to dine with and what foods are available to you is determined by your ability to cook, as measured in a single 1-hr cooking exam when you turned 21.
• Article 3: No one shall be subjected ... to inhuman or degrading treatment ... .

• Article 14: The enjoyment of the rights and freedoms ... shall be secured without discrimination on any ground such as sex, race, ... or other status.
Part II Article 2:
No person shall be denied the right to education.
The Pleasure Principle
Mathematics Education

This Principle emerges from mathematics being a cultural activity. We do what we like doing. Therefore:

“If our students do not like mathematics or learning, then we are unlikely to be able to teach them much”

“Pleasure” as motivation is not momentary pleasure—it includes reward for persistence, challenge, new ideas, shared achievement.

But I fear that rather than creating opportunities for these experiences, we often create a monster on the “dorsal spine” of mathematics education.
Talking to mathematicians, none I know would rather being doing something else. 
But we do not let our students do what mathematicians do!

On the contrary, we create (or are complicit in) two monsters ...
The 14-year Apprenticeship

Imagine a carpenter’s apprentice being asked to practice all the basics of carpentry for 14 years before they are allowed to build anything real.

Or a football player, or a musician, or a ...
Frequent High Stakes Testing

*High-stakes testing* is where the results affect major decisions about schools or students. My reading of the research is:

Positive research comes primarily from the tests themselves. Achievements go up, but ... ... not as much as elsewhere, and gains do not transfer.
Frequent High Stakes Testing

*Other research:*

Evidence of diminishing curriculum.

Negative effects on individuals, teachers, and schools.

Decreasing critical thinking.
If I were a carpenter ....

- Annual national examination ...
- Results available to friends and family if not publically ...
- Visible consequences in your work.

Or a professor, landscape gardener, bank manager, Minister of Education, .....
An ICMI Responsibility?

Over the last decade ICMI has moved from an international professional organisation to taking the lead in development activities: CANP takes half our budget. Jill Adler will build and strengthen this direction.

But perhaps it is also time to be a strong (political) voice for making clear the consequences of certain practices, based on our combined research and experience.
A Personal Responsibility?

And it is certainly time for us all to be more than ever aware of how (possibly unintentionally) we reinforce poor, discriminatory, or destructive practices.

On many issues, our research is clear, and yet, by doing nothing or by staying quiet, we reinforce an immoral status quo.

We are each responsible.
Thank you for your attention

Please approach me at any time to discuss any issue from my talk.

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