PANEL

TRANSITIONS IN MATHEMATICS EDUCATION

ICME 13 Hamburg July 2016

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Hamburg 2016

Panelists

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France

Andrea A diSessa,
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Berkeley, USA

Oh Nam Kwon,
Seoul National University
Korea

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Katholieke Universiteit Leuven
Belgium

Transition questions

Springer website search (in Mathematics Education, books, book chapters, articles): “Transition(s)”

2680 results: 4 books, 1180 book chapters, 1496 articles

Some titles:

The role of arithmetic structure in the transition from arithmetic to algebra

Making the transition to formal proof

E-learning in secondary–tertiary transition in mathematics: for what purpose?

Transitions Between Contexts of Mathematical Practices

Mathematics Learners in Transition

Mathematics Teachers in Transition

….

Choosing a focus

A focus on two kinds of transitions

(a) conceptual change and learning as a transition process

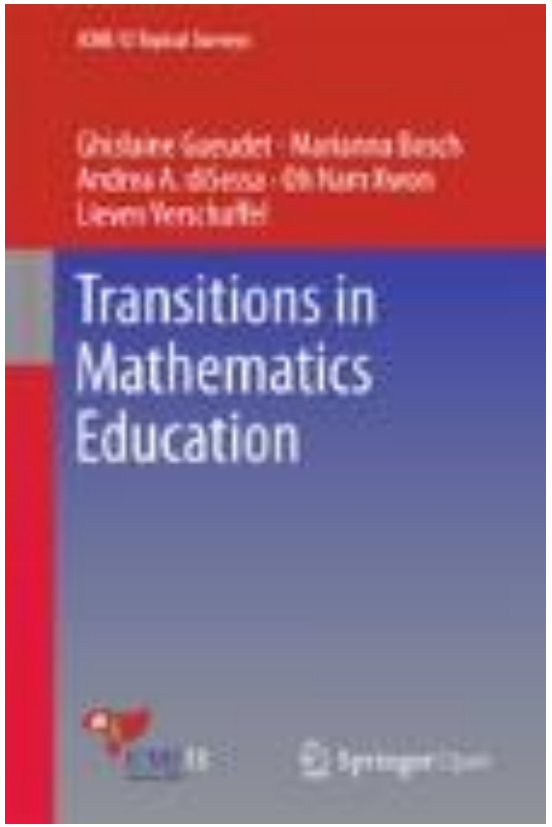
(b) transitions as people move between social groups or contexts with different mathematical practices

Choosing perspectives

Different perspectives (articulated)

- An epistemological perspective, studying changes happening within the mathematical content
- A cognitive perspective, studying changes in students' learning, in thinking modes
- A socio-cultural perspective, studying changes between different mathematical practices

A survey



Invitation to discuss

http://python.espe-bretagne.fr/transition-panel/

Outline of the panel

Individual interventions

1. Andrea A diSessa, *Continuity versus Discontinuity in Learning Difficult Concepts*
2. Oh Nam Kwon, *Double discontinuity between Secondary School Mathematics and University Mathematics*
3. Marianna Bosch, *Transitions between teaching institutions*
4. Lieven Verschaffel, *Transitions between in- and out-of-school mathematics*

Panelists discussion

Conclusion

CONTINUITY VERSUS DISCONTINUITY IN LEARNING DIFFICULT CONCEPTS

Andrea A. diSessa
University of California at Berkeley
USA

Introduction

Focus – Learning Difficult Concepts ("Conceptual Change")

- Rational number ("multiplication makes things bigger")
- Law of Large Numbers ("more samples → closer to expected value")
- Force ("movement requires a force")

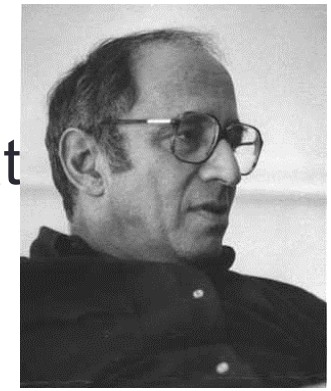
NOT: Skill acquisition; memorizing; "intellectual maturity"

Perspective: Cognitivist - individuals and their ideas

The controversy – Are there intransigent, major obstacles to learning, or is learning best construed as an extended, incremental process?

Discontinuity vs. Continuity: A (few) big "Aha(s)!" or many little "I see's"

History (Science)



Thomas Kuhn & Scientific Revolutions

- Incommensurability: "Everything must change at once"
- Gestalt switch
- Followers: S Carey; M Wiser; S Vosniadou

CONTRAST TO

Stephen Toulmin & Conceptual Ecology

- Opposed to: "the cult of systematicity"
- "Moving picture view"; Conceptual Ecology
- Followers: J Minstrell; Knowledge in Pieces Community: (A diSessa, B Sherin, D Hammer, A Elby)

History (Mathematics)

Kuhn → Gaston Bachelard & "Epistemological Obstacles"

- Core, persistent, unavoidable problems
- "Misconceptions" sub-perspective (wrong→right)
- Followers: A Sierpinska, D Tall

CONTRAST TO

Toulmin → ??

- "Pieces and Processes": Moving picture View; Conceptual Ecology
- KiP Community (J Wagner, M Levin, A Izsak) "Affiliates" (R Noss, C Hoyles, D Pratt)

Resolution (?)

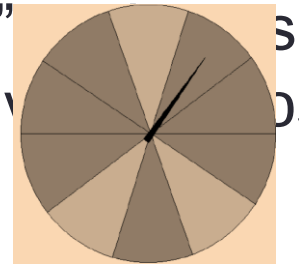
Microgenetic Learning Analysis: From "snapshot" to "moving picture"

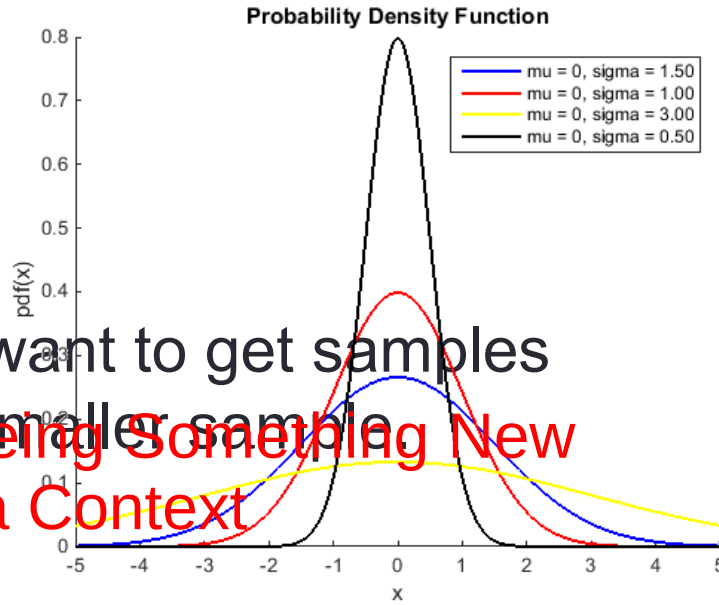
Examining, in detail, the full moment-by-moment process of learning, using all available empirical data.

M Learns the Law of Large Numbers

Incremental learning across many contexts (J Wagner)

Ma[illegible] "isomorphic" [illegible]s: If you want to get samples far[illegible] expected [illegible]ose the smaller sample.





Seeing Something New in a Context

Accuracy!

Extracting and Transporting an Idea

Final Words

I am a partisan!

Prediction: “The continuist side will win.”

Implications:

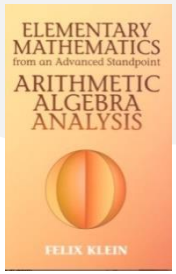
- Many **resources**, not just obstacles and misconceptions
- Incremental, multi-context learning
- **Handling diversity** in students (conceptual ecology)
- **Choice of resources for learning paths!**

Double Discontinuity
Between Secondary School Mathematics and University Mathematics

Kwon, Oh Nam
Seoul National University, Korea

Double Discontinuity

"The young found himself, at the outset, confronted with problems which did suggest, in any particular, the things with which he had been concerned at. Naturally he **forgot these things quickly and thoroughly.** When, after finishing his course of study, he was scarcely able, unaided, to **discern any connection between this task and his university mathematics**, he soon fell in with the time honoured way of teaching, and his university studies remained only a more or less pleasant memory which had **no influence upon his teaching."**



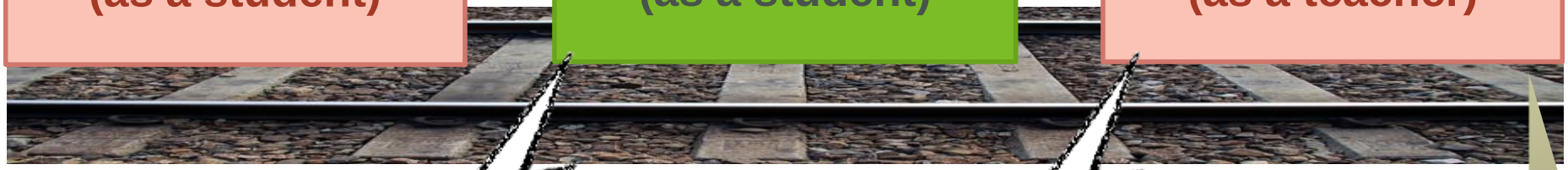
Felix Klein, *Elementary Mathematics From An Advanced Standpoint, I* (1908; English translation from 3rd edition of 1924)

Double Discontinuity

Secondary school mathematics **(as a student)**

University mathematics **(as a student)**

Secondary school mathematics **(as a teacher)**



Convergence of x_n

Concrete Intuitive

Abstract Formal

Concrete Intuitive

If n becomes larger and larger, then $\{x_n\}$ goes closer and closer to x

$$\forall \boldsymbol{\varepsilon} > \mathbf{0}, \exists \boldsymbol{N} \in \mathbb{N}$$
$$\boldsymbol{s.t.}\ |\boldsymbol{x_n} - \boldsymbol{x}| < \boldsymbol{\varepsilon},\ \forall \boldsymbol{n} > \boldsymbol{N}$$

If n becomes larger and larger, then $\{x_n\}$ goes closer and closer to x

Theorizing Teacher Knowledge

PCK(Pedagogical Content Knowledge)
- "Subject knowledge for teaching" (Shulman1986)

MKT(Mathematical Knowledge for Teaching)
- Specialized knowledge of mathematics situated in the context of teaching (Ball et al., 2005, 2009; Hill et al., 2008)

SRCK(School Related Content Knowledge)
- A type of content knowledge applied in a school context for the teaching purpose (Heinz, Lindmeier & Dreher, 2015).

- **MMTsm(Mathematical Meaning for Teaching secondary mathematics)**

- Knowing-to and knowing-why as the most important from of knowing for teachers (Thompson, 2015).

Large-scale Studies on Mathematics Teacher Knowledge

MT21(Mathematics Teaching in the 21th Century)

- *Compare prospective teachers' knowledge and beliefs about teaching and learning in six participant countries*
- *Confirm a gap in teacher preparation across countries (Bulgaria, Germany, Korea, Mexico, Taiwan, & US)*

TEDS-M(The Teacher Education and Development Study: Learning to Teach Mathematics)

- *Provide the opportunity to examine the outcomes of teacher education in terms of teacher knowledge and teacher beliefs across countries*

COACTIV (Cognitive Activating Instruction, and the Development of Students' Mathematical Literacy)

- *Mathematical knowledge needed for comprehensive-oriented instruction*
- *Content knowledge and pedagogical content knowledge can be distinguished structurally*

Developments in Teacher Preparation

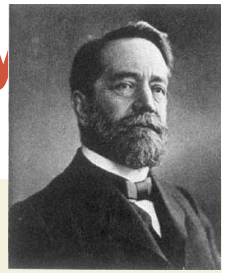
- *A weak intervention compared to one's own school experience and later professional socialization*
- *Future mathematics teachers do not succeed in acquiring the deeper mathematical knowledge needed to dismantle school-related misconceptions and solve elementary mathematical problem competently*
- *Not essentially different from the curricular for mathematics majors*
- *Fail to provide student teachers with adequate learning opportunities for CK and PCK in depth.*
- *No connection between the teacher education curriculum and actual teaching practice.*

Criticism

Bridging Approaches

- *To support student teachers connecting university and school mathematics (typically run in the first or second semester) focus directly and explicitly on connecting the mathematics experienced in different environments.*
- *To offer mathematics in a way intended to address the Klein's double discontinuities*
- *To develop courses on explicitly integrating CK with PCK in mathematics and the didactic of mathematics*
- *To develop "capstone course" in that prospective teachers learn the way to approach school math in a deeper, insightful, and more autonomous way*

How to dissolve the issue of double discontinuity



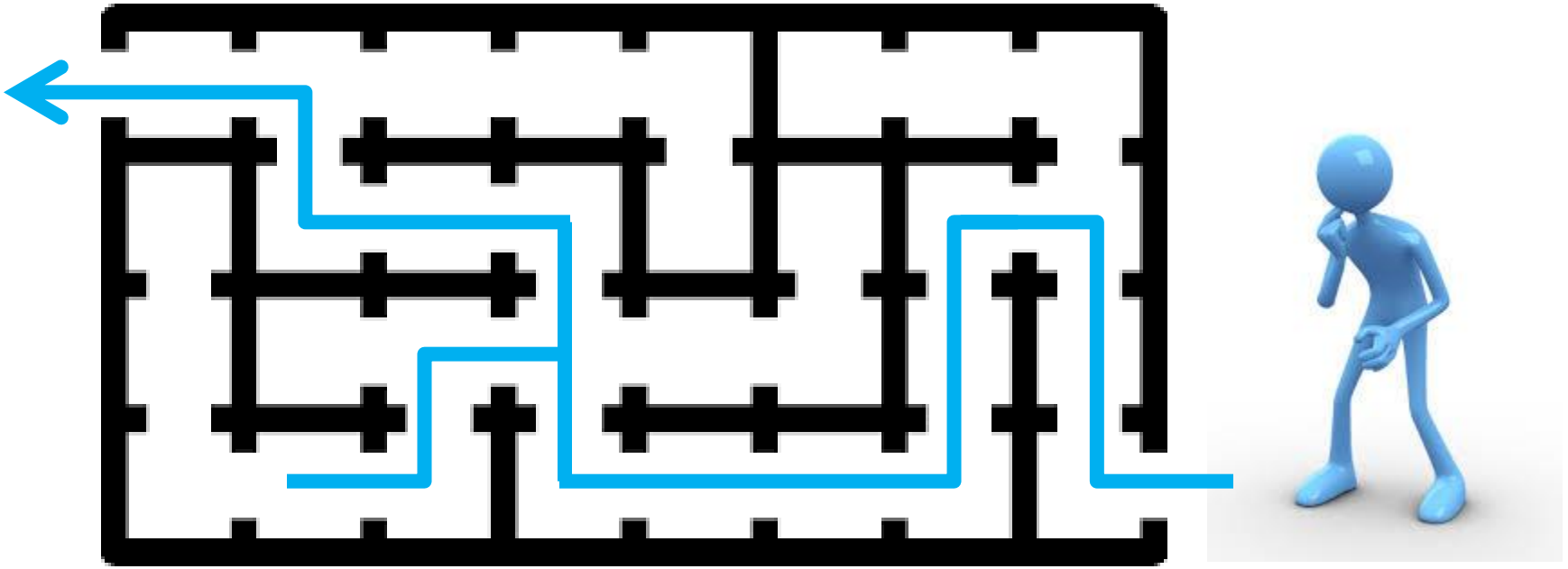
- *How to facilitate the transtion from secondary to tertiary schooling on specific topics?*
- *What could be done in school in order to enable students to find easier ways to learn abstract and rigorous mathematics at university?*
- *How to transfrom the advanced university mathematics into school mathematics without distorting the nature of the content?*
- *How to find a proper balance among different kinds of knowledge of mathematics teachers in teacher education?*

Transitions between teaching institutions

Marianna Bosch
University Ramon Llull, Barcelona, Spain

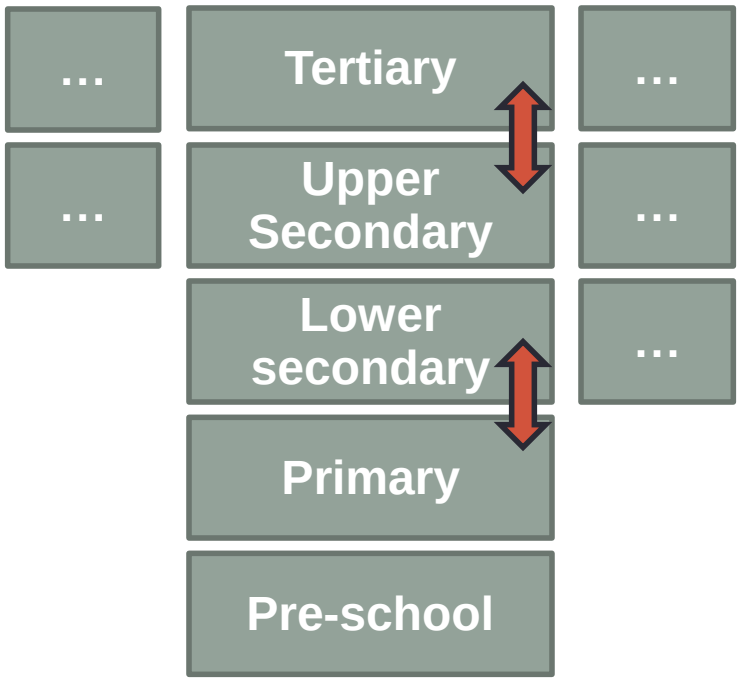
Learners and institutions

Individuals' trajectories (capacities, abilities, difficulties) are shaped by the institutional activities and settings they enter



Broad notion of "institution": primary education, a class, a group of students, the society, a family, a research community, a teachers association, etc.

Transitions and teaching institutions



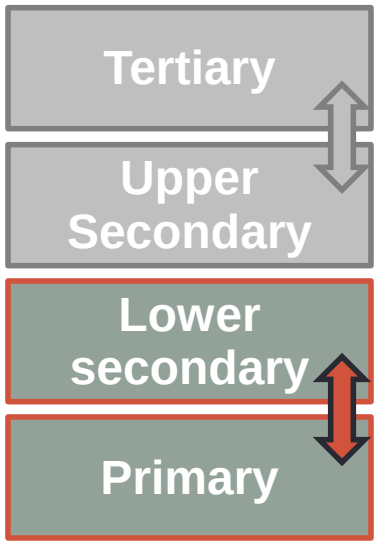
Simplified schema of transitions between institutions

Different levels of analysis:

Society	Citizenship, selective, vocational education
School	Small or big groups/rooms Outdoors activities
Pedagogical	Transmissive – inquiry oriented
Discipline	Maths for all or Maths for the university
Domain	Early algebra - Elementary algebra - Abstract algebra
Block of contents	Functions - Calculus - Analysis
Theme	…

Primary – secondary transition

Literature review



Differences found:

PEDAGOGY

-Less interaction teacher-students at S

-Increase in the students' autonomy

-Passage from more active to more transmissive pedagogies

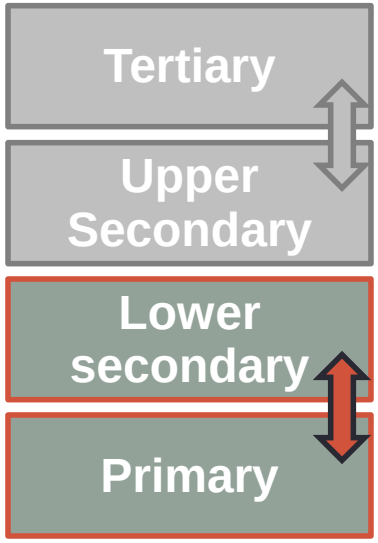
DISCIPLINE

-Separation between disciplines

-Specialist teachers vs. generalist

Primary – secondary transition

Literature review



Proposals to smooth the transition:

-Strengthening teachers' relationships

-Promoting more open activities at S

-Less separation between disciplines

-Not many proposals to modify the curriculum (except Early Algebra)

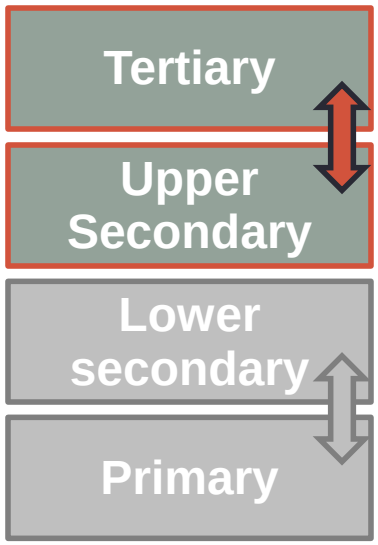
→ ***MAIN FOCUS ON THE PEDAGOGICAL AND SCHOOL LEVELS***

→ ***BRING LOWER SECONDARY CLOSER TO ELEMENTARY EDUCATION***

Compulsory secondary education remains in an uncomfortable position between "education for all" and preparation for the post-compulsory "education of a few"

Secondary – tertiary transition

Literature review



Surprisingly, similar difficulties:

PEDAGOGY

-Less interaction teacher-students at U

-Increase in the students' autonomy

-Passage from more active to more transmissive pedagogies

DISCIPLINE

-Separation between disciplines/domains

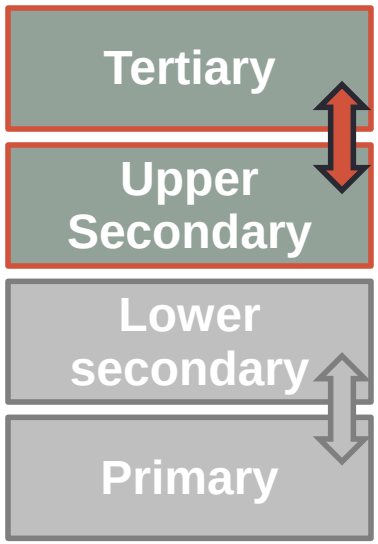
-Lecturers = Teachers & Researchers

More research on specific topics:

- Calculus → Analysis
- Algebra & Geometry → Linear algebra
- Procedural → Formal organization of knowledge

Secondary – tertiary transition

Literature review

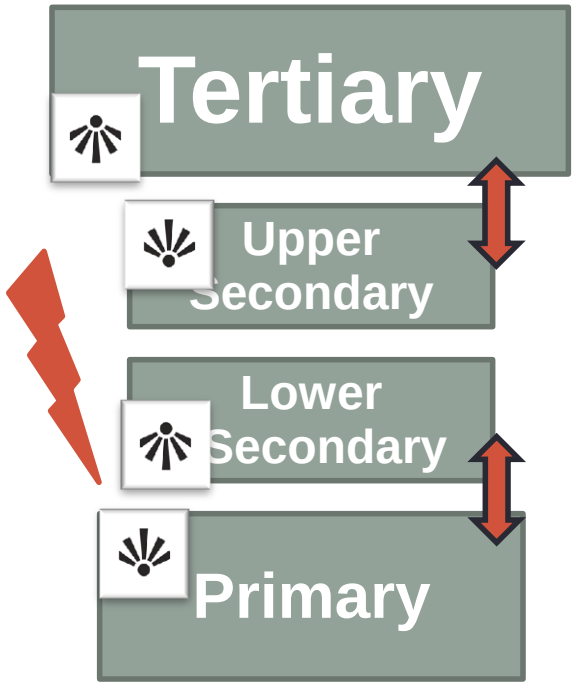


Proposals to smooth the transition:

- Secondary teacher profesional development
- " Bridging courses " *(rites of passage)*
  - trying to reinforce basic knowledge and introduce students to a more theoretical approach to mathematics
  - can be interpret as specific institutional coups de force
- No proposals to take the mathematics of secondary as a basis for further theorisation

Again, modifications proposed affect mostly secondary education (especially in what concerns mathematical issues). University mathematics are rarely discussed or questioned…

Questions to be pursued



- What **institutional perspectives** (and values) are we assuming/questioning?
- How to carry out a **critical analysis** of the "higher" ones? → role of theoretical frameworks and research communities
- How to approach the ***specific*** levels (mathematics) and the **teaching *ecology*** to make **realistic proposals**?

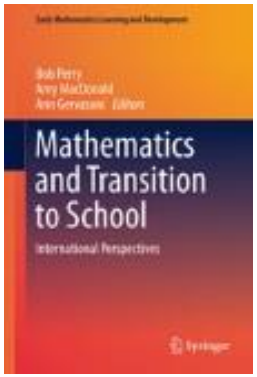
How to be **cautious with the university perspective** that appears as the most legitimate and avoid reinforcing the **propaedeutic function** of the first levels of education over their role in **preparing students for citizenship**?

TRANSITIONS BETWEEN IN- AND OUT-OF-SCHOOL MATHEMATICS

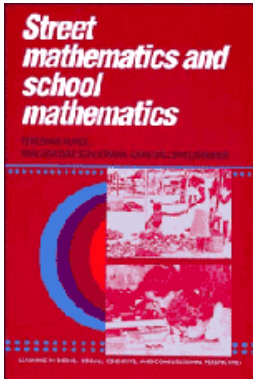
Lieven Verschaffel
University of Leuven, Belgium

Transitions…

1. From prior-to-school to school mathematics



2. Between out-of-school and school mathematics



3 themes

1. Out-of-school mathematics, in comparison to mathematics practiced and learnt in school

2. Descriptions of *transitions* between in- and out-of-school mathematics

3. Attempts to facilitate and exploit these transitions

2 caveats

- Insufficient attention to *diversity* in all its forms
- No attention to impact of *technology*

1. Out-of-school math vs. math practiced and learnt in school

- Much mathematical knowledge is practiced, acquired and transmitted outside school

- Dichotomous descriptions of in- and out-of-school thinking, learning and teaching - showing a strong tendency to “romanticize” the latter as more “authentic”, “meaningful”, and “effective ” - have been replaced by
  - Less caricature-like characterizations of these different contexts of mathematical practices
  - Increased attempts to think beyond this dichotomy and interests in what actually happens at the boundaries of both, in a variety of “intermediate” settings

2. Descriptions of *transitions* between in- and out-of-school mathematics

- Transition from school to out-of-school mathematics
- Transition from out-of-school to school mathematics

Example of a P-item with its typical non-realistic reaction

- Problem: A man wants to have a rope long enough to connect two poles 12 meters apart, but he only has pieces of rope of 1,5 meters long. How many of these would he need to tie together to connect the poles?

- Typical non-realistic reaction: 12 : 1,5 = 8 pieces of rope (without any realistic query about the amount of rope needed for making the knots)

- % of realistic reactions among Flemish 5th graders: 0 %

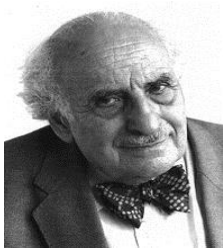
2. *Transitions* between in- and out-of-school mathematics

- Classical psychological notion of (lack of) transfer replaced by alternative conceptualizations such as: “boundary crossing”, “situated abstraction”, “subjectification”

- Analyses of people continuously bridging and integrating math-related learning experiences in constructive and positive ways

3. Attempts to facilitate and exploit transitions

- Bringing the out-of-school reality into the math curriculum to encourage more meaningful and purposeful activity



- Strengthening the transitional links between learners' mathematical home and school culture, by setting up productive forms of home-school exchange and collaboration

- Looking for ways to change school mathematics so that it is more relevant for work and other everyday activities

Discussion

With the perspective you presented, can you say something (and what) about ...

- The transition from arithmetics to algebra?
- What are appropriate or promising “boundary objects” that can play a contributing role in helping students to make the transition?
- What about learning technical, procedural work in the acquisition of concepts? How does it contribute to the continuity/discontinuity of the learning process?
- What is the possible role of the students in helping to ease transitions?

Conclusions

Challenges for research on transition in mathematics education

- Go beyond the initial state/ final state analysis: building methods allowing to grasp the transition process itself in all its complexity;

-Identify commonalities between different contexts, and opportunities offered by transitions;

-Develop resources to support transition: communicate the research results, design teacher education programs, foster communication between the actors in different contexts…

Thank you for your attention!

For further discussion…

http://python.espe-bretagne.fr/transition-panel