The Legacy of Felix Klein




First Hour: Panel (15.00–16.00 h)

1. **Introduction:** What is and what could be the legacy of Felix Klein?
   Hans-Georg Weigand (Würzburg, GER)
2. **Felix Klein - Biographical Notes**
   Renate Tobies (Jena, GER):
3. **Introduction** to the Two Hour Session (16.30–18.30 h):
   - **Strand A: Functional thinking**
     Bill McCallum (Arizona, USA).
   - **Strand B: Intuitive thinking and visualization**
     Michael Neubrand (Oldenburg, GER).
   - **Strand C: Elementary Mathematics from a higher standpoint**
     Marta Menghini (Rome, IT) & Gert Schubring (Bielefeld, GER).

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3. **Introd** h):
   - **Stra**
     Bill Mc
   - **Stra**
     Micha
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What is and what could be the legacy of Felix Klein?

Three basic questions:

1. Which situations and which problems at the end of the 19th and the beginning of the 20th century can be seen in analogy to present situations?
2. How did Felix Klein react to these problems and which solutions did he suggest?
3. What do we know nowadays about the effect of the answers and solutions provided by Felix Klein 100 years ago?

The "Double Discontinuity" (1908)

… 100 years ago … and … today

University

School

School

And today?

Felix Klein as an empathetic person …

… recognized problems:

… thought about solutions

… suggested organisational changes

… suggested content-related changes

… was driven by external requests

… changed his mind

Felix Klein …

1872: Professor in Erlangen

… recognized problems:

… thought about solutions

… suggested organisational chan

… suggested content-related char

… was driven by external request

… changed his mind

Felix Klein …

1872: Professor in Erlangen

… recognized problems:

Inaugural Address: "Lack of knowledge of mathematics!"

… thought about solutions

"Schools do not develop a proper feeling for mathematical operations or promoting a lively, intuitive grasp of geometry" (1872)

… suggested organisational changes

… suggested content-related changes

… was driven by external requests

… changed his mind

Felix Klein …

… recognized problems:

… thought about solutions

"If we better educate teachers, then mathematic instruction will improve by itself." (1872)

"better":
… beyond the contents of school level
… present state of mathematics science

… suggested organisational changes

… suggested content-related changes

… was driven by external requests

… changed his mind

Felix Klein …

… recognized problems:

… thought about solutions

… suggested organisational changes

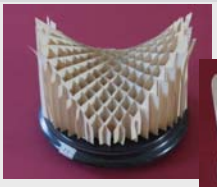
- Student exercises at the university
- General elementary lessons and a later specialization (BA a. MA)
- Scientific Homework (MA thesis)

… suggested content-related changes

… was driven by external requests

… changed his mind

Felix Klein …



… recognized problems:

… thought about solutions

… suggested organisational changes

- Visualization
- Education in applied mathematics
- Introduction of new technologies

… suggested content-related changes

… was driven by external requests

… changed his mind

Felix Klein …

… recognized problems:

… thought about solutions

… suggested organisational changes

… suggested content-related changes

… was driven by external requests

… changed his mind

- 1900: Prussian ministry

→ Commitment to high school mathematics (1900)
→ Meran reform (1905)
→ President of ICMI (1908)

ICME13 Hamburg 2016

Felix Klein …

The Legacy of Felix Klein

… recognized problems:

… thought about solutions

… suggested organisational changes

… suggested content-related changes

… was driven by external requests

… changed his mind

- Meaning of Applications
- Importance of subjects: Calculus

Th@nk you!

weigand@dmuw.de
www.dmuw.de

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Biographical Notes

Felix Klein
and his comprehensive program to promote mathematics, its applications, and mathematical instruction

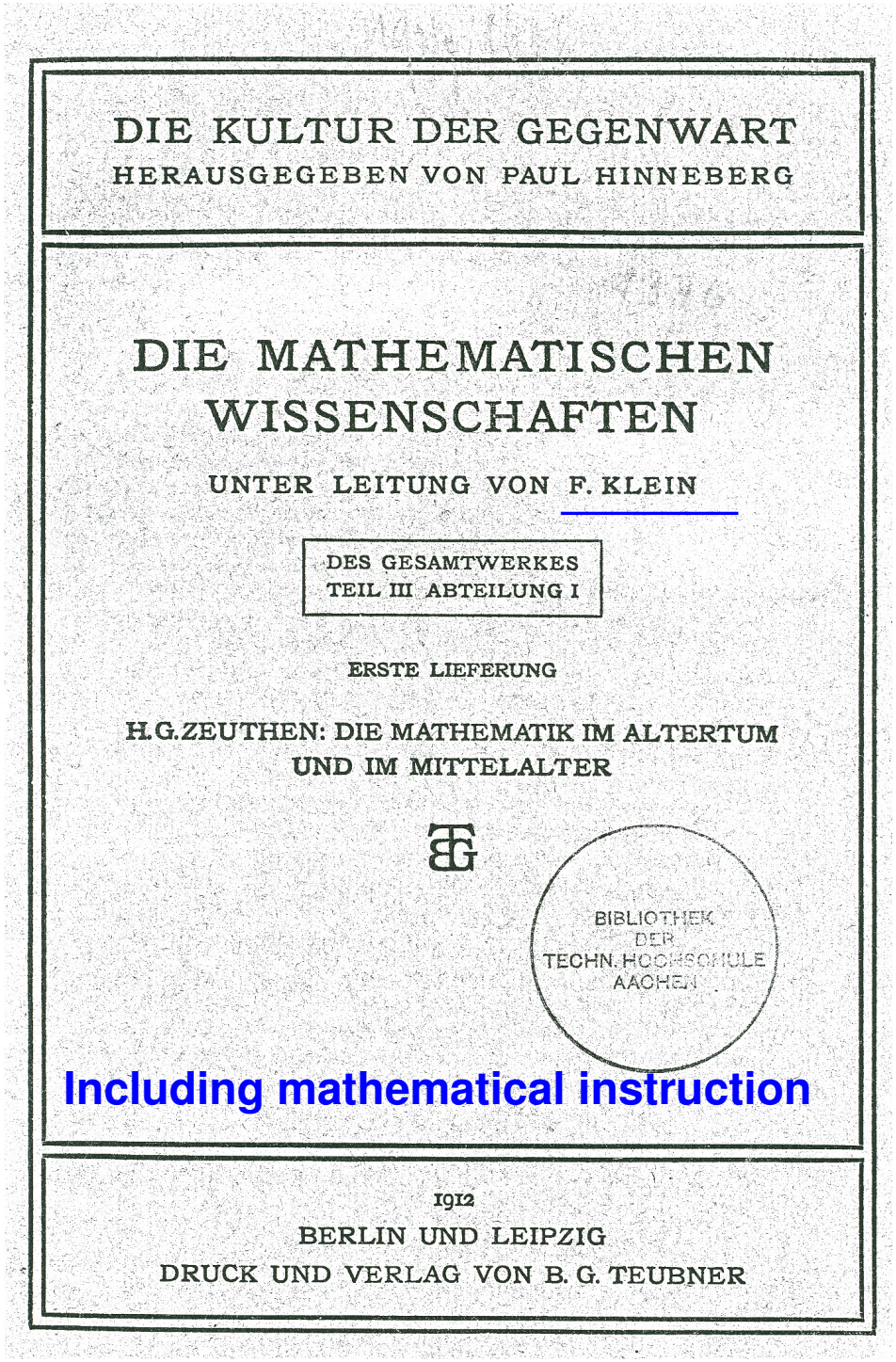
Renate Tobies

Friedrich-Schiller-Universität Jena

seit 1558

ICME, Hamburg, 27 July 2016

Mathematics and Culture

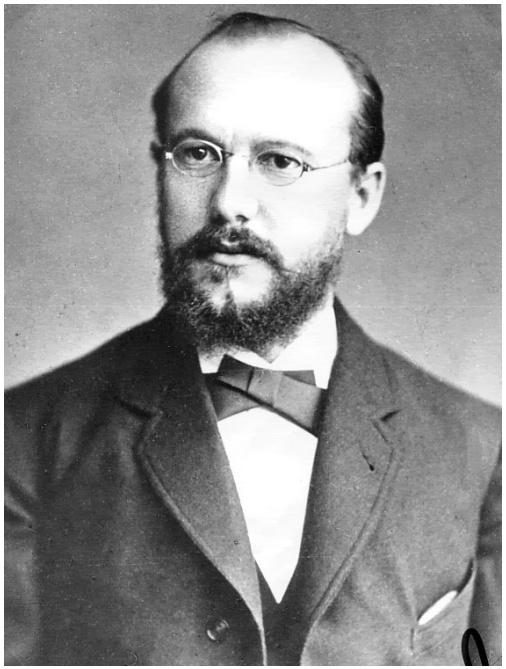


Including mathematical instruction

R. Tobies

Felix Klein's comprehensive program:

to promote mathematics, its application,
and mathematical instruction;
including history of mathematics; mathematics for all;
reform from the kindergarten to university level



1901 Hamburg

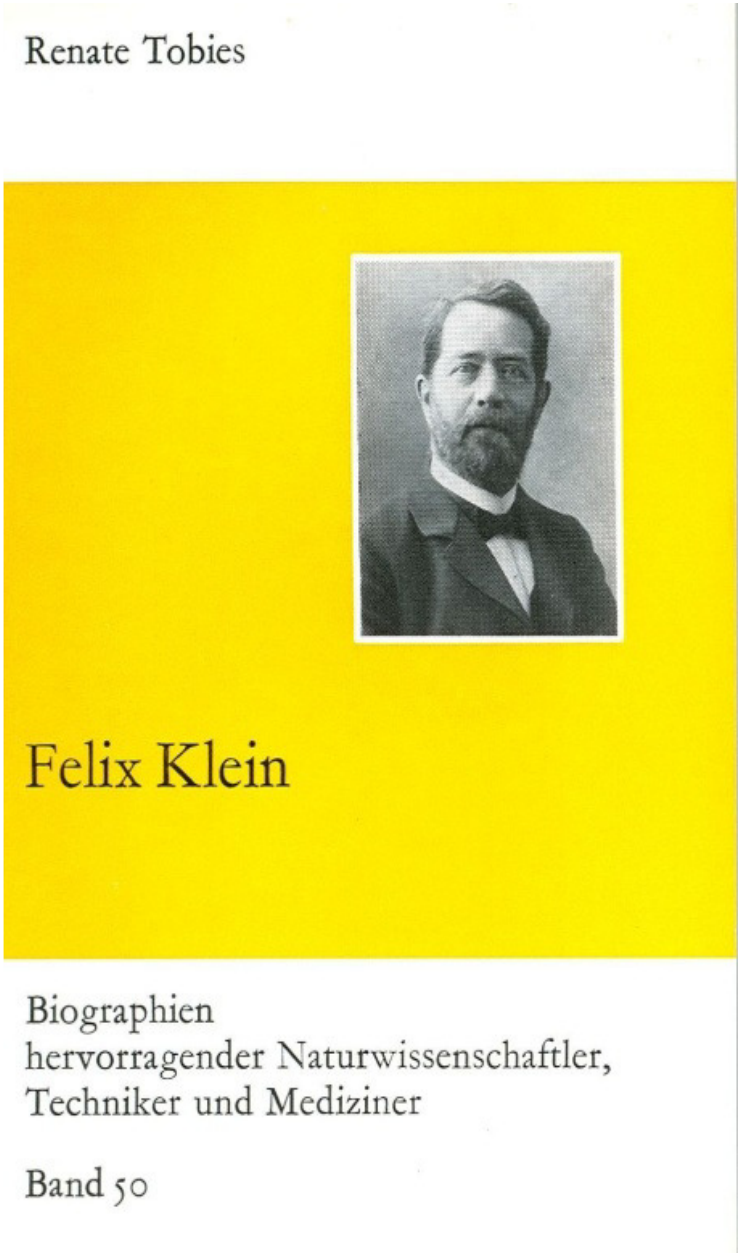
.....

1904 Breslau

(Teaching Commission)

Karl Kraepelin, Hamburg
1848-1915

Felix Klein, Göttingen
1849-1925



Felix Klein (1849-1925)

1865 Abitur in **Düsseldorf** (Prussia)

1868 PhD Univ. of **Bonn** (Prussia)

1869-70 Postdoc **Berlin; Paris**

1871 Habilitation Univ. of **Göttingen**

1872 Prof. Univ. of **Erlangen** (Bavaria)

1875 Prof. Technical University (TH) of **Munich** (Bavaria)

1880 Prof. Univ. of **Leipzig** (Saxony)

1886 Prof. Univ. of **Göttingen** (Prussia)

Felix Klein
painted by
Max Liebermann in 1912

* 25 April 1849 Düsseldorf

† 22 Juni 1925 Göttingen

Mathematician

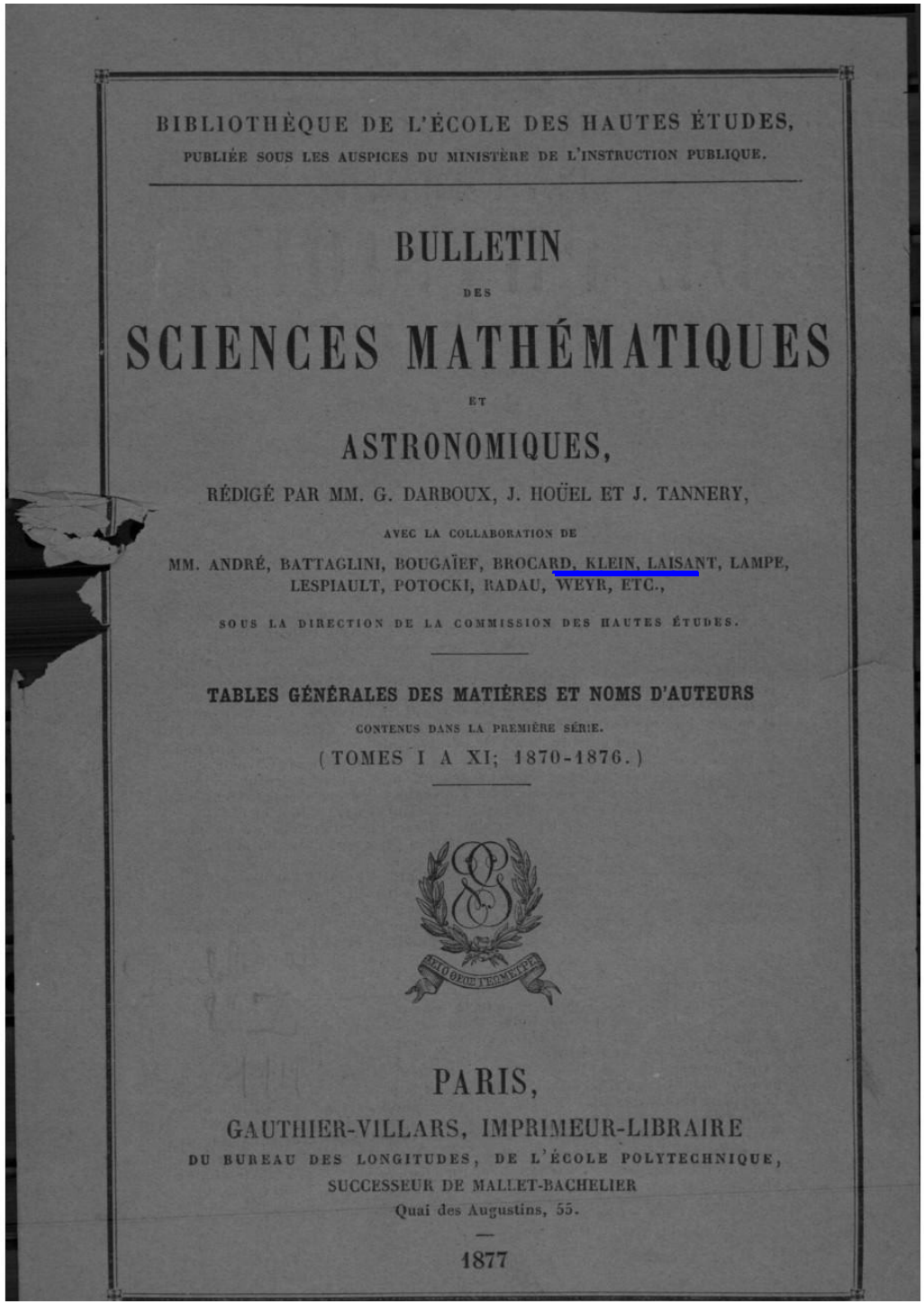
Science organizator

Motor of teaching reform

Basis for his sussecc:

Internationality

R. Tobies


BIBLIOTHÈQUE DE L'ÉCOLE DES HAUTES ÉTUDES,
PUBLIÉE SOUS LES AUSPICES DU MINISTÈRE DE L'INSTRUCTION PUBLIQUE.

BULLETIN
DES
SCIENCES MATHÉMATIQUES
ET
ASTRONOMIQUES,
RÉDIGÉ PAR MM. G. DARBOUX, J. HOÜEL ET J. TANNERY,
AVEC LA COLLABORATION DE
MM. ANDRÉ, BATTAGLINI, BOUGAÏEF, BROCARD, KLEIN, LAISANT, LAMPE, LESPIAULT, POTOCKI, RADAU, WEYR, ETC.,
SOUS LA DIRECTION DE LA COMMISSION DES HAUTES ÉTUDES.

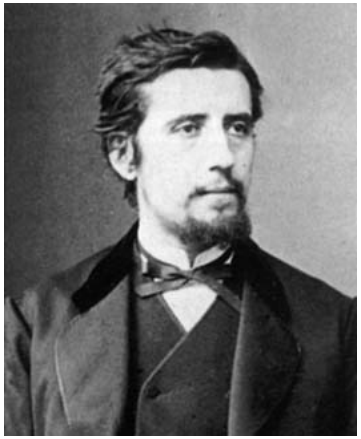
TABLES GÉNÉRALES DES MATIÈRES ET NOMS D'AUTEURS
CONTENUS DANS LA PREMIÈRE SÉRIE.
(TOMES I A XI; 1870-1876.)

PARIS,
GAUTHIER-VILLARS, IMPRIMEUR-LIBRAIRE
DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE,
SUCCESSEUR DE MALLET-BACHELIER
Quai des Augustins, 55.
1877

Internationality

Klein's first translated paper:

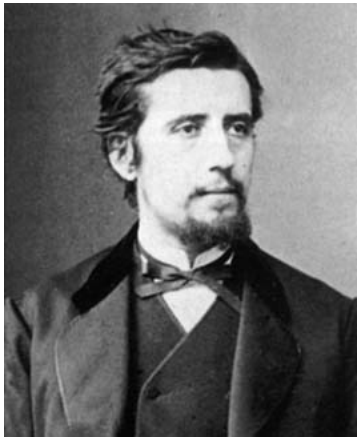
Klein, F.: "**Sur la géométrie dite non euclidienne**". *Bulletin*, 2 (1871) pp. 341-351)



R. Tobies

Internationality

R. Tobies



Gaston Darboux
France
(1842-1899)

Paris 1870

(1849-1925)

Berlin1869/70
Paris 1870

Sophus Lie,
Norway
(1842-1899)

Berlin1869/70
Göttingen 1871

Otto Stolz,
Austria
(1842-1905)

Rom… 1874

Luigi Cremona
Italy
1830-1903

UK 1873

Arthur Cayley
United Kingdom
(1821-1895)

H. G. Zeuthen
Denmark
(1839-1920)

A. A. Markov
Russia
(1856-1922)

R. Tobies

Winter term 1893/94

Klein‘s Lecture: Hypergeometric Function

Emanuel Beke
1862-1946

Seminar: Wednesday, 10.1., 17.1., and 24.1.1894:
Über konvergente lineare Differentialgleichungen.
Dr. Em[anuel]. Beke

Results: 2 papers

Beke, E.: Die Irreducibilität der homogenen linearen Differentialgleichungen. Math. Ann. 45 (1894)

Beke, E.: Die symmetrischen Functionen bei den linearen, homogenen Differentialgleichungen
Math. Ann. 45 (1894)

→ **1906 chairman of the Teaching Reform Commission in Hungary,**
→ **1908 He voted for Klein in Rome**.. .

R. Tobies

Winter term 1893/94

Klein‘s Lecture: Hypergeometric Function

Mary F. Winston
1869-1959

Frederick S. Woods
1864-1950
Textbooks
pedagogical concerns

Virgil Snyder
1869-1950
Textbooks

Grace E. Chisholm
1868-1944

Beginner‘s book on geometry

Impressed by teacher training courses ←**Paper 1894**; *Enc.*

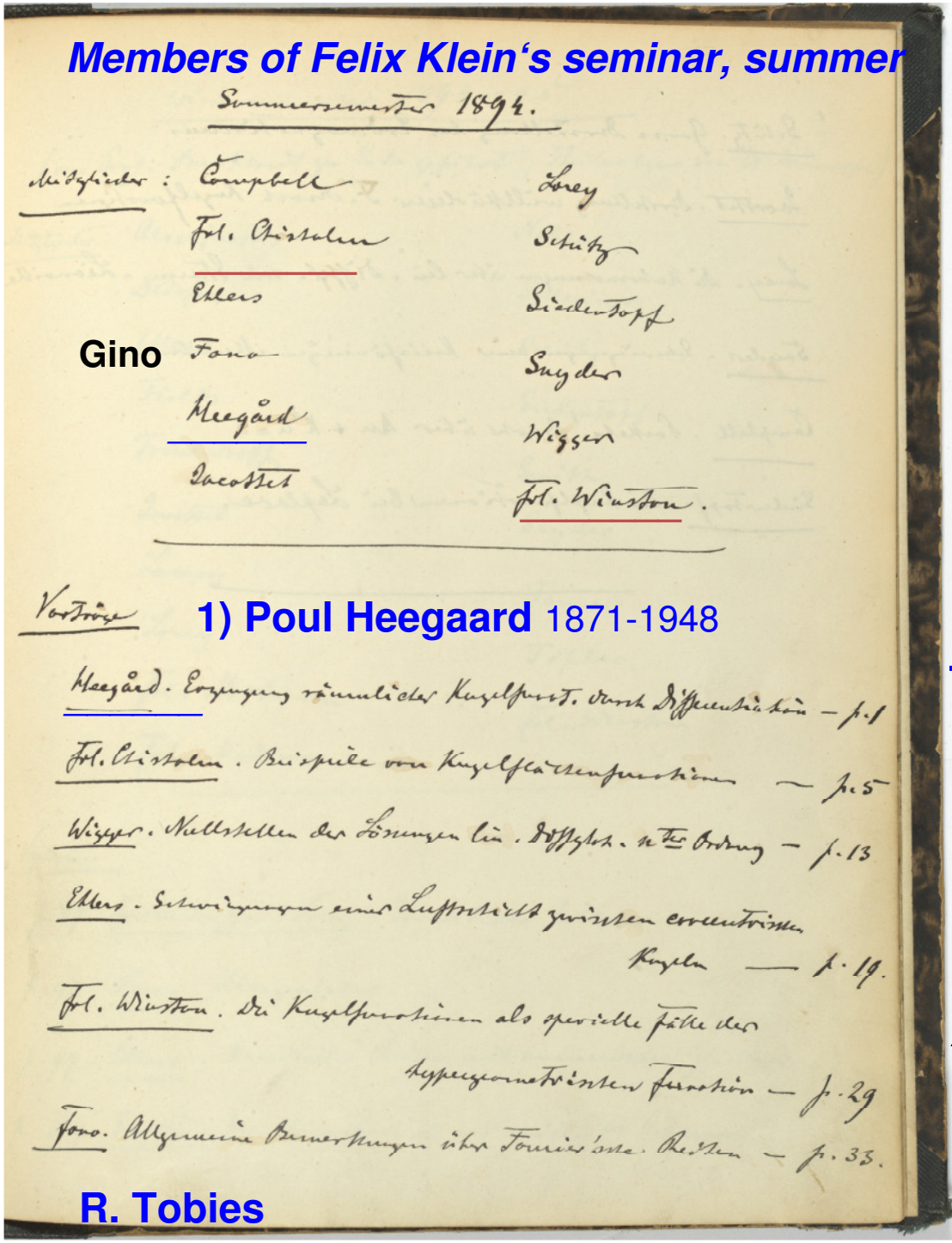
Gino Fano
1871-1952
Mathesis Association,
references to Klein

Wilhelm Lorey
1873-1955

Emanuel Beke
1862-1946
Math. Ann. 45, IMUK

Metzler, Campbell, Jaccottet, Furtwängler

Members of Felix Klein's seminar, summer 1894, University of Goettingen



Sommersemester 1894.

Mitglieder: Campbell, Fol. Christolen, Ehlers, Fano, Heegård, Jacobsthal, Lorey, Schütz, Siedentopf, Snyder, Wigger, Fol. Winston.

Vorträge

Heegård. Entwicklung sämmtlicher Kugelfunct. durch Differentiation — p. 1

Fol. Christolen. Beispiele von Kugelflächenfunctionen — p. 5

Wigger. Nullstellen der Lösungen lin. Diffgln. nter Ordnung — p. 13

Ehlers. Schwingungen einer Luftmasse zwischen concentrischen Kugeln — p. 19.

Fol. Winston. Die Kugelfunctionen als specielle Fälle der hypergeometrischen Function — p. 29

Fano. Allgemeine Bemerkungen über Fourier'sche Reihen — p. 33.

Schütz. Ganze Darstellung des Erdmagnetismus

Jacobsthal. Darstellung willkürlicher F. durch Kugelfunctionen

Lorey. Die Naturnotwendigen über lin. Diffgl. von Sturm-Liouville

Snyder. Schwingungen einer kreisförmigen Membran

Campbell. Forsyth's Buch über $\Delta u + k^2 u = 0$

Siedentopf. Die Kugelfunctionen bei Laplace.

R. Tobies

1) Poul Heegaard 1871-1948

Heegaard:

Klein had me give two lectures in the 'Mathematische Gesellschaft' with a summary of Zeuthen's work on enumerative geometry.
He also discussed with me the idea that would later form the basis for my dissertation.
Altogether, there was a scientific atmosphere which stimulated me very much – stronger than anything I have ever met again.

Poul Heegaard (*1871 Kopenhagen, †1948 Oslo)

ICTM: **Vice-President 1932-1936**

Summer Semester **1894**: **studied with Klein in Göttingen**,

- ***Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*.** B.G. Teubner: Leipzig, Berlin 1898-1935
- (French Edition Gauthier-Villars: Paris since 1904)

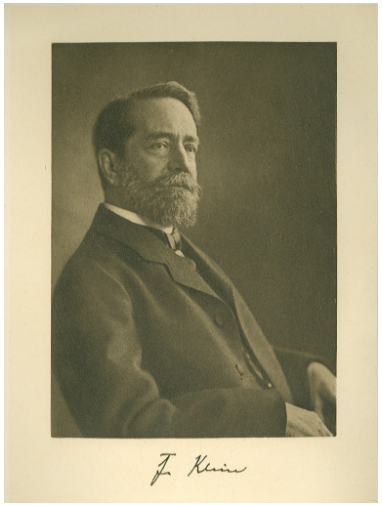
- Bd. I Arithmetik und Algebra
- Bd. II Analysis
- Bd. III Geometrie ←(Dehn/**Heegard**: Analysis Situs, 1907)
- Bd. IV Mechanik
- Bd. V Physik
- Bd. VI Geodäsie, Geophysik, Astronomie

- Bd. VII **(Plan)** Geschichte, Philosophie und Didaktik [History, Philosophy and Didactics]

R. Tobies

World's Columbian Exposition 1893

The Chicago World's Fair



- **Prussian Exhibition of (mathematical) Instruction, Mathematicians' Congress**
- *Conférences sur les Mathématiques*, faites au Congrès de Mathématiques, tenu à l'occasion de l'Exposition de Chicago,
  par Félix Klein ... Recueillies par le professeur Alex. Ziwet,
  traduit par M. L. Laugel. Paris: A. Hermann 1898.
- **Charles Hermite:** « **Il avait passé une heure comme dans le ciel** »
  when he read Klein's papers … (Hermite initiated the translation.)

R. Tobies

Report to Althoff, Prussian Ministry of Education, 1893

(experience from the U.S.):

- Training of prospective teachers has to be changed
- women‘s study
- mathematics and its applications

→ **financial support by engineers and industrialists**

R. Tobies

Royal Commission on Technical Instruction:
Report on a visit to Germany, London 1896:

- „Our foreign rivals are convinced, that **the nation which has the best schools is the best prepared for the great industrial warfare** which lies before us, and **no money appears to be grudged** for the creation, equipment, and maintenance of educational institutions of all grades, and especially of the science laboratories which, as we have seen, are being multiplied in Germany.“

Henry Th. v. Böttinger
Chemical industry
President

Friedrich Althoff,
from the Prussian
ministry of education

Felix Klein,
mathematician,
Vize president

The Göttingen Association for the Promotion of Applied Physics and Mathematics,

28 February 1898

Members:
industrialists;
Professors of math., physics, chemistry at the University of Göttingen

R. Tobies

Satiric picture postcard, 1908

- **Anton v. Rieppel** (1852-1962), bridge construction engineer, enterpreneur, and a **founding member of the "Göttingen Association for the Promotion of Applied Physics and Mathematics"** wrote that Klein set forth the following goals for this association:

  1. To strive above all for the improved Training of future teachers.

  2. To enhance the research conducted in the applied sciences.

  3. To influence the politics of higher education in such a way that universities restore their former concern with practical exigencies.

  We agreed unanimously that the first point was the most important […].

→ ***That was:***

- **teaching in applied mathematics at universities** (graphical, numerical, and instrumental methods);
- **new subjects at secondary schools**: calculus, analytical geometry; concept of function,

Examination Requirements for prospective secondary school teachers: applied math as a new subject in 1898

1898 **September, 12**	1914 (memorandum)	1921
1.Descriptive geometry up to the study of central projection and the proficiency to draw 2.Technical Mechanics: mathematical methods, esp. **graphical statics** 3. Surveying and elements of geodesy together with **theory of probability** **R. Tobies**	Being proficient in **graphical and numerical methods** (descriptive geometry, graphical arithmetic, calculus of observations) and their use in at least one of the following fields: 1. Astronomy 2. Geodesy 3. Meteorology and Geophysics 4. Applied Mechanics 5. Applied Physics 6. Mathematical statistics and Actuarial Science	Familiarity with those applications of analysis that are most important, in particular: **numerical, graphical and instrumental methods**, descriptive geometry, mechanics (including graphical statics and kinematics), calculus of probabilities and calculus of observations; deeper practical and theoretical studies at least in one of the following fields: 1. Astronomy 2. Surveying 3. Meteorology and Geophysics 4. Applied Mechanics 5. Applied Physics 6. Financial Mathematics, Mathematical Statistics and Actuarial Science 7. Technical Sciences (i.e. **Electrical Engineering** or Engineering Thermodynamics or Aeronautics or Statics of Building Construction)

R. Tobies

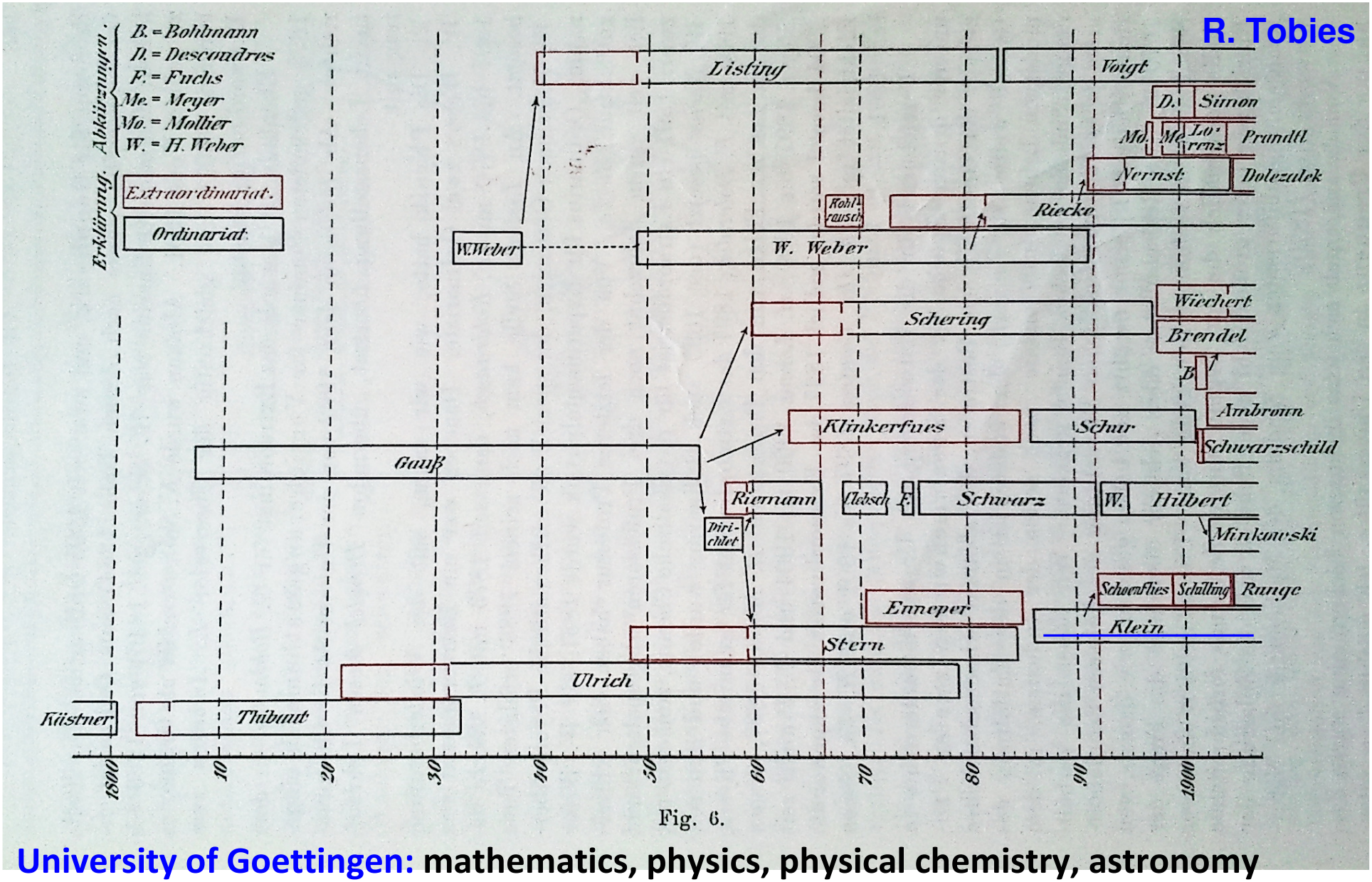


Fig. 6.

University of Goettingen: mathematics, physics, physical chemistry, astronomy

- associate and full professors from the year 1800 to 1904

- **1898-1908 from 5 to 10 full professorships in math and physics**

•1908 Rome:

•Official organ of the ***ICTM*** *(International Commission on the Teaching of Mathematics*

L'ENSEIGNEMENT MATHÉMATIQUE

REVUE INTERNATIONALE

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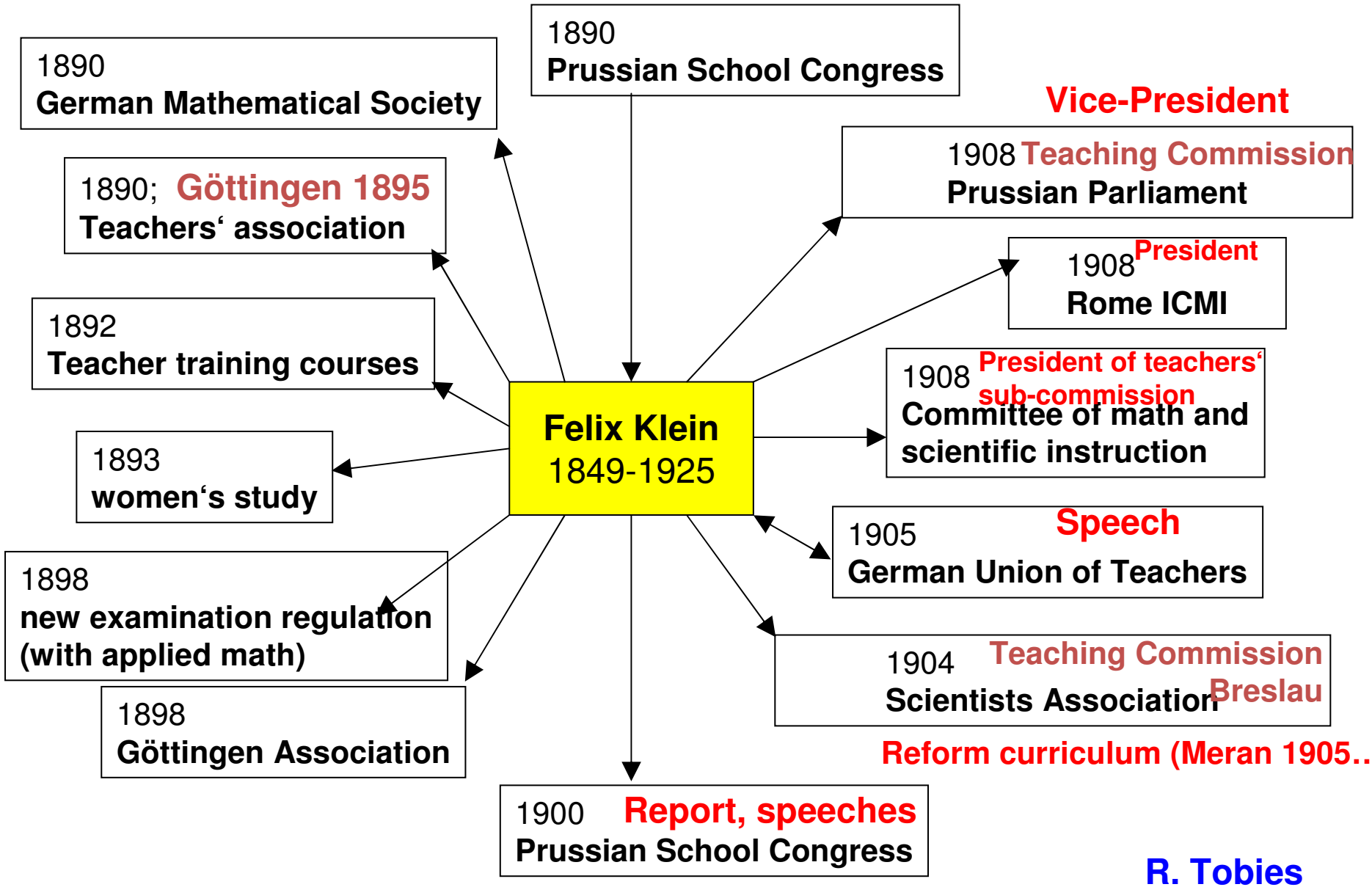
A. VASSILIEF (Kasan). — A. ZIWET (Ann-arbor, Michigan, U. S. A.).

1re Année. 1899.

R. Tobies

Reform of mathematical instruction from the kindergarten to university level



FAMOUS PROBLEMS

OF

ELEMENTARY GEOMETRY

THE DUPLICATION OF THE CUBE
THE TRISECTION OF AN ANGLE
THE QUADRATURE OF THE CIRCLE

AN AUTHORIZED TRANSLATION OF F. KLEIN'S
VORTRÄGE ÜBER AUSGEWÄHLTE FRAGEN DER ELEMENTARGEOMETRIE
AUSGEARBEITET VON F. TÄGERT

German original, F. Klein 1895

BY

WOOSTER WOODRUFF BEMAN
PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF MICHIGAN

AND

DAVID EUGENE SMITH
PROFESSOR OF MATHEMATICS IN THE MICHIGAN STATE NORMAL COLLEGE

Historian of mathematics;
maths pedagogue

BOSTON, U.S.A., AND LONDON
GINN & COMPANY, PUBLISHERS
The Athenæum Press
1897

CONTENTS.

INTRODUCTION.

PART I.

The Possibility of the Construction of Algebraic Expressions.

R. Tobies

David Eugene Smith (1860-1944)

- *To his initiative: Foundation of the*

  International Commission on the Teaching of Mathematics (ICTM),

  1952: International Commission on Mathematical Instruction (ICMI)

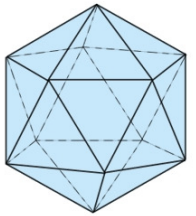
  D.E. Smith: - Vice-President 1912-1920,
  - President **1928-1932**

- **Felix Klein**: 1908-1920 President,
  [Greenhill; Fehr]

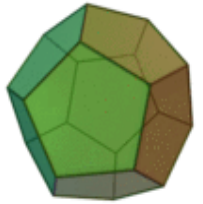
- 1920-1928: no commission

R. Tobies

„Klein‘s Reform“



- **New Curriculums for mathematical instruction from primary schools to the Universities including women‘s secondary schools**



  - „Anschaulicher“ instruction (three-dimensional models for a better understanding), applications of math.
  - The concept of function
  - Graphical representations
  - Analytical geometry
  - Calculus
  - **History of mathematics** (should be a key stone)

R. Tobies

Thank You For Your Attention!

R. Tobies