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CONCEPTUALISATION OF THE ROLE OF COMPETENCIES, KNOWING AND KNOWLEDGE IN MATHEMATICS EDUCATION RESEARCH

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This talk presents the outcomes of the work of the ICME 13 Survey Team on 'Conceptualisations and the role of competencies, knowing and knowledge in mathematics education research' and is given on behalf of and in collaboration with the team members. These are Regina Bruder, Darmstadt, Germany; Kyungmee Park, Seoul, Korea; Núria Planas, Barcelona, Spain; Ross Turner, Melbourne, Australia; and Jhony Alexander Villa-Ochoa, Medellín, Colombia, with Mogens Niss, Roskilde, Denmark as chair.

The talk will be divided into seven sections. The points made and the arguments presented in each section will be illustrated by concrete cases and examples from different parts of the world.

In the *first* section, the overarching *problématique* of the theme of the Survey Team will be outlined. The point of departure is the question 'what does it mean to master mathematics?' This question has a number of ramifications, such as 'what does it mean to possess *knowledge* of mathematics?'; 'to *know* mathematics?'; 'to have *insight* in mathematics?'; 'to be able *to do* mathematics?'; 'to possess *mathematical competence* (or proficiency)?'; 'to be well versed in *mathematical practices*?'. Varying with time and place different answers have been offered to these questions by people with different sorts of backgrounds, positions and points of view. Examples will be provided in the talk. It is interesting to note, however, that oftentimes neither the questions nor answers to them are stated explicitly. This does not mean that they are absent, only that they tend to be taken for granted, and somehow remain on the level of tacit knowledge in current mathematics education research.

However, before attempting to answer these questions, we should answer another question, 'why are the initial questions significant?'. This is the topic of the next – the *second* - section of the talk. Whether explicit or implicit, answers to the initial questions determine at least three key components in mathematics education, including the ultimate purposes and the specific goals of mathematics education ('what do we wish to accomplish?'), the ensuing criteria for success of mathematics teaching and learning ('how, and when, do we know whether we have accomplished what we strive at?' and 'what means of formative and summative assessment are suitable for assessing the outcomes of mathematics education?'), and the structure and organisation of mathematics teaching ('what is going to happen in and outside the mathematics classroom?'; 'what activities are teachers and students supposed to be engaged in?'; and 'what materials for teaching and learning are (should be) available to teachers and students?'). Different answers to these questions and to the ones posed in the first section give rise to very different kinds of mathematics education. In fact, one may well argue that one of the most important reasons why mathematics education around the world is so diverse is exactly the diversity of answers to this set of rather fundamental questions.

Answers to the questions above can be used in two different ways, in prescriptive / normative or in descriptive / analytic ways, the topic of the *third* section. The *prescriptive / normative* use focuses on *what ought to be* the case, e.g. in specifying the goals and aims of mathematics education, in defining and designing curricula and teaching-learning activities, as well in designing modes and instruments of formative and summative assessment, including tests and exams. In contrast, *descriptive* and *analytic* uses focus on *what is* the case, e.g. by uncovering what is actually on the agenda in various curricula, what is actually happening in mathematics teaching and learning in different settings and contexts, what the outcomes actually are, how students progress through the stages of mathematics education, and how well they "survive" the transition from stage to stage or from one type of institution to another within the education system. It should be kept in mind that "what is the case" is context dependent, so that different answers emerge from the different contexts in which the related questions are posed. Such uses may also deal with judging whether some ways of orchestrating teaching and learning are superior to others when it comes to pursuing the goals and meeting the criteria for success in mathematics education. Implementing these kinds of use typically requires a non-negligible amount of research and development.

The issues dealt with so far are all to do with the importance of considering the question of what it means to master mathematics, whether from an academic / intellectual point of view or from a policy or practice oriented point of view. In the next – the *fourth* section – we shall take a closer look at the actual answers that have been given by different people and agencies in different places and contexts.

Classically, the focus of attention has been the *knowledge* of mathematical *facts* ("knowing what" concerning concepts, terms, results, rules, methods) as well as procedural *skills* - "knowing how" – i.e. the ability to carry out well-delineated and well-rehearsed rule-based operations fast and without errors. At later stages in the history of mathematics education (roughly in the 1940s and 1950s), *process oriented aspects* were given emphasis based on one or more of the following views of mathematics. 'Mathematics is what professional mathematicians do'; 'mathematics is what users of mathematics do in their workplace'; 'mathematics is what ordinary citizens do in their private, social and societal lives'; and 'mathematics is what mathematics teachers do'. Later still (roughly since the 1960s), some mathematicians and mathematics educators began to pay more attention to what is involved in the very *enactment* of mathematics, i.e. working within and by means of mathematics in intra- and extra-mathematical contexts. An early example of this is the first IEA study, later to be named FIMS (as a precursor to TIMSS). Since the 1990s much work has been done to develop notions such as mathematical competenc*e* and competenc*ies*, fundamental mathematical capabilities (PISA 2012), mathematical proficiency, and mathematical practices, in addition to their slightly more distant relatives mathematical literacy, numeracy and quantitative literacy.

A non-negligible amount of research on mathematical competencies and their nearer or more distant relatives has attempted to come to theoretical grips with the conceptual aspects of these notions. 'What are the core constituents of these notions?' and 'what are the similarities and differences between them?' are key questions to such research. Empirical research has typically concentrated on identifying the presence and role of mathematical competencies in students' work, especially pivotal competencies such as modelling and reasoning competencies. Also the possibilities and challenges involved in assessing students' possession and development of the competencies are in focus, both from a holistic and from an atomistic perspective, where a holistic perspective considers complexes of intertwined competencies in the enactment of mathematics, whereas an atomistic perspective zooms in on the assessment of the individual competency in contexts stripped, as much as possible, of the presence of other competencies. These issues form the centerpiece of the *fifth* section of the talk.

The *sixth* section is parallel to the previous one, focused on research, in that it deals with the state of the art as regards implementation of mathematical competencies and their relatives in practices of mathematics teaching and learning. The degree to which competencies have been put into practice varies greatly with place, educational context and educational level. So far, the implementation has primarily concerned curriculum planning and design, pre-and in-service programmes for teachers – where it has been found to be challenging for teachers to come to grips with notions of mathematical competence / competencies and their relatives and, not the least, with their implementation. The same is true of the design and implementation of modes and instruments of assessment and evaluation of competencies. A general observation is that in most cases in which competencies or their relatives have been put to use in concrete contexts, the original notions and definitions have been modified or simply re-defined to suit the purposes and boundary conditions of that particular context. It also deserves to be mentioned that in some cases the introduction and implementation of competency oriented notions in educational systems or sub-systems (i.e. particular segments - such as streams, levels or institutions - of an overarching educational system) have been of a rhetorical (i.e. 'lip service' like) rather than of a substantive nature.

Another significant issue to be dealt with in the section on practices of mathematics teaching and learning is the educational backgrounds and competencies teachers need from pre-service education or in-service programmes in order to engage in the implementation of competency-oriented mathematics education.

The talk will finish by an attempt, in the *seventh* section, to identify and chart current trends in discourses on mathematical competencies and in the emphases encountered in research and practice, and to identify significant issues and topics that ought to be considered in future research, development and implementation projects. Such discourses reflect the particular features that pertain to research, development and practice in different contexts.