Invited Lecture

Why Language Diversity Matters in Mathematics Education

Richard Barwell[1]

ABSTRACT I examine the question of why language diversity matters in mathematics education, offering four responses, illustrated with examples drawn from my research. The four responses look at the nature of language diversity, its role in learning and teaching mathematics, its connection with social stratification, and its connection with the ecological crises faced by our planet.

Keywords: Mathematics learning; Mathematics teaching; Language diversity; Sociolinguistics; Ecojustice.

1. Introduction

There is now a broad understanding that mathematics classrooms often feature learners who speak more than one language or who may be learning the language of instruction. In fact, despite widespread evidence (including our own experience), mathematics education as a field has not yet recognized that language diversity is the more common state: that is, in the vast majority of mathematics classrooms around the world, some degree of language diversity is present. Nevertheless, a growing body of work has examined different features of teaching and learning in mathematics classrooms in the context of language diversity (see Barwell et al., 2016; Barwell et al., 2017). This work has, for example, examined the challenges experienced by mathematics teachers, the different language practices used by learners and teachers for mathematical meaning-making, and the impact on learners' performance. I have made some contribution to this literature over the past 20 years, through ethnographic studies of learners' participation in mathematics classroom interaction, particularly in second-language contexts (i.e., those in which the language of instruction is a second or additional language for some or all learners).

My interest in language diversity in mathematics classroom arose first as a mathematics teacher. I grew up in the UK and completed my mathematics teacher education in west Wales, where there are many Welsh speakers. My university offered the mathematics teacher education program in both Welsh and English. Later, I went to work in northern Pakistan. I worked in English-medium schools, but my students spoke a local language, Burushaski, at home, as well as Urdu, the national language. I

[1] Faculty of Education, University of Ottawa, Ottawa, ON, K1N 6N5, Canada.
E-mail: richard.barwell@uottawa.ca

was impressed at how my students could learn mathematics through a combination of three (or more) languages and I spent much time thinking about what it meant for my teaching. I now work in the Ottawa region of Canada, in a bilingual (French-English) university in a society that features Indigenous languages, two official languages (French and English), as well as the languages of many immigrant communities. Language diversity is the norm in mathematics classrooms in this part of the world, as in most places.

For this paper, then, I return to the question I first thought about as a mathematics teacher in the 1990s: How does language diversity matter in mathematics education? I answer this question four times from different angles. In my first response, I examine in more depth what language diversity means and why it is relevant for mathematics educators. Second, I review key findings from the literature about the difference language diversity can make in mathematics classrooms. Third, I show how language diversity links mathematics classrooms to broader social forces. And fourth, I connect language diversity in mathematics education to the future of Planet Earth. Throughout these responses, I include examples from my research to illustrate the different points.

2. What is Language Diversity? What does It Have to Do with Mathematics Education?

Language refers to self-organized systems of semiotic interaction used to coordinate human activity. Humans use combinations of sounds, symbols, gestures and so on, to coordinate social action, whether it be organizing a party, running a political campaign, bringing up children or teaching and learning mathematics. Language enables information, values and learning to be shared across time and space. *A* language, meanwhile, is a form of language shared by a group of people. Urdu is a language (shared by most people in Pakistan, many people in India, as well as their diasporas). Cyrillic is not a language (it's a writing system). From this perspective, mathematics is not a language (it is embedded within languages and uses a symbolic system).

My thinking about language is influenced by the writing of the literary theorist M. M. Bakhtin (e.g., 1981), for whom language is fundamentally and inherently diverse, captured by the term heteroglossia. Think of the many forms of language and the many different languages that might be heard in a street market (the languages of vegetable sellers and fishmongers and snack stalls), or in hip-hop (from different countries, by different artists, in different traditions), or on the television news (the language of political reporting, economics, sport, the weather). The infinite variety of language means that even within a market, the variety is constantly changing: language changes over time, and individual speakers have their own versions of a language.

In similar vein, we can see that there is no single, unified language of mathematics (or mathematics register or mathematical discourse). Apart from the fact that mathematics may be conducted in many different languages, we must also note the diversity of languages within mathematics (of geometry, of algebra, of probability, etc.) and in different contexts (in school, in an undergraduate class, in an academic seminar, in a popular media article). Again, each individual speaks their own version of mathematical language, which changes over time. Language diversity is diverse.

Once we accept this view of language diversity, it is apparent that the category 'language' is analytically unhelpful: language is hard to pin down, even if, of course, there are commonalities that can be discerned. As an example, consider the following observation, taken from fieldnotes from an upper elementary school mathematics class for new immigrants to the province of Quebec, Canada, in 2010, in which the language of instruction is French, a language the learners are in the process of learning:

> Luis ajoutait qu'« un losange ce n'est pas un carré qui tourne, je le sais déjà en anglais. Les angles d'un losange sont plus grands (grandé) à deux côtés. »

In English, this says:

> Luis added that "a diamond is not a square that turns, I already know it in English. The angles of a diamond are bigger (grandé) on two sides."

In this brief fragment, we see that Luis, who is a Spanish speaker, talks about mathematics in French, clearly influenced by Spanish in his accent and word choice, while referring to English, a language with which he also has some familiarity.

If language is always diverse, there is also a tendency to standardize different features, such as in the idea of correct speech. This ideal of standard forms of language, known as unitary language in Bakhtin's work, is ideological in nature and is related to European enlightenment ideals of unified peoples in nation states. This ideology informed European colonialism not only in their language policies but even in the way in which peoples and nations were identified (Makoni, 2011). In mathematics, the idea of standard formal mathematical discourse, also known as the mathematics register, reflects a unitary language perspective. Hence, while the notion of language has weak analytical power, it is politically powerful:

> The traditional idea of 'a language' is an ideological artifact with very considerable power — it operates as a major ingredient in the apparatus of modern governmentality; it is played out in a wide variety of domains (education, immigration, high and popular culture etc.), and it can serve as an object of passionate personal attachment. (Blommaert and Rampton, 2011)

In contemporary sociolinguistics, heteroglossia is linked to superdiversity: the condition of multiple affiliations, identities, and hybridities in the context of global migration and communications. This work has led to some commonly used concepts being challenged. For example, the notion of native speaker proficiency is problematic, since the idea that a standard form of a language can be determined is ideologically loaded. The notion of native speaker proficiency instead serves deficit-based evaluations, with speakers being seen as reproducing with greater or worse fidelity some standard form of language. An alternative view is to see that speakers draw on complex repertoires of language practices. Different parts of an individual's repertoire are brought into play according to the needs of the situation. We all operate with broad repertoires, usually drawing to some extent on several different natural languages.

A final powerful idea introduced by Bakhtin (1981) is that there is constant and inherent tension between unitary language and heteroglossia. This tension is described in terms of the metaphor of centripetal and centrifugal forces. Centripetal forces represent the drive to standardize language, while centrifugal forces represent the opposing tendency for language to diversify through use. For Bakhtin, the tension between these two forces shapes every interaction. This idea explains well many of the findings in mathematics education research that expose tensions and challenges for teachers, learners, policymakers and families (Barwell, 2012). For example, research in South Africa has shown how an official policy of using one language in mathematics classrooms is not reflected in classroom interaction in which many languages may be used (e.g., Setati, 2005). Or teachers have reported challenges relating to the fact that learners use many diverse informal ways to talk about mathematics while the teacher aims to ensure that the use 'standard' mathematical discourse (Adler, 2001). These tensions are illustrated in the diagram in Fig. 1.

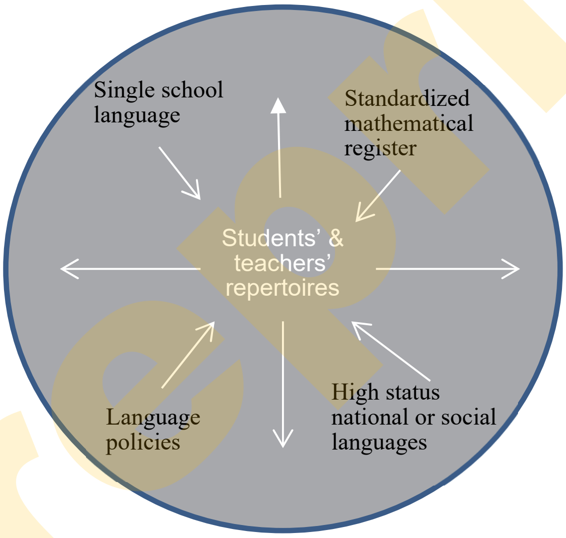

Fig. 1. Centripetal and centrifugal forces in mathematics classrooms

To sum up, language diversity is itself diverse. Multiple language practices are present in any given milieu. Mathematics classrooms feature speakers with diverse repertoires. Language practices are constantly changing across time and space. Hence language diversity matters in mathematics education because:

- Every mathematics classroom is a site of language diversity;
- Every mathematics learner has a different repertoire of language practices;
- Every mathematics teacher has a different repertoire of language practices;
- Every mathematics teacher must adjust their teaching in relation to the different repertoires of their learners;

- Interaction in every mathematics classroom is shaped by the tensions between a unitary language ideal of standard mathematical discourse and the heteroglossia of all language.

3. What do We Know about Language Diversity and Learning and Teaching Mathematics?

What difference, then, does language diversity make to learners and teachers? Research on language diversity and attainment in mathematics shows that in many situations, learners of mathematics who are learners of the classroom language under-achieve. In some situations, however, learners of mathematics who are learners of the classroom language match or exceed expected levels of attainment. A key factor is the learner's proficiency in the different languages they speak (see, for example, Clarkson, 2007). For example, learners who have high proficiency in any language (e.g., the classroom language, or a language they use at home) will have comparable mathematics attainment to monolingual learners, while learners who have high proficiency in two languages will tend to out-perform monolingual learners.

A major problem with this kind of research is its basis in a unitary language ideology. Research that compares multilingual learners with monolinguals assumes that monolingualism (usually in a globally dominant language like English) is the norm. It tends to assume that populations and speakers are homogenous in how they use language in mathematics. Moreover, such work relies in written tests which tend to favor 'native speakers' of the official classroom language (see Barwell, 2003a, for a discussion). We can conclude, therefore, that language diversity does make a difference to learners' performance in mathematics assessments, but these differences are relative to the assumptions about language embedded in the assessment instruments.

Research on language diversity has also examined learners' different practices in mathematics classroom activities to show how they may draw on varied and complex repertoires of languages and language practices to participate in mathematical meaning-making. Some of the language practices that have been identified in the research literature include (Barwell et al., 2017):

- Using multiple languages
- Code-switching, translanguaging
- Drawing on the features of different mathematics classroom genres (e.g. worksheets, proofs)
- Gesturing (pointing, tracing shapes or curves)
- Working with diagrams
- Using deixis (context-based words like 'this' or 'there')
- Using mathematical discourse features (ways of explaining, justifying, questioning, etc.)
- Using mathematical vocabulary
- Using everyday forms of expression

Much attention has been given to theorizing these practices and their role in learning mathematics. The prevailing theoretical approach is based on the idea of language as a resource (e.g., Planas, 2018; Planas and Setati-Phakeng, 2014). Code-switching, or gestures, for example, have been cited as resources that learners may use. I have argued that this perspective is based on a view of language that is too static. As an alternative, I have proposed the idea of language as a source of meaning (Barwell, 2018).

An example of learners' diverse repertoires of language practices is provided by in a study conducted in the UK, conducted in a language diverse Year 5 primary school class (Barwell, 2003b). The study revealed four significant sources of meaning (as I would now call them) for learners in relation to mathematical word problems:

- Mathematical structure (i.e., the arithmetic relationships involved in a word problem);
- Narrative (in the form of learners' accounts of their own experience);
- Genre (i.e., the expected features and organization of word problems);
- Language features (such as verb tense, spelling, in this case in English).

Learners drew on these four sources of meaning to construct and interpret arithmetic word problems. In the following exchange, for example, Cynthia and Helena are devising a subtraction word problem (from Barwell, 2003b). Cynthia first went to school in Hong Kong and speaks Cantonese. Helena grew up in the UK, speaks English.

Helena	Cynthia has thirty pounds for/
Cynthia	no/ not for her mum/ if I bought/ for my mum
Helena	for her mum's present
Cynthia	if give my mum thirty pound I bought nothing from her/ that not make sense
Helena	no/ I won't writing for you mother/ I said Cynthia has thirty pounds for her mother's present
Cynthia	thirty pound/ I gave thirty pound for my mum present
Helena	no/ I didn't say give it to her
Cynthia	then how why you
Helena	you have thirty pounds [for your mum's present
Cynthia	[no but/ I think this make sense/ Cynthia has thirty/ pound/ thirty pound/ she bought err something something something/ it's cost something something/ from her mum present/ and how much she left?/ is that make sense little bit

In this exchange between Cynthia and Helena, all four of these sources of meaning are apparent. With respect to *language features*, they negotiate the use of words like 'for', 'from', 'give' and 'have'. Cynthia provides a concise *generic* summary of how a word problem should be organized, complete with blanks ("something something something") where the numerical values can be inserted. She relates their word problem to the *narrative experience* of buying a gift for her mother. In subsequent discussion, she refers to what her mother likes to determine what the gift could be. Finally, the problem has a clear *mathematical structure* and in the full discussion, they discuss how to ensure a mathematically coherent subtraction problem.

Another strand of research has focused on how mathematics teachers deal with language diversity in their classrooms. Research has shown, for example, that teachers may struggle with working with language diversity, facing various challenges and dilemmas (e.g., Adler, 2001). Faced with such challenges, teachers may initially focus on mathematical vocabulary more than other aspects of language. The findings referred to above, however, suggests that vocabulary is just one source of meaning for learners. Additional research shows that mathematics teachers can and do develop skillful and supportive practices. Promising approaches include: those which draw on students' full repertoires and help them to make connections between different parts of these repertoires; those which have a strong focus on mathematical meaning and thinking; and those which integrate language learning with mathematics learning (Moschkovich, 2018). Design-based approaches hold much promise as a systematic way to identify and develop productive teaching strategies along these lines (Prediger, 2019).

Taken together, research clearly demonstrates that multilingual learners, including those learning the classroom language, can participate successfully in mathematics classroom activities in the right conditions. I call them *language positive classrooms*, since they feature practices that acknowledge and incorporate learners' languages. I recently described some of the potential characteristics of such a classroom, based on findings from an ethnographic study of four second language mathematics classrooms in Canada (Barwell, 2020). The four classrooms were: one of Indigenous learners, one of recent arrivals to Canada, one in a French immersion program, and a mainstream classroom featuring some second language learners. The study examined socialization practices across the four classrooms resulting in a distinction between language positive and language neutral classrooms. In language positive classrooms, I observed the following common practices:

- Students' home languages were regularly heard; students reference or use home languages during mathematical discussion.
- "Non-standard" accents, pronunciation, spelling or punctuation were present and explicitly related to norms.
- Explicit attention was given to features of mathematical discourse: learners actively participated in socialization into mathematical discourse.
- Relations between more formal and more informal mathematical discourses were made visible.
- Working with mathematics classroom genres was inclusively supported through specific socialization practices.
- Gestures in mathematical interaction were actively used by learners and teachers; explicit links were made between gestures and other aspects of mathematical discourse.
- Explaining mathematical thinking was actively supported

- Learners participated actively, taking extended turns or sequences of turns and initiating exchanges.

To sum up this section, language diversity matters in mathematics education because:

- It can affect positively or negatively learners' mathematics attainment;
- It affects how learners participate in mathematics class, including with respect to specific subdomains of mathematics;
- Learners bring diverse repertoires of language practices;
- It affects how teachers teach mathematics, including with respect to specific subdomains of mathematics;
- Some mathematics classroom conditions are more favorable in relation to some kinds of language diversity.

4. How is Language Diversity in Mathematics Classrooms Connected to Wider Social Structures?

Research on political aspects of language diversity and mathematics has shown how the relative status of different languages influences the choices of students and teachers in mathematics classrooms. Setati (2008), for example, showed how families in South Africa often preferred English as the classroom language even though it could make learning mathematics more challenging, since English was perceived as valuable for accessing better educational or employment opportunities. It is also clear that learning and teaching mathematics are organized in relation to the politics of language. In Canada, for example, there is still no widespread support for the use of Indigenous languages in education. In many countries, language diversity is not represented in mathematics policy or curriculum, which often privilege one dominant language. It is also worth noting that the discourse of mathematics education is itself also organized in relation to the politics of language. The papers in this proceedings, for example, are written in English, which favors researchers used to working in that language (see Barwell, 2003a).

The political organization of language diversity can be theorized using Bakhtin's ideas. As noted above, centripetal language forces represent the societal pressure to standardize language. These forces tend to align with dominant groups or ideologies. Meanwhile centrifugal language forces indicate the opposing tendency for language to diversify, apparent in the heteroglossia of human interaction. Centrifugal forces tend to be driven by the whole of society and as such create the space for its marginalized members. These two forces are in tension and produce a stratification of language in society, whereby accepted forms of unitary language are preferred over divergent forms, and hence speakers who use these accepted forms are able to access the dominant groups. This theorization explains the tension that Setati (2008) recorded in South Africa. Stratification occurs through, among other things, patterns of indexicality, which refers to the symbolic value of different forms of language:

> Ordered indexicalities operate within large stratified complexes in which some forms of semiosis are systemically perceived as valuable, others as less valuable and some are not taken into account at all, while all are subject to rules of access and regulations as to circulation. That means that such systemic patterns of indexicality are also systemic patterns of authority, of control and evaluation, and hence of inclusion and exclusion by real or perceived others. (Blommaert, 2010, p. 38)

Thus, learners who can display valued forms of mathematical discourse are more likely to be seen as knowledgeable. Conversely, many learners may be marginalized by discourses found in mathematics classrooms.

As an example, consider the discussion shown below, recorded in the class of Indigenous Cree learners as part of the ethnographic study mentioned in the previous section. Most of the students are from the James Bay region, far to the north of the city in which they attend school. The discussion took place between me, Curtis and Ben, as they worked on a problem that included the following text (see Barwell, 2014):

> Every year Ottawa holds a world-renowned tulip festival in the month of May. There are different gardens in various locations, one of which is on Parliament Hill. The Canadian Tulip Festival was established to honour Queen Juliana of the Netherlands, in 1953. It is the largest tulip festival in the world, making this flower the International symbol of friendship and the beauty of spring. This festival receives thousands of tourists every year from North America, Europe, and Asia and has an economic impact of approximately $50 million on the Ottawa region.

In the following extract from the discussion, we read through part of the problem, which positions the reader as a gardener and asks them to complete a pattern of increasing square designs composed of tulips:

RB	You're going to draw poppies=well its about tulips haha ok (.) so: maybe we can read out that part again (.) ^what do you think^ Ben can you read it out alright Curtis?
Curtis	you are the gar(.)dener (.) for tulips
Ben	tulips
Curtis	ah what's that?
RB	**bulbs**
Curtis	bulbs (.) for Canadian tulip festival in may
RB	so you are a **gardener** (.) do you feel like a **gardener**?
Curtis	^what^
RB	are **you** a gardener Ben?
Ben	^what?^
RB	are you a gardener (.) do you know what a gardener is=yeah it says in this problem you have to think you are a gardener okay (.) to plant tulip bulbs (.)

In a second extract, from a few minutes later, I support Curtis to write a suitable account of his reasoning:

RB	so (.) that's a good beginning (.) but you need to explain like the calculations that you did (.) you need to say what kind of calculations you did
Curtis	times
RB	yup but precisely what did you times what did you add
Curtis	I timesed seven (.) times seven (.) six times (.)
RB	right right
Curtis	seven plus thats it
RB	so like when you worked out for purple
Curtis	I did five times five
RB	uhum
Curtis	plus one
RB	right so I would write purple and then exactly what you just said

My analysis of the full text of the tulip problem, as well as of the interaction relating to the problem, shows that it indexes certain standard ways of languaging mathematics in relation to the world and thereby alienates the two students in particular ways, as shown in Tab. 1.

Tab. 1. Indexicality and alienation in relation to a word problem

Indexed by the Tulip problem	Alienated by the same problem
A nation (Canada)	The students' Cree nation
A region (Ottawa)	The students' home region of James Bay
An event (the tulip festival)	Events in the students' communities
Speakers of a language (English)	Speakers of Cree
A register (school mathematics)	Informal mathematical expression in English and Cree
A genre (word problems)	Everyday and Cree genres
Some mathematics (geometry and arithmetic)	Mathematics in Cree activities

This example illustrates how the tension between centripetal forces, as demonstrated by the left-hand column of Table 1, and centrifugal forces, apparent in both columns, result in the marginalization of the two students. A very concrete instance of this marginalization is the fact that they are unfamiliar with tulips, since they do not grow in James Bay.

In this section, I have shown how language diversity in mathematics education matters because:

- Mathematics learners whose diverse repertoires do not fit the prevailing dominant standard forms of language in mathematics classroom risk being alienated; i.e. if you don't talk and write in the right way, you won't succeed;
- Mathematics classrooms risk maintaining a social hierarchy based on (forms of) language.

5. How is Language Diversity in Mathematics Education Connected to the Future of Planet Earth?

Although much of my research has focused on language diversity in mathematics classrooms, I have also written about environmental sustainability and climate change in mathematics education (e.g., Barwell, 2013; Barwell and Hauge, 2021; Hauge and Barwell, 2017). This work is a response to the multiple crises facing our planetary ecosystem, such as climate change, mass extinction, pollution and ecosystem degradation. It turns out that there is a connection between language diversity and biodiversity: a recent report shows how regions of the world with a high degree of language diversity are also regions with a high degree of biodiversity (see Fig. 2) (IPBES, 2018).

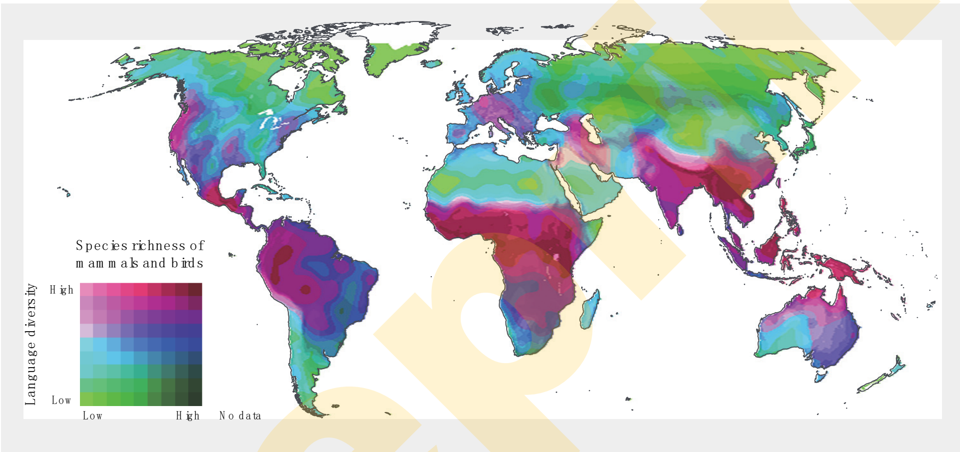

Fig. 2. Language diversity and biodiversity are spatially associated
(IPBES, 2018, Assessment report on land degradation and restoration, p. xxxiv. Copyright © 2018, Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services)

The report highlights evidence that loss of language and culture, specifically Indigenous language and cultures, is associated with loss of ecosystem knowledge, including knowledge about different species, the ecosystem and its various interconnected relations, as well as sustainable practices for maintaining a healthy ecosystem. For example, the report states:

> Alienation of indigenous peoples and local communities from the land often leads to the irreversible loss of accumulated knowledge on how to manage land. In most cases, land management practices based on indigenous and local knowledge have proven to be sustainable over long time periods and offer alternative models to the currently dominant human-nature relationship. (IPBES, 2018, p. xxxv)

There is also a clear association with economic and political marginalization, as well as individual and community well-being:

> Land degradation causes a loss of sense of place and of spiritual connection to the land, in indigenous peoples and local communities as well as in urban residents living far from the affected areas. (IPBES, 2018, p. xxxiv)

The report refers to the many Indigenous or local knowledge systems which recognize the interdependent relationship between humans and other species, and that assume a stance of relational ethics rather that one of technological progress of economic growth (p. xxxv). Research in mathematics education is beginning to incorporate similar perspectives (e.g., Boylan, 2016; Coles, 2017; Gutiérrez, 2017).

A perspective of interdependence and relationality stands in contrast to prevailing ways of thinking in mathematics education and in dominant economic systems such as neoliberal capitalism, which tend to be based on ordering, hierarchy and dominance. As I discussed in the previous section, mathematics classrooms are connected to social stratification. Such orderings in relation to language diversity in mathematics education are also apparent at a macro scale. For example, looking across Canada as a whole, there is a clear ordering in terms of which languages are valued in mathematics classrooms: 1. English (the official language of the majority of Canadians); 2. French (the official language of a minority of Canadians); 3. Languages of immigrant communities, such as Chinese or Spanish; 4. Indigenous languages, such as Cree or Inuktitut. This kind of ordering contributes to the disappearance of minority languages around the world. These hierarchies can be translated into the specific orderings of language in mathematics classrooms. In Canadian mathematics classrooms, the ordering is: 1. English mathematics register; 2. French mathematics register; 3. other mathematics registers. Of course, there are regional variations (in Quebec, French is dominant, followed by English, for example), but overall, there is a hierarchy, in which learning and teaching mathematics is inscribed and which it reinforces. Similar orderings can, I suggest, be found in most mathematics classrooms in the world.

Hence, mathematics education, perhaps inadvertently, reinforces an ordering way of thinking. This way of thinking has been identified as a "root metaphor" (Bowers, 2001) underpinning many aspects of the environmental crises we currently face (Martusewicz et al., 2014). In relation to biodiversity, common scientific, mathematical and societal discourses are related to orderings (of species) and alienation (of humans from other species). For example, one ordering of species might be: 1. humans; 2. farm animals (e.g., chickens, sheep); 3. agricultural crops (e.g., rice, wheat); 4. wild animals (e.g., wolves, seals); 5. wild plants (e.g., forests); 6. insects; 7. viruses. Such orderings are generally in relation to how useful a species is to human exploitation. In a similar way, orderings of languages are often justified in terms of how useful a language is within the dominant economic system, as found in Setati's (2008) research, for example. Such orderings, as shown in the previous section, also alienate; in particular, they alienate speakers of languages that are positioned as marginal or peripheral, with respect to the dominant, unitary language forms. These unitary language forms are related to colonial, often Eurocentric, ideologies that privilege Eurocentric ways of thinking and make it difficult to build relational practices and ways of thinking. Teaching mathematics most often involves reinforcing these ways of thinking, and requires innovative, decolonizing approaches to disrupt the prevailing order (see Parra and Valero, 2022, for an example of such work in Columbia).

Language diversity therefore matters in mathematics education because:

- Ordered patterns of thinking about languages and ordered patterns of thinking about biodiversity use the same underlying way of thinking, and result in the same pattern of alienation;
- Ordered hierarchies reflect a Eurocentric, colonial, unitary perspective on language and on species;
- The ordering way of thinking makes ethical relationality difficult;
- Many languages and many species are threatened with extinction.

6. Conclusion

In this paper, I have examined several different reasons why language diversity matters in mathematics education. First, and fundamentally, language diversity matters because mathematics classrooms around the world are diverse. Moreover, the language diversity that is present in mathematics classrooms is itself diverse and constantly changing. Second, language diversity matters because learners of mathematics draw on many aspects of their repertoires of language practices to participate in mathematical meaning-making. Teachers need to adopt strategies that are responsive to language diversity, such as those found in language positive classrooms. Third, language diversity matters because it is one of the links between mathematics classrooms and wider society. Mathematics classrooms are sites of social stratification, linked to the stratification of language. Failure to acknowledge such links risks mathematics classrooms reproducing patterns of marginalization and alienation that are present more widely. Finally, language diversity matters in mathematics education because the way we think about language diversity often reflects deeply embedded ordering discourses that reflect Eurocentric, colonial ways of thinking. These discourses are also implicated in the ecological crisis facing our planet, through a dominant ordering of humans with respect to other species.

There are some clear implications arising from these points. First, language diversity should be more widely recognized in mathematics education research and in mathematics curriculum and policy. Most research in the field, for example, ignores the presence of language diversity among research participants and does not consider the possible implications of this diversity on research design, data collection or analysis and interpretation of data and findings. By extension, it would make sense to incorporate language diversity into theories of learning and teaching mathematics. Similarly, strategies for teaching mathematics in contexts of language diversity should be the basis for any general guidance for teachers, since all contexts potentially feature such diversity. While centripetal and centrifugal language forces are always present and the tension between them shapes every utterance, research, policy and teaching can all work to ensure that unitary language ideologies are made visible and that there is space for heteroglossia; in effect, a dialogic approach to language diversity is more likely to avoid rigid stratification and consequent alienation. Another way to say this is to argue for an approach informed by ethical relationality with respect to learners, teachers, languages and other species that recognizes the interdependence between them (and us) all.

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