Invited Lecture

Textbook Transformation as a Form of Textbook Development: Approaches, Issues, and Challenges from a Social and Cultural Perspective

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ABSTRACT In this article, I use the term "textbook transformation" to refer to the development of a textbook or a series of textbooks based on another selected pre-existing textbook(s), which leads to the formation of a new textbook(s). By mainly drawing on my own mathematics textbook research and development experiences, particularly in transforming a popular Chinese mathematics learning resources series, One Lesson One Exercise, or Yi Ke Yi Lian in Chinese, to the English learning resource series the Shanghai Mathematics Project Practice Books and developing the Zhejiang Secondary Mathematics Project Textbooks over the last two decades, I argue that the means of textbook transformation can be classified into five types: translation, adaptation, revision, rewriting and a combination of them, based on the selected pre-existing textbook(s). Following this classification, the article analyzes and discusses the approaches, issues and challenges in textbook transformation by using concrete examples from available textbooks, and illustrates in particular how social and cultural factors play an essential role in textbook transformation and its significance in international exchange and collaboration in the development of school mathematics textbooks.

Keywords: One lesson one exercise; Shanghai Maths Project; Textbook research; Textbook development; Textbook transformation; Zhejiang Secondary Maths Project

1. Introduction

1.1. Textbook research in mathematics education

Over the last two decades, issues in mathematics textbooks have received increasing attention internationally. This trend can be partly seen from the fact that many academic and research conferences which focused on mathematics textbooks have received a considerable amount of attention from international mathematics education community. In 2011, the International Conference on School Mathematics Textbooks

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(ICSMT) was held at East China Normal University and chaired by Jianpan Wang, focusing on the comparison of different textbooks across different countries. In 2013, The journal, *ZDM — International Journal of Mathematics Education* (hereafter ‘ZDM’), published its first special issue on the theme of “textbook research in mathematics education” (Fan, et al., 2013). The First and Second International Conferences on Mathematics Textbooks Research and Development (ICMT), held in the UK in 2014 and Brazil in 2017, respectively, attracted a large number of participants from over 30 countries (Jones et al. 2014; Schubring et al. 2018). Subsequently, a second ZDM special issue on the theme of “recent advances in mathematics textbook research and development” was published in 2018 (Schubring and Fan, 2018). The Third International Conference on Mathematics Textbook Research and Development (ICMT-3) was held in Germany in 2019. The third ZDM special issue with the theme “Mathematics Textbooks as Instruments for Change”, will be published soon, and the Fourth International Conference on Mathematics Textbook Research and Development (ICMT-4) will be held in Beijing next year3.

In a comprehensive review article I and my co-authors published in the ZDM’s special issue in 2013, I proposed the following five directions for future textbook research (Fan et al. 2013).

1. To establish more solid conceptualization and theoretical underpinning of the role of textbooks and the relationship between textbooks and other variables in a wider educational and social context, and view the existence of textbooks from a broader perspective.
2. To have more confirmatory research about the relationship of the textbook and students’ learning outcome.
3. To have more research directly focusing on the issues about the development of textbooks [emphasis added].
4. To employ more advanced and sophisticated methodology in textbook research.
5. To have more research on the use and development of electronic textbooks in mathematics.

For ICMT-3, the conference themes included content and its presentation in textbooks, both traditionally printed or more recently digital, use of textbooks, historical perspective on textbooks, comparative studies of digital textbooks or traditional textbooks, textbook and policy, research on textbooks, and the development of textbooks. Here, the development of textbooks includes concepts, task design, learning-teaching-trajectories, methodological approaches, quality, design-based research, etc.

3 [post-congress note]: The third ZDM special issue on mathematics textbooks was published in 2021 and ICMT-4 was held in 2022. The proceedings of all the ICMTs, from ICMT-1 to ICMT-3, are available at https://acme.ecnu.edu.cn.
However, as researchers have pointed out, among the available studies in this area, there has been notably a lack of research directly addressing the issues concerning the development of mathematics textbooks or how textbooks are produced, and more research in this direction is highly needed (Fan, 2013; Johnsen, 1993). This article (presentation) focuses on the development of mathematics textbooks. More specifically, by mainly drawing on my own mathematics textbook research and development experiences, particularly in transforming a popular Chinese mathematics learning resources series, *One Lesson One Exercise, or Yi Ke Yi Lian* in Chinese, to the English learning resource series, *the Shanghai Mathematics Project Practice Books* (Collins, n.d.), and developing *the Zhejiang Secondary Mathematics Project Textbooks* (Zhejiang Education Publishing Group, n.d.) over the last two decades, my aim is to propose and look into a new form of textbook development: textbook transformation, with a main focus on a social and cultural perspective.

1.2. *The Shanghai Maths Project*

The Shanghai Maths Project is an international collaborative effort between HarperCollins in the UK and East China Normal University Press in China. It is based on the latest edition of the award-winning series of *One Lesson One Exercise* (or *Yi Ke Yi Lian* in Chinese) published in China (Editorial Team, 2014). After adaptation following the English National Curriculum, its English series *Shanghai Maths Project Practice Books* were published by HarperCollins in the UK (Fan 2015, 2016), and there are 11 books in total.

The project and its main product, i.e., Shanghai Maths Project Practice Books, has attracted much attention both in China, the UK, and internationally (e.g., see Farrington, 2015; Fan, Ni, et al., 2018; Fan, Xiong, et al., 2018; “WITMAS International School”, n. d.).

Since I started working on the project as its director, I have been frequently asked a variety of questions and I summarized them into the following five key questions:

1. Is it a direct translation?
2. Is it an adaptation?
3. Is it a rewriting?
4. Is it related to a Shanghai maths learning series?
5. What is it?

Here come some simple answers based on my experiences. Firstly, the project is not completely a direct translation; some questions and ideas were indeed a direct translation, but many are not. Secondly, the project is partially an adaptation, and partially a rewriting, as about 30% of the tasks were virtually brand new. The project is, essentially, a form of textbook development, related to and based on the Shanghai maths learning series, *One Lesson One Exercise*. Therefore, I eventually named it a “transformation”, which is across different social and cultural settings.
1.3. The Zhejiang Secondary Maths Project

The Zhejiang Secondary Mathematics Project is aimed to develop a series of secondary mathematics textbooks from Grade 7 to Grade 9, which have been published by Zhejiang Education Publishing Group and widely used by students in Zhejiang province of China, probably about 90% or 95% of the student population.

From the initial edition developed in 2004 to the current latest edition, which is still being used, two editions of the textbooks were developed following different Chinese primary and secondary mathematics curriculum standards issued by the Chinese Ministry of Education at different times. In addition, the later edition was slightly revised in 2019, which was necessary and helpful.

The development of the Zhejiang Secondary Mathematics Project Textbooks from its first edition to the latest is certainly not a translation, adaptation, or rewriting. It can be also viewed as a kind of transformation, although within the same social and cultural backgrounds.

2. Definition of Textbook Transformation

I use the term “textbook transformation” to refer to the development of a textbook or a series of textbooks based on another selected pre-existing textbook(s) based on the pre-existing textbook(s), which leads to the formation of a new textbook(s). So, it is not a start from scratch nor is the textbook(s) a brand new work.

Based on my experience in research and textbook development, at least five types of textbook transformation can be identified in order to understand the different means of textbook transformation. Those means can be translation, adaptation, revising, rewriting, or a combination of the above approaches. Table 1 shows a framework about textbook transformation. The left column is added to describe the transformation of textbooks from an international perspective.

<table>
<thead>
<tr>
<th>Means</th>
<th>Description</th>
<th>Languages before and after</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Translation</td>
<td>Direction translation; the difference between pre-existing and the newly formed are in the languages used only.</td>
<td>Different</td>
</tr>
<tr>
<td>2. Adaptation</td>
<td>The pre-existing and the newly formed are tailored to the different target users with different requirements, conditions or backgrounds, no matter whether the pre-existing and the newly formed use the same language or different languages.</td>
<td>Different or the same</td>
</tr>
<tr>
<td>3. Revision</td>
<td>Usually for the same target users using the same language, but for different requirements or conditions of users and often at different times.</td>
<td>Same</td>
</tr>
<tr>
<td>4. Rewriting</td>
<td>It is for different requirements or conditions of users in the same language. It can be for the same or different targeted users.</td>
<td>Same</td>
</tr>
<tr>
<td>5. Combination of the above</td>
<td>It is a combination of two or more of the above means to produce a new textbook based on a pre-existing one.</td>
<td>Different or the same</td>
</tr>
</tbody>
</table>
It should be emphasized that all the textbook development activities listed in the table are based on selected pre-existing textbooks and will lead to the formation of new textbooks. It is a formation of the newer by transformation of the older.

There are abundant historical examples of textbook transformation both within and across nations and educational settings for mathematics textbook development. For example, Mathematics 1 shown below (Fig. 1) was edited by the University of Chicago School Mathematics Project or UCSMP, which was a translation of Japanese textbook for grade 10 from Japanese into English and published by UCSMP. Russian Grade 2 Mathematics shown below (Fig. 2) was a translation of a Russian mathematics textbooks from Russian to English, also published by UCSMP. More information about the UCSMP textbook translation can be found its website (UCSMP, n.d.)

The Shanghai Maths Project Practice Books were based on Shanghai’s One Lesson One Exercise, as mentioned earlier, were jointly published by Collins and East China Normal University Press, as said earlier. The most recent example I have been involved, within the same educational setting, is the Zhejiang Secondary Mathematics textbooks (see more information below).

3. Cases of Textbook Transformation: Issues, Challenges and Approaches

Below I introduce two studies I have conducted in relation to textbook transformation, with some concrete examples when appropriate, to illustrate the issues, challenges and approaches concerning the transformation of mathematics textbooks.
3.1.  **Case 1: Shanghai Maths Project**

One of the basic principles of the transformation from the Shanghai series to the English series in the Shanghai Maths Project was to follow the English national curriculum. It is not simply a reduction or an adaptation. While basically retaining the original pedagogy in Shanghai, which is a main reason for the existence of the project, there are clearly changes made in various aspects from the original Shanghai series to the English series. The issues and challenges I encountered can be, to a large extent, classified into the following categories: curriculum-related, language-related, culture-related, and context-related.

3.1.1.  **Curriculum-related issues.**

There are similarities as well as a considerable number of differences in compulsory mathematics curriculum between Shanghai and England.

Tab. 2 provides an overview about where different topics of statistics are introduced in the two series, both of which must follow their different curricula.

<table>
<thead>
<tr>
<th>Topic in Statistics</th>
<th>Shanghai series</th>
<th>English series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical tables</td>
<td>Grade 2</td>
<td>Grade 2</td>
</tr>
<tr>
<td>Bar charts</td>
<td>Block diagram without coordinates in Grade 2.</td>
<td>Bar chart based on the knowledge of pictogram and block diagram in Grade 2.</td>
</tr>
<tr>
<td></td>
<td>Bar chart with vertical axis in Grade 3.</td>
<td>Bar chart with concepts of horizontal and vertical axes in Grade 3.</td>
</tr>
<tr>
<td>Broken line charts</td>
<td>Broken line chart, with concepts of horizontal and vertical axes in Grade 4.</td>
<td>Grade 4.</td>
</tr>
<tr>
<td>Mean number and its application</td>
<td>Grade 5, Term 1.</td>
<td>Grade 6.</td>
</tr>
<tr>
<td>Pie diagrams</td>
<td>Grade 6, Term 1.</td>
<td>Grade 6.</td>
</tr>
</tbody>
</table>

For the table, we can see that the Shanghai series introduced bar charts in Grades 2 and 3, which is similar with the English series. However, the English national curriculum introduces block diagram and pictogram, while the Shanghai curriculum does not. Another example we can see is that the mean number and its application in the English series are introduced in Grade 6, while they are introduced in grade 5 in Shanghai.

In addition, in Shanghai, the initial introduction to the idea or concept of fractions is in the 2nd semester of Grade 3, while in the English series it is in Grade 2 and Grade 3. The comparison of fractions, the addition, and the subtraction of fractions are also introduced earlier in the English curriculum than in the Shanghai’s, which might be beyond many people’s expectation.

Understanding the differences in the coverage and sequence of mathematics contents in the two curriculum is certainly a challenge for the textbook developers.
involved. I would argue that having experienced and knowledgeable team members or external consultants proved to be most helpful to tackle the challenge.

3.1.2. Language-related Issues.

There are tons of examples of language-related issues. For example, considering “1001” in English, we pronounce it as “one thousand and one”, while in Chinese the pronunciation is “yi qian ling yi” in Mandarin or “one thousand zero one” in English literally. So, in the Chinese textbooks, students were asked “Do you have to pronounce zero when you read 1001?” But if it is directly translated into English, it does not make any sense.

Another example is dengshi (equation). In Chinese, a dengshi is a statement of equality, e.g., $7 + 3 = 10$. That is different from a fangcheng, which contains at least one variable or unknown, e.g., $7 + 2x = 10$. But in English, a dengshi and a fangcheng can both referred to an equation. So, communication will be a problem if one does not distinguish the difference between a dengshi and a fangcheng.

By the way, I noted that some Chinese textbooks or research articles translate zhengshi into integral expressions, which is incorrect and could not be understood for English readers as it was expected to understand. The reason is that, in Chinese, a zhengshi as a mathematical term means either a monomial or a polynomial expression. However, there is no equivalent term in English.

In short, a direct translation from one language to another language, like in this case, does not make sense, which is a real challenge, and one has to either rewrite or drop such questions for the English series.

3.1.3. Culture-related issues.

There are also culture-related issues or challenges, as mathematics teaching and learning are often contextualized, and teachers and students all have certain culture imprints and are not culture-free. The examples in Tab. 3 depict such a difference and the transformation made.

Tab. 3. Two related questions in the Shanghai series and the English series

<table>
<thead>
<tr>
<th>Chinese series [for the 2nd year]</th>
<th>English series [for year 3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xiao Qiao went to the Shanghai Book City with 200 yuan. She bought a Xinhua Dictionary for 79 yuan and a set of Fairytales World series for 114 yuan.</td>
<td>Joan went to a bookstore with £200. She bought two dictionaries for £79 and a set of fairy tale books for £114.</td>
</tr>
<tr>
<td>① Please estimate whether Xiao Qiao had enough money.</td>
<td>(i) Estimate whether Joan had enough money for the dictionaries and books she bought.</td>
</tr>
<tr>
<td>② If she had enough money, how much change should she get? If she did not have enough money, how much was she short?</td>
<td>(ii) If she had enough money, how much change should she get? If she did not have enough, how much was she short?</td>
</tr>
</tbody>
</table>

(Source: Fan, et al., 2018)

In the example, seven items relating to cultural factors, as highlighted, were changed from the Shanghai series to the English series. Those changes include from
familiar to Shanghai students, to a bookstore, and from the most popular Chinese dictionary Xinhua Dictionary to two dictionaries, and so on.

Another example is that, in China, there are many national holidays, such as the Tomb Sweeping (or Qingming) Festival, Dragon Boat Festival, and National Day, introduced as question contexts in mathematics textbooks which I think is not suitable to use as background for English students to learn mathematics, as an English student would not understand nor need to understand them as a learner of mathematics. Therefore, it is necessary to replace the traditional Chinese festivals with those familiar to English students in textbook development.

In the Shanghai Maths Project, we changed, for example, International Labor Day, Dragon Boat Festival, Woman’s Day, etc. which are celebrated in the Chinese culture to New Year’s Day, Good Friday, Christmas Day, Boxing Day, and so on in the English series. So students in the United Kingdom or other English-speaking countries could understand the meaning of the question without cultural-related challenges in questions’ backgrounds.

A case study I and my colleagues conducted on the manifestation of cultural influences in the formation of mathematics textbooks suggested that culture-related differences should not be underestimated, and the results revealed that most adaptations between the Chinese series and the English series are related to ‘ways of behaving and customs’ and ‘artifacts, flora and fauna’, followed by ‘identities’ and ‘geography’, and the least are related to ‘organisations’ and ‘history’ (Fan et al. 2018).


In addition to culture-related differences, there are also context-related issues, often more complex than one might expect. It should be pointed out, not everything is culture, and some are more context-related, though culture and social context are sometimes intertwined.

For example, there is a question in the Shanghai resource book series:

There are 205 yellow cattle. There are 4 times as many water buffalo as there are yellow cattle. How many water buffalo are there? [In the Shanghai series, Grade 3 book]

As yellow cattle and water buffalo are not commonly seen in English, for helping students learn better it is beneficial to change them to some more popular animals, such as goat and sheep, which are more familiar to English students. Below is the “transformed” question in the English series:

There are 205 goats on a farm. There are 4 times as many sheep as there are goats. How many sheep are there? [In the English series, Year 3 book]

Another example is related to buildings. Many landmark buildings are used as problem contexts in the measurement topics in the Shanghai series, such as Shanghai Centers (632 meters), Global Financial Center (492 meters), East Pearl TV Tower (468 meters), and Jinmao Tower (428 meters). For Shanghai students, it is familiar and
helpful. But for English students, it may not be so helpful to learn mathematics using the Shanghai contexts. So, those contexts in the questions were changed into Canary Wharf Tower (244 meters), One Churchill Place (156 meters), and Broadgate Tower (178 meters) with the help of my English colleagues.

Nevertheless, there are some other problems in the Shanghai series for which we could not find comparable contexts for English students. For example, we could not find any buildings in London or England which are more than 400 meters high. So, to help students learn big numbers in the English series, we had to change the questions or use other types of contexts for the questions.

3.2. Case 2: Zhejiang Secondary Maths Project

The Zhejiang Secondary Mathematics Project, in which I was also privileged to serve as the project director, is another typical case reflecting the idea of textbook transformation, although within the same educational settings, as aforementioned. A basic principle from the first edition to the second edition in the project is that the level of standards needs to be maintained the same. Apart from this, there have been systematic changes which have been necessary and helpful. The dimensions of the transformation from the first edition to the second edition were reflected in, and can be classified into, curriculum-related, content-related, pedagogy-related, and context-related issues, but basically not language-related or culture-related, as it is in the same language, i.e., Chinese, and the users are from the same cultural background.

There are a great number of changes which are curriculum-related. According to the national educational policy and the law, school mathematics textbooks in China except Shanghai which had its own city-wide official curriculum, must follow the national curriculum. In the first edition of the Zhejiang textbooks published in 2004, we introduced “Special parallelogram and trapezium” for the 2nd semester of Grade 8. However, the topic of trapezium had to be removed in the second edition, since the newer curriculum no longer required students to learn the topic. Besides the curriculum requirement, some contents in the textbooks also need to be changed to reflect the development of mathematics and the society, in addition to the curriculum.

By integrating teachers’ feedback, some improvements were made to the pedagogy in the second edition. For example, some basic-level questions designed as Group A exercises for students’ homework to develop their basic knowledge and skills in the first edition were reclassified and moved to Group B in the second edition because according to the teacher’s feedback, they were too difficult for many students. In addition, the second edition also reflected, to some extent, the new development of pedagogy in mathematics teaching and learning, for example, by providing more opportunities for students to do hands-on activities and solve more open-ended problems in their learning of mathematics.

Many changes, probably more than any other kinds, were context-related from the first edition to the second edition, which also reflected the fast social, economic and scientific development in China during the period of time.
Adopting PISA’s well known “Personal, Societal, Occupational and Scientific context framework” (OECD, 2019), I and my colleagues examined the context-related items including worked examples and exercise questions in the textbooks for Grades 8 and 9. More specifically, we compared the differences between the second edition published in 2013 and 2014 respectively and it is revised edition published in 2019 in terms of the number of context-related items. The results revealed there were some changes, but not as significant as one might expect, as shown in Fig. 3 for Grade 8 and Fig. 4 for Grade 9, with inter-rater reliability being 0.93 and 0.89, respectively.

4 It is not called the third edition as it was a relatively “minor” revision, which can also largely explain why the distributions were quite similar.
More important changes were found when we further examined the contexts of the questions themselves between the first edition, the second edition and its revised edition. Below are two examples for illustrating the change of contexts in the categories of societal and scientific contexts using the PISA framework. The reasons for such changes are virtually self-explanatory (Note: the main changes were highlighted)

**Societal context: Hangzhou Bay Bridge** (In Section 1.1, “From natural numbers to fractions”, First semester, Grade 7).

First edition (2004):

Please read the following: The longest cross-sea bridge in the world, the Hangzhou Bay Bridge, was laid on June 8, 2003, and it was planned to be completed and opened to traffic in 5 years. This 6-lane highway cable-stayed bridge with a designed daily traffic volume of 80,000 vehicles, with a designed speed up to 100 kilometers per hour and a total length of 36 kilometers will be the first cross-sea bridge in the Chinese mainland. What numbers did you see in the above text? What kinds of numbers do they belong to?


Please read the following: The Hangzhou Bay Bridge was opened to traffic on May 1, 2008. This 6-lane highway cable-stayed bridge has a designed daily traffic volume of 80,000 vehicles, with a designed speed to 100 kilometers per hour and a total length of 36 kilometers, and a service life of 100 years. It was the longest cross-sea bridge with the largest engineering workload in the world at that time. What numbers did you see in this report? Please find out these numbers and explain which of them are for counting and measurement, and which of them are for labeling or sorting.

**Scientific context: Supercomputer** (In Section 3.1, “Multiplying powers with the same base”, Second semester, Grade 7).

Second edition (2012/13)

The measured calculation speed of the “Tianhe-1A” supercomputer in China has reached 2.566 million billion per second. If it works at this speed for a whole day, how many times of calculations can it perform?

Revision of second edition (2019)

The measured calculation speed of China’s “Shenwei - Light of Taihu Lake” supercomputer has reached 93 million billion times per second. If it works at this speed for a whole day, how many times of calculations can it perform?

It should be mentioned that about the Hangzhou Bay Bridge, when we wrote the first edition, the bridge was only a plan to be the longest bridge in the world. But for the second edition, the bridge was already completed in 2008, and moreover, many longer bridges were built or planned to build, so it was no longer the longest at all after it was completed. Therefore, it must be changed for the second edition. A similar approach was taken for the context of supercomputer to reflect the rapid scientific development in China during those years.
There are also many changes related to personal contexts and occupational contexts. For example, the textbook for the first semester of Grade 8 in the first edition used the price of the residential water rate in a city as 1.2 yuan/m³ while in the latest version of the textbook revised in 2019, the price was changed to 2.9 yuan/m³ to reflect the increase of the water price. Another example is that the hourly wage was 6 yuan/hour in the first edition while it was changed to 25 yuan/hour in the second edition. This change was also necessary as the standard of monthly minimum wage is increased yearly, and the minimum hourly wage was increased from 4.2 yuan/hour in 2005 to 12 yuan/hour in 2013 (Zhejiang Province Government, 2012). Without change, a wage of 6 yuan/hour would be not only unrealistic but also illegal.

4. Summary and Concluding Remarks

To summarize, in this article, “textbook transformation” is defined to refer to the development of a textbook or a series of textbooks based on another selected pre-existing textbook(s), which leads to the formation of a new textbook(s).

By mainly drawing on my own mathematics textbook research and development experiences, particularly in transforming a popular Chinese mathematics learning resources series, One Lesson One Exercise, to the English learning resource series, the Shanghai Mathematics Project Practice Books, and developing the Zhejiang Secondary Mathematics Project Textbooks for its different editions, the article argues that the means of textbook transformation can be classified into five types: translation, adaptation, revision, rewriting and a combination of them, based on the selected pre-existing textbook or textbooks.

Furthermore, the main issues or challenges in textbook transformation can be classified into curriculum-related, language-related, culture-related, context-related, content-related, and pedagogy-related, to different extent and depending on the types of textbook transformation.

A related question is, why should we take on the challenges? From the macro-level or external factors, I would argue it is because of social development, mathematical development, pedagogical or educational development. Furthermore, because of globalization and international exchanges, we all want to develop ourselves by learning from others. From the micro-level or internal factors, it is one of the most efficient and effective ways to develop textbooks by not only learning from others but also learning from the past. This is particularly clear when we consider that developing a set of textbooks starting from scratch is truly complex and time-consuming.

Finally, regarding approaches to tackling the challenges, I would argue collaboration of textbook developers with different expertise and backgrounds is a key to ensure the quality of the transformation, and exchanges in terms of research are also important. Unfortunately, there are not many studies available yet in this line, and I hope this article can make a meaningful contribution to it.
Acknowledgement

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