Invited Lecture

Experimentations in Mathematics Education with Art and Visuality

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ABSTRACT "Art and Mathematics" has been considered in mathematics education primarily for the possibility of teaching and learning mathematics through art. Many reasons are implied in this: to give meaning to mathematics; to motivate or contextualise teaching; to broaden mathematical visualization, among others. However, neither colonizing art by mathematics nor instrumentalizing mathematics by art, we have been considering this pair for the experimentations that can happen in the exercise of thinking. Taking this into account, in this presentation, first, I introduce the idea of visuality in differentiation with the concept of visualization in mathematics education to point out some theoretical concepts of the research. Then, I present some research works I have been developing, especially those that have been effects of the production of a methodological stance that occurs at the interface between paintings, visuality and mathematics education. Finally, I draw some conclusions, outlining an ethical, aesthetic, and political stance for teaching mathematics with arts.

Keywords: Philosophical perspective; Historical critical attitude; Experimental ethos.

1. Introduction

This paper reports on an empirical study that aimed to legitimize the visuality's perspective for mathematical visualization (Flores, 2013; Flores, 2012), as a frame for research on visualization in mathematics education. This emerges as an analysis strategy and, consequently, as an extension of research on visualization and art, discussing how our look was historically constituted, and thinking about other relations between art and mathematics for teaching.

One of the connections between art and mathematics has been considered in mathematics education for the possibility of teaching and learning mathematics through art. Many reasons are implied in this: to give meaning to mathematics; to motivate or contextualize teaching; to learn about geometric terms and basic shapes; to improve retention of key concepts and vocabulary; to connect math with other disciplines; to develop mathematics skills, such as mathematical visualization.

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Actually, the point is: Once you see the relationship between the two, art and mathematics, you will — no doubt — begin to see opportunities everywhere to use art in your math classroom.

So, the question is no longer: What connects mathematics and arts for mathematics teaching? It seems that there is more to it than the fact that mathematics underlies patterns and perspective, for instance.

Having this in mind, I have divided my presentation (and this paper) into three parts. First, I am going to talk about the idea of visuality in contrast with the concept of visualization in mathematics education to point out some theoretical concepts of the research. I find this point of the visual interesting because it puts us in another relationship with art.

Then, I present some research works I have been developing, especially those that have resulted from the production of a methodological stance that occurs at the interface between paintings, visuality, and mathematics education. Briefly, those works are developed as workshops with elementary school children, carried out by our research group, GECEM[2].

Finally, I draw some conclusions, outlining an ethical, aesthetic, and political stance for teaching mathematics with arts.

2. Visualization and Visuality

The role of visualization in mathematics teaching has been considered important for learning since the 1990s. Many researchers have emphasized this importance of visualization and visual reasoning for mathematics teaching and learning (Presmeg, 1989; Zimmerman and Cunningham, 1991; Dreyfus, 1991; Arcavi, 1999); others explored concrete examples of visualization and visual reasoning in the context of problem solving (Zimmerman, 1991; Goldenberg, 1991; Tall, 1991); some others defended the idea that mathematical technologies and software play a fundamental role in the process of visualization, contributing to the development of the student's ability to visualize in mathematics (Nemirowsky and Noble, 1997; Borba and Villareal, 2005); and others considered teachers and their beliefs on the role of visualization (Biza, Nardi and Zachariades, 2010).

Taking this into account, Flores, Wagner and Buratto (2012) analyzed Brazilian research dealing with visualization in mathematics education from 1998 to 2010. The authors found that most of those research works use the term visualization to refer to a visual capacity that would be necessary to foster, encourage, teach and educate through visual activities. Among these activities, the arts have served as a powerful mechanism to meet the expectations of learning and teaching mathematics.

Also, art in mathematics education has been deemed a powerful instrument to be associated with aspects of visualization in mathematics as a whole, for learning

[2] GECEM: Group of Contemporary Studies and Mathematical Education. Group registered at the Portal of the National Council for Scientific and Technological Development — CNPq and at the Federal University of Santa Catarina. [www.gecem.ufsc.br]. Leadership by Prof. Dr. Cláudia Regina Flores.

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concepts. Flores and Wagner (2014) analyzed research conducted in Brazil on art and mathematics education from 1987 to 2013. The authors found that most research works are based on constructivist models of cognition and representation. In this case, visualization has been considered as a set of cognitive activities that lead to the understanding of information from the images. As such, teaching and learning mathematics through arts is either to repeat experiments already carried out, or to study the rules and postulates discovered by science. Consequently, art has served as a mere support to a very well-defined end, which is to teach mathematics.

2.1. *From the perspective of visuality*

When operating on a theoretical — conceptual displacement, the term visualization opens up to the questions of visuality which, transported from the studies of visual culture (Brennan and Jay, 1996; Sturken and Cartwright, 2001; Dikovitskaya, 2005), comes to be seen as the result of the sum of a multiplicity of visual and discursive practices in the scope of history and culture. The image is considered a focal point in the process through which meaning is made in a cultural context (This subject is neither independent nor autonomous; rather, it is a deeply interdisciplinary activity).

Therefore, this means that, when we learn to see socially, i.e., when our visual retina is articulated amid experiences and codes of recognition, we are part of a system of visual discourses that organize and indicate how we should see and how we see. For example, if we are given the world in perspective, it is because the technique of the perspective appears to us as a model of vision that produces a three-dimensional space, as much as a rationalized view of the perspective space.

From this vantage point, the art is considered a poetic component, an opening of creativity that produces an intimate connection between thinking and being. Thus, with art, instead of identifying and repeating mathematical concepts, one thinks with it. Thinking about concepts, about education, about teaching, about learning, where thinking would be a type of learning that happens by experimenting with mathematical formulations, mathematical thinking and mathematical language.

Conceptually comparing both terms, visuality and visualization, we could say that the first leads to a deconstruction of the founding principles of the sense of sight and perception, instead of being mostly concerned with the learning of concepts of geometry and visual skills.

Tab. 1. Conceptual differences between visualization and visuality

Visualization	Visuality
Process of construction and transformation of mental images	It is the sum of the discourses that inform how we see
Learning concepts and developing visual skills	Discusses visual practices in the context of history and culture

Going further in this comparison, I could add that visualization implies representation, cognition, perception, identification, visual abilities, while visuality

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implies historicity, narrativity, the process of creation, problematization, imagination, and so on (see Fig.1).

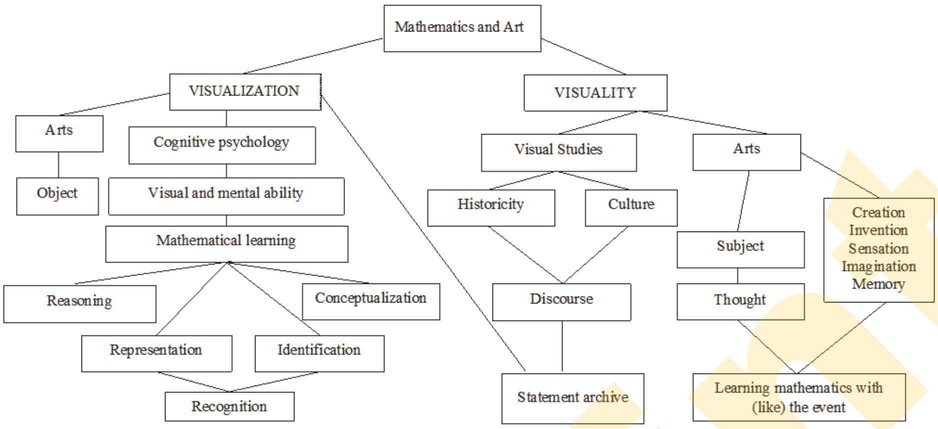

Fig. 1. Map of comparison between visualization and visuality

Therefore, in my research, I have proposed a perspective of visuality for mathematical visualization (Flores, 2013; Flores, 2012), which analyzes briefly how paintings have effects on a way of looking, specifying that we are engaged by mathematical thinking. It also provides the study of further forms of educational practices that could be possible by understanding the role of art in shaping a particular mathematical gaze. Then, my works invite mathematics education researchers to open spaces of reflection on research about visualization in mathematics education, particularly in relation to the mathematical thinking, but also suggests that bodily experience of mathematics may be fundamental for learning.

To put it simply, it will be necessary to question about how, with the arts, other ways of teaching and learning mathematics could be possible. For this purpose and inspired by the visuality point of view, together with my group of students, I've mentioned in the beginning of this paper, we have been working on practices of looking and exercises of thinking about mathematics with arts at the Elementary School.

3. Laboratories Settings: Experimenting with Mathematics and Art

By the perspective of critical educational research, I have found that the main problem addressed above could be developed since explains "the way in which we could help students to arrive at a more open, better, more critical, emancipated or liberated view." (Masschelein, 2010, p.274). According to the author, the critical education research is about educating the gaze, and

> it is related to this idea of being 'present in the present'. It is related to an understanding of education not in the sense of 'educare' but of 'e-educare' as leading out, reaching out. E-ducating the gaze is not about getting at a liberated or critical view, but about liberating or displacing our view. (Masschelein, 2010, p.277).

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Furthermore, the basis of this theoretical perspective is aligned with Foucault's idea that we must try to proceed with the analysis of ourselves as beings who are historically determined, opening up a realm of historical inquiry. It means that

> The critical ontology of ourselves […] has to be conceived as an attitude, an ethos, a philosophical life in which the critique of what we are is at one and the same time the historical analysis of the limits that are imposed on us and an experiment with the possibility of going beyond them. (Foucault, 1984, p.47).

By means of a historical-critical attitude in the work of philosophy and critical educational research, and considering the perspective of visuality to mathematics education, workshops, that are laboratory settings, have been created and developed with Elementary School children, by our research group, in Brazil, in order to analyze critically both mathematical visuality practices and also mathematics research under a philosophical perspective. A laboratory can be conceptualized:

- "as an experimental system that should allow for (new) things to happen, to appear as such, […], emphasizing the practice of making as trying to call into presence." (Masschelein, 2012, p. 367).
- "It is about registering, seeing, illuminating, bringing into play, penetrating, inviting, inspiring, experimenting; it is about exposing oneself and trying the words and verbs again." (Masschelein, 2012, p. 368).
- They are a place for exposing and registering, while gathering students, teachers and researchers around the questions of our present time (how mathematical thinking organizes the way we represent and look at the world). (Masschelein, 2012).

As an example, let me consider three works carried out by our research group. For all of them, the activities were developed with children from the 5th grade of Elementary School, from the College of Application of the Federal University of Santa Catarina, Brazil.

3.1. *The case of experiencing mathematics with cubist art*

Inspired by Cubist art, a workshop was carried out. The first one, was developed by Bruno Francisco (2017), a master student. The workshop that he carried out was formed by both the photographs of the children and the cubist painting of portraits. The experience itself was constituted by putting clippings of photographs that were taken from each of the children inside yellow and blue boxes. Each one received their own little box containing pieces of their photos. The pieces were cut by the researchers in various shapes, which could be regular but also irregular, an invitation to strangeness, exercise and study. The question was: How could they redo their own photographed images if these were not a jigsaw puzzle of geometric figures?

Well, a normal photo was not possible because not all the parts of the same photo were there. The solution was to make a portrait similar to cubist portraits. By doing that, children experienced an inventive process of assembling, disassembling and reconstructing their own image. The discomfort, the de-regulation, the strangeness and the disorganization are some of the statements that were reverberated from that experience, and which co-emerge in a cubist way of thinking. Thus, such performances, which expand mathematics education's habitual use, contribute not only for a contingency of knowledge, but also for an exercise of thought. To get an idea of that, look at Fig. 2 which is a set of images from a video of Francisco's work.

Fig. 2. Set of images from a video of Francisco's work.
Available in <https://youtu.be/Q286va8Sung>. Accessed on Jan 14, 2022.

3.2. *The case of experiencing mathematics with surreal art*

Starting from a larger project, entitled "De-dramatizing Education (Mathematics): Experiences with Art Workshops in Elementary School"[3], we created a workshop[4] inspired by the art of Salvador Dali. Jéssica Souza (2018) invented a story in which narrated characteristics of a surreal world. After listening carefully to this story, the children were invited to build their own world, making collage from clippings from various magazines in egg cartons. During the workshop, narratives of children linking the deformation of the figures to something wrong or ugly, and attempts to non-deform the figures through different strategies were observed. The collage on the corrugated surface of the box caused disquiet and strangeness for not being a flat surface.

From the analysis of this workshop, that:

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[3]Developed with support from CNPq, Brazil, from March 2017 to February 2020, aiming to recognize art workshops as spaces of freedom to experiment with mathematics.
[4]Developed and applied by the scientific initiation scholarship holder, CNPq, Jéssica Juliane Lins de Souza, from July 2017 to June 2018.

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(i) The real has to do with proportion: something too big or too small is not part of the real world; it needs a different function or name to be part of some invented world.

(ii) The real has to do with form: and form has to do with beauty. It is beautiful and real the thing that maintains its original shape, without deformations. Deformed and twisted figures are weird, crazy and ugly.

(iii) The real has to do with organization and method: things seem to be more real when they are organized and categorized. Scrambled and mixed images make the world seem confused and weird.

(iv) The real has to do with reason: and reason has to do with the correctness. What escapes our reason and our sense causes strangeness and is associated with something wrong, that needs to be corrected, done otherwise or undone.

(v) The real has to do with a model: the real is the representation of what we see, reproduction of the world as it presents itself to us. Anything that we don't recognize escapes reality.

(vi) The real has to do with Euclidean geometry: to represent the real, the use of a flat surface that does not cause disturbances in the images is indicated. Objects represented in another geometry are not part of reality. (SOUZA, 2018, p.62)

Look at Fig. 3 which is a set of images from a video of Souza's work for getting an idea that.

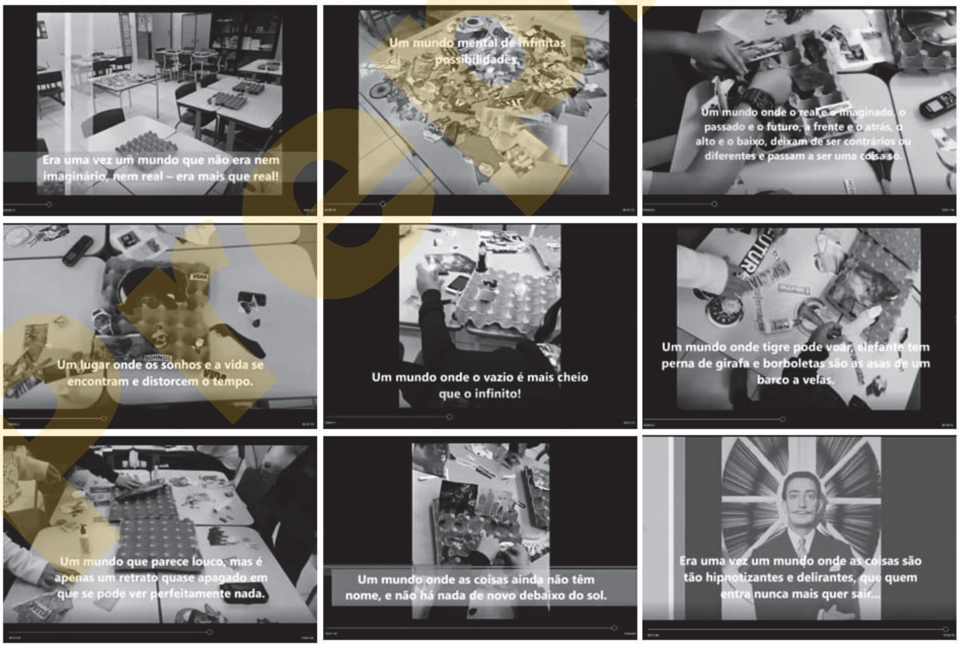

Fig. 3. Set of images from a video of Souza's work.
Available in <https://youtu.be/jTa8I74jcj4>. Accessed on Jan 14, 2022.

3.3. *The case of experiencing mathematics with abstract art*

How Euclidean geometry gives way to topology, that is, a topological plan that has no inside and no outside, has no beginning and no end? From this question Mônica Kerscher (2018), a master student, developed a workshop taking the geometric abstract art. With the workshop the children experienced knowledge of Euclidean geometry, such as geometric shapes, division of equal parts of a two-dimensional plane and so on, but also, the idea of continuity and the infinite came up.

One of the activities was: the children received colored ribbons that took on different shapes: a drop, a wheel, a zero, an eight turned, a Möbius ribbon, or even, the shape of infinite. From these forms of tape, a hole-line was made on the plane, whereby the child would cut the length of the tape, but they must follow the order of not being able to divide it in two parts. The question raised was: how far could you cut the tape without dividing it? To have an idea of this work, look at Figure 4 which is a set of images from a video of Kerscher's work.

Fig. 4. Set of images from a video of Kerscher's work.
Available in <https://youtu.be/TRBs47FuSoc>. Accessed on Jan 14, 2022.

With these three examples, I can summarily say that the space created by the workshop made it possible to teach and learn mathematics from the senses of experience, problematizing others connections with art and mathematics. A space not to see what we think, but to experiment with: mathematics, learning, culture, visuality, history.

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4. Final Remarks

From these work that I have showed, in relation to what is proposed to teach mathematics with art, apparently, for some it could be considered as a brilliant or innovative perspective, perhaps an unregulated attempt to teach mathematics, where mathematics appears here or there without substance, or foundation. However, I would like to draw attention to the fact that the space created by the workshop with art and children, made it possible to teach and learn mathematics from the senses of experience, problematizing connections with the two areas of knowledge: mathematics and art. Then, a space not to see what we think, but to experiment with mathematics, learning, culture, visuality, history.

Apart from this, they may think that if there are suspended rules, the place is occupied by habitus or repetitive practices. However, the way that we are connecting art and mathematics takes us to see not what a painting means, but ask about how it works: in the way we look, we create thoughts in which mathematics appears as an effect and agent of a way of being in the world and, based on this, re-create or re-think teaching and learning practices in which visual and mathematics are connected.

Finally, I would like to stress that all of this leads us to outline another ethical, aesthetic, and political stance for teaching mathematics with arts. It implies letting oneself be affected by resonances, dismantling devices, creating and experimenting with others, and ethically, aesthetically and politically questioning the truths. That is, an *ethos* that asks more than is affirms, and makes of the research practices as if it is multiple and multiform spaces of producing collective knowledge with: the child, researcher, mathematics, art.

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