Invited Lecture

Challenging Deficit Perspectives in Developing Countries: Teachers' Explanations of Fraction Concepts

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ABSTRACT Dominant discourses in teacher development often posit teachers as being lacking in knowledge, beliefs, or skills, thus justifying the "need" for further development and for educational reforms. This perspective shaped the analysis of Filipino teachers' explanations of fraction concepts using the constructs of content knowledge and pedagogical content knowledge, leading to an interpretation that reinforced deficit narratives about teachers. However, there are increasing contestations of these deficit research narratives (Adiredja 2019) that neither acknowledge the larger context that contributes to the ways teachers perform nor highlight the productive resources that teachers may draw upon in their teaching. This paper aims to illustrate a reconceptualization of the research away from focusing on what teachers lack towards identifying the ways by which teachers' fractional explanations reflect their constructed perception of ideal mathematics teaching as shaped by the broader system where education takes place. This is my attempt to acknowledge my own participation in the deficit perspective and challenge the narrative about education in a developing country.

Keywords: Developing countries; Preservice teachers; Fractions; Anti-deficit.

The scholarly literature is replete with reports of teachers' knowledge of fractions. Olanoff et al. (2014) found at least 43 papers that focused specifically on teachers' fraction knowledge. Since then, further studies on teachers' fraction knowledge were published (Bansilal and Ubah 2020; Chinnapan and Forrester 2014; Depaepe et al. 2015; Depaepe et al. 2018; Lee 2017; Lemonidis et al. 2018; Van Steenbrugge et al. 2014). Majority of the studies were carried out in developed countries or in countries with average to above average mathematics achievement based on international benchmarks such as PISA (OECD 2019).

The initial objective of the current study was to perform a similar assessment of preservice teachers' content and pedagogical content knowledge of fractions in the context of a developing country (the Philippines). An implicit assumption was that knowledge gaps of teachers in developing countries may be more serious than those in developed countries, which is why this study needed to be conducted. This perspective

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typified the “fixit” approach (Graven 2012) to professional development. Breen described the prevailing mindset regarding teacher training.

> There is something wrong with mathematics teaching world-wide, and that we, as mathematics educators, must fix it... Mathematics teachers need someone to fix them, and mathematics educators need someone to fix … This culture is based on judging what is right and wrong, paying little attention to what mathematics teachers are actually doing (since it is wrong anyway) in their classrooms, and looking outside themselves for the ‘right’ way, the newest ‘fix’. (Breen 1999, p. 42)

Deficit master narratives about preservice teachers are much entrenched in the scholarly literature. They exist even in reputable mathematics education journals, and even in carefully designed studies that focused on the development of understanding (Adiredja 2019). Specifically, deficit master narratives also abound in studies that focus on fraction understanding. Based on teachers’ performance in various fraction assessments, they are described in terms of what they “could not” do; their knowledge is often described as “inadequate”, “weak”, “insufficient”, “limited”, or as having “gaps” and “misconceptions” (Bansilal and Ubah 2020; Chinnapan and Forrester 2014; Depaepe et al., 2015, Klemer et al. 2017; Putra and Winsløw, 2018; Rosli et al., 2020; Şahin et al., 2016; Van Steenbrugge et al., 2014). I also acknowledge my own contribution to these deficit master narratives through claims of teachers’ poor reasoning strategies (Verzosa 2020). Were it not for a wise suggestion and for a further review of the literature, I would probably remain unaware and continue to be entrenched in a deficit view.

Deficit research narratives neither acknowledge the larger context that contributes to the ways teachers perform nor highlight the productive resources that teachers may draw upon in their teaching (Adiredja, 2019). This paper aims to illustrate a reconceptualization of the research away from focusing on what teachers lack towards identifying the ways by which teachers’ fractional explanations reflect their constructed perception of ideal mathematics teaching as shaped by the broader system where education takes place. This is my attempt to challenge the narrative about education in the Philippines.

1. Anti-deficit Perspectives

It can be argued that claims of the teachers’ weak or inadequate knowledge in fractions in the previous section are based on data. However, the interpretation of the data was shaped by a deficit master-narrative and failed to recognize the teachers’ sense making. Previous studies have demonstrated how the same data can be analyzed through a deficit or anti-deficit lens. For example, Lewis (2014) offered two contrasting explanations for the persistent understandings found in her case studies of two children. In the first approach, the two children’s persistent understandings could be thought of as an indication of their inability to mentally represent and manipulate numbers. Lewis argues that this interpretation does not provide much value to the identification or remediation of students with mathematical learning disabilities. In the alternative anti-deficit approach, Lewis offered a Vygotskian perspective and interpreted the children's understandings as resulting from the inaccessibility of mediational tools. Within this interpretation, the reason for the two children's persistent understandings was not because they could not mentally represent and manipulate numbers but because they understood the representation of a fractional quantity in unconventional and atypical ways.

In another study, Adiredja (2021), students' claims about the temporal order of epsilon and delta in the formal definition of the limit was investigated. In this study, four questions regarding the temporal order of epsilon and delta were asked—what depends on what? Which one do you think comes first? Which one do you think is set first? How would you arranged the four variables, epsilon, delta, x and f(x) in order? Only one student correctly answered "epsilon first" on all questions. Half the students answered "epsilon first" in at least one question. A deficit perspective would immediately lead to an interpretation that most students did not understand the temporal order. However, the study demonstrated that a single context/question cannot capture students' understanding. The fact that some students answered epsilon first on some, but not all, questions revealed that the students' justifications were context sensitive. Different questions influenced the cueing priority of certain knowledge elements (e.g., functional dependence). These knowledge elements (which are useful in other contexts) were found to be resources that students possessed during their sense-making of the temporal order of the limit definition.

These studies demonstrate that a deficit perspective focuses on what individuals lack (including misconceptions), rather than the resources that they draw upon (Adiredja 2019). It locates students' knowledge as a problem to be fixed, and not as a resource for learning. Further, a deficit perspective views certain individuals as if they possessed deficiencies, which may be biological or cultural (Settlage 2011). Some individuals may be thought to be "biologically inferior" or deficient by virtue of being raised in a certain culture. A deficit perspective does not consider that education occurs within a sociopolitical context (Adiredja 2019) where forces may systematically undo the efforts of teachers or schools (Settlage 2011).

2. The Philippine Context and Research Questions

Nebres (2009) stated that problems of mathematics education in the Philippines include two types: micro problems or problems internal to mathematical education (curriculum, teacher training, textbooks, etc), and macro problems or issues arising from pressures from other sectors of society. These macro problems very much exist in the Philippines.

In the Philippines, teaching is not an attractive career choice, and entry standards are typically lower than in other degree programs (Tatto et al. 2012). In some cases, education is chosen as a field of study because it is relatively cheap (no equipment is required), or because it accepts students who are unqualified in more attractive programs. Additionally, teachers are known to be overworked and underpaid. Public school teachers teach a maximum of six hours per day, and are also expected to write very detailed lesson plans, fill up official forms, and complete reports (Bautista et al., 2008). They may be asked to perform other non-teaching related duties in a highly centralized environment where information, opinions, and teachers' options are tightly controlled (Bernardo and Garcia 2006; Nebres, 2009).

Teaching is generally not an option among graduates of the top universities. To encourage the best students to teach, even just for a fixed term such as two years, initiatives such as Teach for the Philippines have been devised (Sodusta and De Leon, 2019). The words of a promising graduate turned public school teacher, sums up the aversion to teaching as a career:

> During my first year as a public school teacher, ahhh, there are three reactions that I always get: surprise, amazement, dismay. It's like, 'What? Why are you teaching there?' 'Oh you come from a prestigious university but you're a public teacher? What a waste'. It's always like that. (Sabrina Ongkiko, quoted by Sodusta and De Leon 2019, p. 12)

The study reported in this paper is grounded in the assumption that the macro problems emerging from the contextual realities of the educational system can contribute to teachers' training and preparation. It asks the following research questions. How do preservice teachers reason about fraction comparison and operation tasks? What forms of reasoning are associated with correct responses?

3. Method

3.1. Sample

The participants were 405 preservice elementary and 157 preservice secondary teachers. The preservice teachers were enrolled in six universities, spread across the northern, central, southern, and capital region of the Philippines. Moreover, the universities had varied ranking in terms of the passing rate in the September licensure examination for teachers (ranging from 40 to 93% for elementary teachers, and from 54 to 94% for secondary teachers). The preservice teachers were all in their third or fourth year, and had completed all mathematics courses within their program of study. No personal details were collected, but it can be assumed that most preservice teachers in the sample were between 19 and 21 years old.

3.2. Materials

The instrument consisted of four explanatory tasks, three of which involved fractions, and were included in this paper. For each of the three tasks, the respondents were asked to provide an answer and describe their solution to a student. The tasks were as follows: (1) Which is larger, 1/5 or 1/8? (2) What is 2/3 + 1/2? (3) What is 1 ÷ 2/3? For Tasks

2 and 3, the respondents were specifically asked to explain their solutions using a drawing. The concepts (comparison, addition, and division) were chosen because they represented basic knowledge in elementary mathematics, and there was sufficient literature to provide task-specific frameworks to guide this study's analysis (Bansilal and Ubah, 2020; Chinnapan and Forrester, 2014; Geller et al., 2017; Kaasila et al., 2010; Lee, 2017). Although the medium of instruction in mathematics from Grade 4 onwards is English, the teachers could write whole or part of their explanations in the regional language to lessen obstacles brought about by using a second language to explain academic content. Participants were given 30 minutes to complete the tasks, but most were finished before the time allotment.

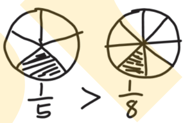
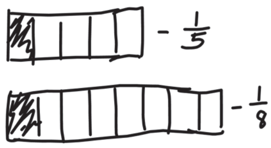
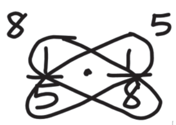
3.3. Data Analysis

Responses were initially coded as correct or incorrect. The preservice teachers' explanation strategies were derived inductively from the responses. From the raw data, repeating responses were noted and assigned codes. Similarities among codes were identified, which resulted in an intermediate list of codes. The data was coded and re-coded until the codes were finalized. With the final set of codes, the data were coded a second time as a manner of checking.

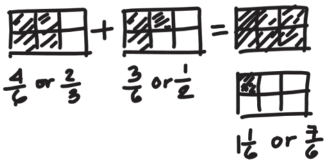
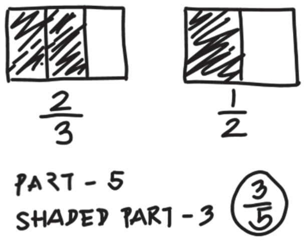
Because I wanted to capture the all the reasoning strategies provided by each preservice teacher, I coded all strategies. If the preservice teacher gave more than one reasoning strategy, responses were coded more than once.

Tab. 1, Tab.2, and Tab. 3 show the codes for each of the three tasks in this study. Sample responses for each code are also provided.

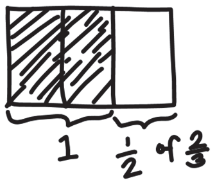
Tab. 1. Response codes for task 1 (which is larger, 1/5 or 1/8?)

Correct picture	Incorrect picture	LCD/decimal
		"8/40 > 5/40; 1/5 is larger." 1/5=0.2; 1/8=0.125; 1/5 is larger
Cross multiply	**Denominator**	**Real life**
To know the larger fraction, the two denominators will be multiplied to the opposite numerator. 	As long as the denominator increases, the value decreases. Simply compare the denominator. The larger fraction is the one with smaller denominator.	In our quiz everyday, if we get 1/2 the half of paper is what we need and in 1/4 the half of one-half is what we need then I conclude that the smaller the denominator, the closer is the fraction to the whole so 1/5 is larger.

Tab. 2. Response codes for task 2 (what is 2/3 + 1/2?)

Correct fraction relations	Incorrect fraction relations	Procedure
		First, get the LCD of the two denominators. Divide it from the original ones. 6 divided by 3 is equal to 2. Then 2 multiply by 2 is equal to 4. So the answer is 4/6. Do the same for the other, then add.

Tab. 3. Response codes for task 3 (what is 1 ÷ 2/3?)

Measurement Division	Procedure	Commutativity
	In dividing fractions, the whole number always has 1 in the denominator so the division symbol will be replaced with multiplication. In dividing, we have what we call a reciprocal so reverse 2/3, making it 3/2 then cross multiply.	Any number divided by 1, the answer is also the number.

4. Results

Fig. 1 shows the results in task 1 (which is larger, 1/5 or 1/8?). The white regions represent correct answers. Results show that correct answers were produced through different kinds of explanatory approaches. The use of real-life situations was more prevalent among the preservice elementary teachers, and the use of the least common denominator (LCD) or decimals was more prevalent among the preservice secondary teachers. The strategies LCD/decimal, cross multiply and denominator, considered "non-conceptual" in other studies (Geller et al. 2017), were found to produce correct answers, especially for the preservice secondary group. Most strategies underlying incorrect responses (represented by the dark regions) were incorrect drawings. However, there were 78 preservice teachers who produced an incorrect drawing and a correct answer, implying that an incorrect drawing did not automatically result in an incorrect solution. Further, there were 43 preservice teachers who produced an incorrect drawing but had other resources or strategies from which to draw upon (Fig. 2).

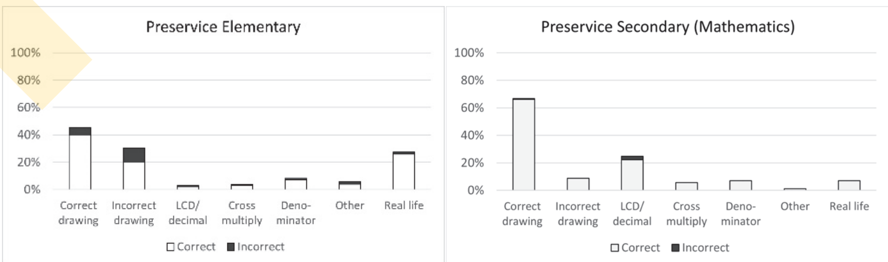


Fig. 1. Explanatory approaches in task 1 (which is larger, 1/5 or 1/8?)

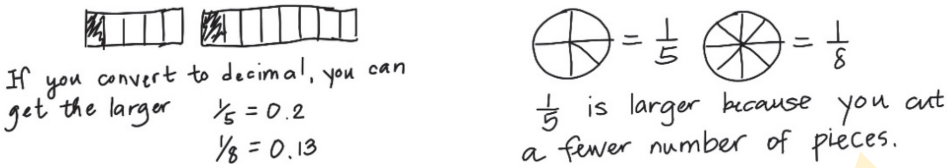


Fig. 2. Sample response with more than one strategy

Fig. 3 shows the results in task 2 (what is 2/3+1/2?). The most common explanation approach for preservice elementary teachers was based on procedures. This approach was effective for the preservice secondary teachers, but had around a 50% success rate among the preservice elementary teachers. For the preservice secondary teachers, the most common explanatory approaches were based on correct fraction relations and procedures. Among those who used correct and incorrect fraction relations, all produced a correct solution. The only exception was one preservice teacher who did not write an answer. The explanation approach based on incorrect fraction relations was only found among the preservice elementary teachers. This strategy of "count the shaded and count the parts" may have been productive in other contexts, such as when recognizing the fraction represented by a drawing, but was incorrectly applied here. Teaching students to understand the contexts where certain strategies are productive remains a learning goal.

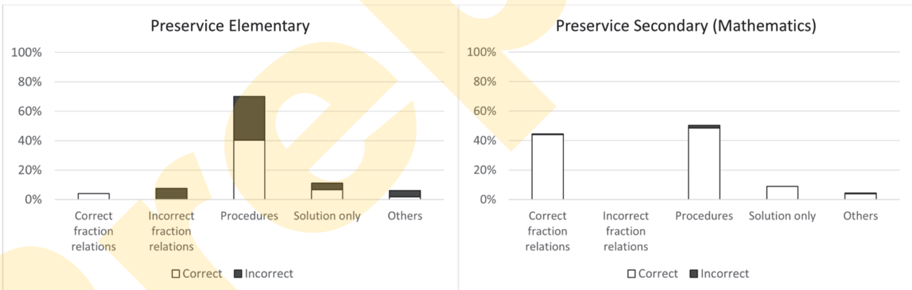


Fig. 3. Explanatory approaches in task 2 (what is 2/3 + 1/2?)

Fig. 4 shows the results in task 3 (what is 1÷2/3?). The most common explanatory approaches for this task mirrored those in the second task. For both cohorts, procedural explanatory approaches dominated. As in the previous task, this approach was effective for the preservice secondary teachers, but had a rather 50% success rate among the preservice elementary teachers. Except for one preservice teacher, everyone who used measurement division as an explanatory approach gave correct answers. Thus, this offers a potential goal for learning. Procedures were not as reliable, as almost half of preservice elementary teachers used an incorrect rule to solve the task. The rules they provided also resembled some of the rules they might have encountered in their mathematical experiences. The explanation approach based on commutativity was only found among the preservice secondary teachers. For these teachers, the strategy of “interchanging the numbers in an operation”, which is productive in addition or multiplication contexts, was incorrectly applied.

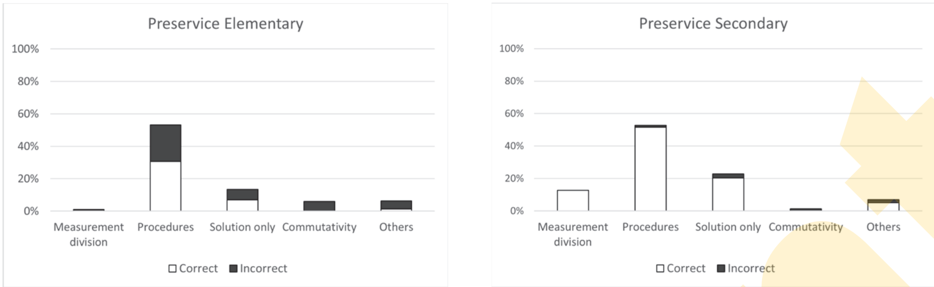


Fig. 4. Explanatory approaches in task 3 (what is 1 ÷ 2/3?)

Although procedures led to correct responses among half of the preservice elementary teachers, most of their explanatory approaches resembled standard rules. To compute 1÷2/3, they utilized elements of the correct solution (such as getting the reciprocal at some point). For example, one preservice teacher said, “We will make 1 the numerator, and then 2/3 the denominator. So that 1 becomes 3/1.”

4.1. Repeating Strategies

As indicated in the Data Analysis section, the explanatory approaches were searched for repeating ideas as a means to finalize codes. One repeating pattern in the responses was the use of words such as “just”, “simply” or “easily”. For example, 33 (7%) respondents included these words in their explanatory approaches. These words convey encouragement to students by suggesting that the task is doable.

Several explanations made use of a stated fact. These did not require much thought — these can be directly conveyed to any student who listens attentively. For example, in task 3, 64 preservice teachers explained that to divide 1 by 2/3, one must write 1 as 1/1. They explained that it is “understood” or “automatic” that a whole number has 1 in the denominator, that there is an “imaginary” or “invisible” 1 in the denominator, or that any whole number has 1 in the denominator.

5. Discussion

The data suggests that procedures are reasonably reliable, particularly for the preservice secondary teachers. Most preservice secondary teachers remembered the procedure while around half of the preservice elementary teachers did not. The preservice secondary teachers had more math courses in their training, which may have increased their opportunities to learn. Another possible explanation is that preservice teachers who enrolled in preservice secondary mathematics teaching are themselves already predisposed to math to begin with. Thus, it appears that procedural explanations are reliable only for students who have reached some threshold of exposure to mathematics.

Knowledge of the fundamental fraction concept of equivalence (Smith 1995) provides a reliable gateway towards a correct solution. Those who reasoned correctly about quantities were able to provide correct answers. The implication is that without the resources offered by thinking about quantities, procedural thinking can be a hit-or-miss — it is either you remember the rule or not. Interviews may reveal the reason why some preservice teachers remembered, while some did not.

The large proportion of explanatory approaches found in this study is consistent with the beliefs that Philippine teachers and teacher educators hold about mathematics learning. In the Teacher Education and Development Study in Mathematics (TEDS-M) study (Tatto et al., 2012), Filipino teachers and teacher educators, on average, believed 90% of the statements related to mathematics as a set of rules and procedures. Bergqvist and Lithner (2012) used Brousseau's (2002) theory of didactic situations to interpret that giving the students a procedure relieves the student of his/her mental work. Whether this is indeed the case for the preservice teachers in this study is up for further investigation. Certainly, the less-than-ideal educational context where students are crammed 80 to rooms built for 50, or where teachers are burdened by too much work makes it reasonable to ensure that rules sounded clear and friendly and to promote the easiest and most straightforward explanatory approach possible.

To highlight the difference and affordances of an anti-deficit approach in the interpretation of the results, I would like to offer two deficit conclusions form the same data set. Admittedly, this deficit model informed my initial analytic framework and the interpretation of the results. The first conclusion stemming from a deficit model is that the preservice teachers, particularly the preservice elementary teachers, lacked content knowledge and pedagogical content knowledge in fractions, given that more than half the teachers did not give a correct answer to Tasks 2 and 3. However, a more detailed look at their explanatory approaches revealed some potential resources from which the preservice teachers drew upon. These included "count the shaded", the notion of "commutativity", and the production of rules that resembled standard rules. Teacher educators may build upon these and elucidate the contexts where such knowledge is reliable. Teacher educators may also design assessments so that these contexts can be clarified. One example to facilitate a discussion of an "equal pieces" model is to ask preservice teachers to draw 1/5 and 1/8 given (a) two similar-sized objects, and (b) given two different-sized objects. These tasks may be a springboard for discussion whether it is possible to compare 1/5 and 1/8 using these models.

A second possible conclusion that perpetuates a deficit master narrative is that preservice teachers promoted learning by rote because most of their strategies relied on procedures. However, this explanation does not recognize the classroom contexts or the broader educational system as major influencers of teachers' actions. Johnson et al. (2000) argued that teachers know more strategies than what they use in the classroom. They argued,

> Whereas the wisdom gained from northern/western contexts suggests it is the teacher that does the selecting, we wish to reverse this wisdom, and suggest that for science teachers who are in educational systems at anything other than the professional stage, it is the environment in which the teacher works that creates the selection. (Johnson et al., 2000, p. 186)

These teacher constraints were expressed by a South African teacher during the time when an innovative assessment was being piloted in South Africa (Bansilal, 2011). This teacher said,

> I found out that you can't do any of the innovative tasks with them, you have to teach the concepts. Because if you don't teach these concepts, then they can't do the activity. You saw for yourself how difficult it was — there are 50 learners in each of my classes. So we have to stand in the front and do as much as we can, even though we ourselves know at the end of the day it doesn't make much of a difference. (Bansilal, 2011, pp. 104–105)

This quote demonstrates how contextual factors such as students' non-readiness for the task or the physical setup of the classroom can constrain teachers' actions. In the Philippines, the same contexts and macro-structures are also present. An interview with a Filipino teacher revealed such pressures (Verzosa et al. 2017). In relation to a mandated focus on critical thinking and exploration which was considerably different from the "spoonfeeding" method that their students were exposed to in their elementary school years, one Filipino teacher mentioned (p. 93), "ikaw na nga gagawa ng activity, ikaw din ang sasagot (you designed the activity but you end up answering it as well)." Further, for the recommended strategies to work, teachers are compelled to provide worksheets and other materials for students at their own expense.

6. Limitations and Future Research

A three-item written assessment cannot access students' full understanding. Interviews with the preservice teachers would have provided the information needed to understand how their choices were shaped by their own experiences. Most studies that focused on students' productions rather than misconceptions utilized interviews in their research design (Adiredja, 2021; Adiredja et al., 2020; Hunt and Empson, 2014; Lewis, 2014; Lewis and Lynn, 2018). As was stated earlier in this paper, this study was initially not designed with an anti-deficit perspective as a supporting framework. If not for the wise guidance of experts in the field, I would have persisted with my initial analysis that focused on errors and downplayed the resources from which the preservice teachers drew upon. By doing so, I would have contributed even further to deficit master narratives about teachers. While I was reconceptualizing my analysis, I began to recognize that many of my initial conclusions produced a deficit story about preservice teachers.

Much of the research framed by anti-deficit perspectives involve students of color or students with mathematical learning disabilities. There are few, if any, published research from non-Western contexts. Thus, I will end this paper by also pushing for the use of anti-deficit perspectives in designing, implementing, and interpreting research, especially in developing countries.

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