Topic Study Group 21

Neuroscience and Mathematics Education / Cognitive Science

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1. State of the Art

Without a doubt, doing mathematics depends on creative, problem-solving thinking. Mathematics didactics provides mathematics with some cloak: What is to be understood is the teaching and learning of mathematics and how the acquired knowledge can be used for teaching processes. In a fundamentally oriented manner, borrowings from cognitive science are taken up and mental processes are examined using neuroscientific methods. Mathematics is a vast field of knowledge full of concepts and tools, mathematical thinking is highly complex. Fig. 1 displays the complex of areas and approaches relevant to TSG-21.

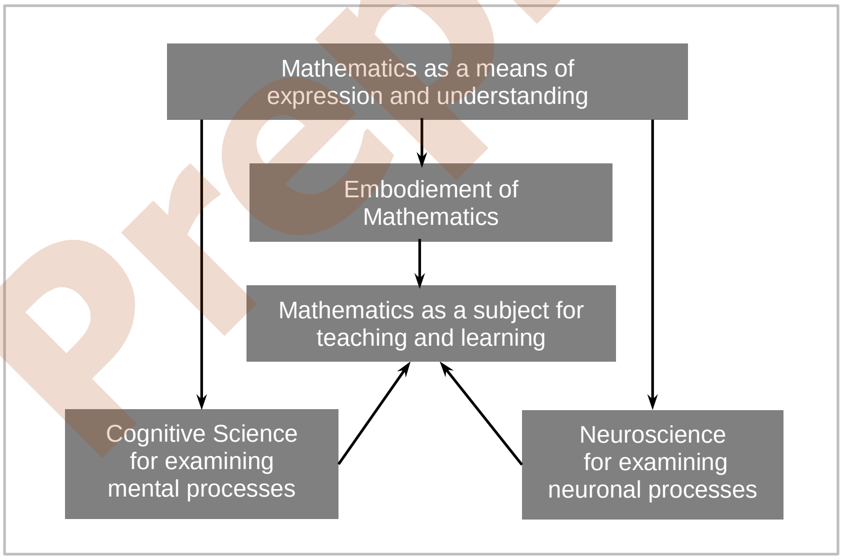

Fig. 1. Relevant complex of areas and approaches

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2. Contributions

The contributions by scientists from six countries (Tab. 1).

Tab. 1. List of papers presented at TSG-21

	Paper and author(s)
[1]	General Spatial Ability Other than Special Mathematical Ability Correlates with Ill-Structured Problems in Junior Students. *Xinlin Zhou, Chunxia Qi, Li Wang, and* ***Chen Cao*** (China).
[2]	Behavioral Processing of Fractions in Adults with and Without Mathematics Learning Difficulties. *Parnika Bhatia, Jessica Leone, Jerome Prado, and* ***Marie-Line Gardes*** (France).
[3]	Consideration of Characteristics of Eye Movement and Brain Activity During Mental Rotation Tasks. ***Tatsuki Kondo****, Naoko Okomato, and Yasufumi Kuroda* (Japan).
[4]	Learning Representations of Mathematical Objects in Computational Models of Mathematical Cognition. ***Trygve Solstad****, Silvester Sabathiel, and Celestino Creatore* (Norway).
[5]	Electrophysiological Characteristics of First-Grade Children at Different Levels of Number Sense. ***Yuqing Zhao****, Feidan Yu, and Zikun Gong* (China).
[6]	Declarative Knowledge and Procedural Knowledge: Learning Processes in the Case of Pound Arithemtic. *Roland Grabner* (Austria), *Stefan Halverscheid* (Germany), *Jochen A. Mosbacher* (Austria), *and* ***Kolja Pustelnik*** (Germany).
[7]	Even Young Children Are Able to Grasp and Apply Logical Rules in Mathematically Structured Environments — the Puzzle of Cognition. ***Inge Schwank*** *and Elisabeth Schwank* (Germany).

Contribution by Zhou et al. (Cao's presentation)[1] falls into the area of cognition-oriented mathematics didactics. By the use of relevant tests the connections between general cognitive and mathematical abilities were examined. Apparently, general spatial ability has a greater impact on ill-structured problem solving than special mathematical ability.

Contribution by Bhatia et al. (Gardes' presentation)[2] also comes from the field of cognition-oriented mathematics didactics and examines, how fraction knowledge and competencies are processed in adults with mathematical learning difficulties. Results indicate that different pathways are utilized for accessing the magnitude of non-symbolic line ratios and symbolic fractions. Mathematical learning difficulties when dealing with fractions had a greater impact on failures in calculation and estimation and less on representing symbolic fractions in verbal form and vice versa.

Konto et al.[3] used cognitive science methods. Teaching spatial geometry is challenging. This contribution's objective is to better understand the basics of cognitive processes in view of spatial geometry. For this purpose, the present study examines the characteristics of eye movement and brain activity during mental rotation tasks. The result shows: The mental rotation tasks that required lesser time involved a high frequency of looking at the same parts of left and right solids and reached max activation time quickly.

Solstad et al.[4] applied to the mathematical modeling of cognitive processes when doing mathematics. The initial question is: Can our mathematical abilities be explained in neuroscientific or computational terms? Some of the properties of representations generated by idealized neural network models for numbers are examined and described. Furthermore, the question of how computational tools can contribute to a greater understanding of the relation between mathematics education and neuroscience is addressed.

Zhao et al.[5] applied a neuroscientific method and examined the electrophysiological characteristics in the first-grade children at different levels of number sense in a number comparison task. In fact, there are differences in brain activity in children with a high level of number sense compared to children with a middle or low level of number sense. The results suggest that children with different levels of number sense show different electrophysiological characteristics during number sense processing.

A joint contribution[6] by German and Austrian researchers comes from the field of cognition-oriented mathematics didactics with the use of mathematical modeling. Learning processes when dealing with a special arithmetic are examined. A distinction is made between "know-how"; i.e.: learning how to solve arithmetic problems and "know-that"; i.e.: learning arithmetic facts. The power-law function is used for the description with a view to its coefficients and any correlations between these coefficients. This worked satisfactorily only with regard to fact learning. Correlations between the two learning processes turn out to be weak.

Finally, Schwank and Schwank[7] came from the field of cognitive mathematics. Based on the theory of functional-logical thinking versus predictive-logical thinking, different scenarios are examined. Access to (first) arithmetic is easier for children dominantly using the approach of functional-logical thinking, mathematically gifted children show special talents here. The development of mathematical thinking can be promoted through the use of special mathematical play worlds as the results from an early support study with preschoolers indicate. Particularly children with difficulties in the area of early mathematics seem to benefit from the support provided by mathematical play worlds.

3. Outlook onto Future Research Activities

There is still a large discrepancy between the possibilities of gaining knowledge based on theoretical concepts as well as experimental methods offered by Cognitive Science and Neuroscience and the needs for knowledge acquisition, which arise due to the complexity mathematical thinking, mathematical knowledge, mathematical problem solving as well as mathematical creativity. Nevertheless, it can be expected — especially if interdisciplinary research is intensified — that the mental processes when dealing with Mathematics will be increasingly understood more thoroughly and that these findings for mathematical teaching and learning processes can be used specifically and successfully. An important issue will be gaining professional mathematicians' cooperation as well as further people, especially children with so-called special needs. The digital transformation of society will play a big role in this advancement. Accessibility issues have never been easier to adapt to the addressee (e.g. in terms of font size, contrast, images, interactions, problem variations). The variety that is thereby made possible in a considerably easier way, sets a whole new potential for research questions and scientific studies. Cognitive Mathematics will be the state of the art (Fig. 2).

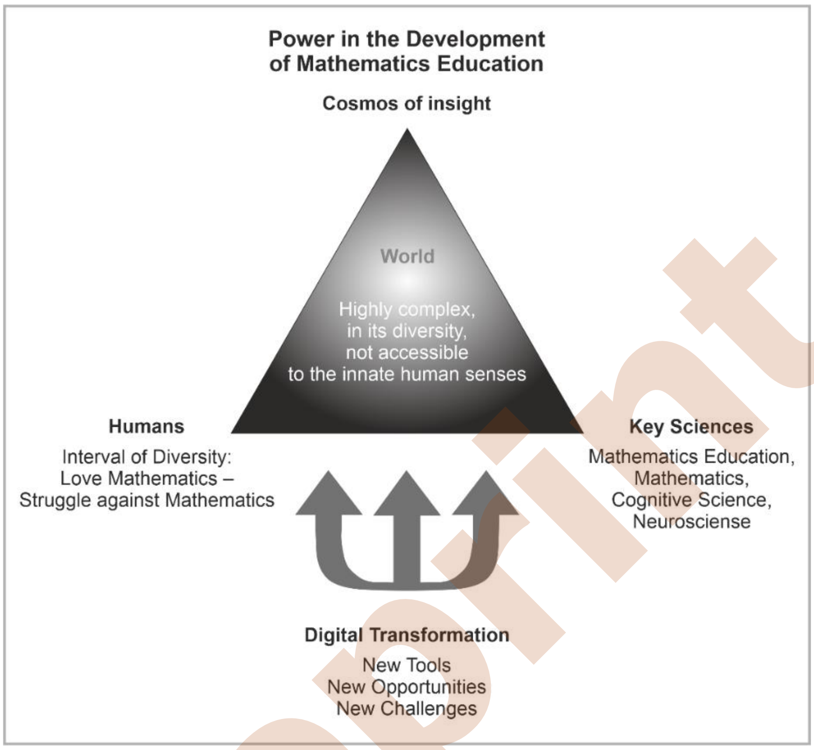

Fig.2. Cognitive Mathematics as a combination of important approaches to the world of understanding, learning, teaching and strengthen mathematics