Topic Study Group 26

The Role and the Use of Technology in the Teaching and Learning of Mathematics at Upper Secondary Level

TSG-26 Working Team[1]

1. Introduction

The TSG-26 counts 17 papers presented, in a double modality: as a long paper (three in the 1st Session), and as a short paper, (fourteen, in the 2nd and 3rd Sessions). The participants to the Sessions have been 24, divided in these countries: Australia, Botswana, Brazil, Canada, Chile, China, Colombia, France, Italy, Mexico, the Netherlands, Philippines, South Africa, United Kingdom, U.S.A.

The call for papers of TSG-26 has been divided into 5 themes:

- Theoretical and methodological aspects: current/new frameworks for developing and analyzing new technology's integration in mathematics teaching and learning from didactical, cognitive and epistemological perspectives.
- Role of emerging devices and technologies, such as tablets, smartphones, virtual learning environments, augmented reality environments, and haptic technologies.
- Interrelations between technology and the mathematics taught at this age level.
- Students' education and the relationships between teaching and learning.
- Teachers' professional development.

Each theme has been articulated in different questions, ideas, suggestions for discussions, research results, and methodologies that guided the authors to the submission of papers and posters.

2. Sessions

The works of TSG-26 took place in three sessions, two of 90 minutes and one of 120 minutes. In the first session, a panel of three long papers was organized with the three presentations followed by a discussion. The other two sessions have been divided into

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two sub-sessions each, around common topics, with presentations of short papers and discussions in the end. Each communication was videotaped and made available for all participants before the sessions. During sessions, presentations were limited to 10 minutes (long papers) and 5 minutes (short papers) in order to allow the maximum time for the discussion within the TSG.

This report is structured according to the sessions and sub-sessions of the TSG.

2.1. Session 1

The first session of the TSG was dedicated to three long papers on the role of the students in a "digital class" at different levels (Tab.1). The presentations of the long papers have been organized as a panel, followed by a discussion. The panel dealt with the different roles that students may have in educational processes: as protagonists of learning processes, as active designers of resources for education purposes, and as subjects evaluated by a system of formative assessment.

Tab. 1 Papers presented in Session 1

Paper and author(s)	
[1]	Students as designers of digital curriculum resources. ***Annalisa Cusi*** *and Agnese Ilaria Telloni* (Italy).
[2]	Formative assessment and technology: an attempt of framework. ***Gilles Aldon*** (France) *and Monica Panero* (Switzerland).
[3]	Straightening the bend: sequencing embodied experiences with high and low-tech designs for the notion of Radian. ***Rosa Annalucia Alberto****, Anna Shvarts, Arthur Bakker, and Paul Srijvers* (The Netherlands).

The first paper[1] presented by Cusi shows an educational programme aimed at involving upper secondary students in the design of digital curriculum resources (DCR) using the GeoGebra software. The study characterizes the praxeologies (made of practices and theoretical reflections on them — Chevallard, 1991), developed by the students in relation to the task of DCR-design, through the analysis of the reflections they proposed during semi-structured interviews at the end of the educational programme. This characterization of students' praxeologies highlighted their awareness both on the characteristics of the DCR that supports students' learning and on the role of the design process in fostering the designers' learning itself.

The second paper[2], by Aldon and Panero, presents a research study on the place and the role of technology in the assessment process, specifically on formative assessment (FA) processes. It addresses the professional development of teachers integrating technology into their practices in order to enhance formative assessment and to give students the awareness and the ownership of their learning. The main claim of this paper is that technology does modify classroom assessment processes, but at the cost of a reorganization of the act of teaching by promoting student ownership of their learning. Technology is not only a facilitator in the implementation of FA-strategies, but more profoundly a modifier of the didactic contract, making teacher and students together responsible for teaching and learning.

Alberto, Shvarts et al.[3] aimed to investigate how to support students in moving from calculation to reasoning strategies in trigonometry by the design of technology-enhanced learning activities inspired by embodied mathematical cognition and embodied design. They conjectured that novel mathematical relations need to be enacted physically to provide opportunities to actively conceptualize these relations. They followed up on this by exploring embodied interactions that foster students' understanding of the input of trigonometric functions in the unit circle and the sine graph. Students appeared to lack physical experience with measuring circular arc lengths and using the radius as a unit of measurement. Low-tech paper materials and practices — like folding and bending — seem to afford sensible enactments better than would digital materials. Embodied approaches and interactions are essential for the emergence of mathematical conceptualization.

In the three presentations, students learn through an awareness of a reversal of responsibilities: this awareness is provoked by the teacher and the situations he/she sets up in the classroom. Technologies and actors' relationships with artifacts are responsible for the awareness of an evolving role for students in their learning processes. The use of digital technologies seems to favor a change of roles in students and teachers and also a change in the dynamics of the classroom: co-responsibility and cross-responsibility foster an active participation, modifying students' praxeologies used in learning processes (design, assessment, conceptualization). Technologies, in these cases, are no longer studied for themselves but rather for the role they play in the new organization of learning.

2.2. Session 2

Session 2 was split into 2 parts, totally 7 papers were presented (Tab. 2).

Tab. 2 Papers presented in Session 2

Paper and author(s)	
Part 1	
[4]	Questions of design research: a technology mathematics lesson framed by the didactical triangle. ***Marie Joubert***, *Geoff Wake, and Marc North* (UK).
[5]	Merlo item as boundary object in teachers professional development. *Ornella Robutti* (Italy), ***Theodosia Prodromou*** (Australia), *and Gilles Aldon* (France).
[6]	Acceptability of the proposed multimedia instructional module in selected pre-calculus topics among STEM students of Muntinlupa National High School. ***Maxima Joyosa Acelajado*** *and Arlene B. Miyas* (Philippines).
[7]	Twitter, emotion and mathematics. ***Mario Sanchez Aguilar*** (Mexico).
Part 2	
[8]	Integrating GeoGebra in classroom teaching of 3D geometry: contrasting a French and a Chinese case. ***Mingyu Shao*** (China/France).
[9]	Mathematics prospective teacher display of technological content knowledge in a GeoGebra-based environment. ***Kim Agatha Ramatlapana*** (Botswana).
[10]	An implementation of technological pedagogical content knowledge framework for analysing the design of tasks in a digital environment. ***Carolina Guerrero-Ortiz*** (Chile).

2.2.1. Session 2 part 1

This sub-session was devoted to the role technology can play in the interactions between teachers and students, but also between teachers and researchers or teacher trainers.

Joubert et al.[4] analyzed these interactions by means of knowledge, technology playing a role of support of interactions between students and knowledge. Joubert's communication concerns the process of design research, within the context of teaching a 'technology' mathematics lesson in Further Education colleges in England. It explains the context, then uses the "didactic triangle" to frame an analysis of the design requirements for a lesson on factors and multiples. Having determined the design requirements, an example of a lesson was given, with an explanation of how it meets the requirements. It concluded that, although the design requirements had been met, the end product perhaps lacked a sufficiently coherent narrative. It ended by speculating on whether a fourth vertex, to represent technology, should have been added to the didactic triangle or whether there was a way of capturing a coherent narrative within the design requirements.

Prodromou et al.[5] considered the technology as a component of a boundary object providing a place to interact between teachers and researchers. The presentation was focused on the possibility to consider Meaning Equivalence Reusable Learning Objects (MERLO) items as a boundary object in crossing the boundary between two communities: researchers and teachers. The boundary crossing was seen as a process of transformation that can influence a modification (more or less stable) in the meta-didactical praxeologies (namely practices and theoretical reflections on them — Arzarello et al., 2014) of the teachers. Primary pre-service teachers were engaged in this experiment, during their professional development. Results were on the possible existing intertwining of their praxeologies and the MERLO items they produce, seen as boundary objects in their evolution over time.

Acelajado et al.[6] presented a tool, the Multimedia Instructional Module, aiming at helping learners to overcome difficulties in understanding mathematics. The authors developed and analyzed this new technological tool whose goal was to help students with mathematics learning difficulties, specifically on basic concepts of calculus. Statistically significant differences in the post-test meant scores between the two groups reveal that the experimental group performed better and that the participants of this group perceived the MIM to be "Highly Acceptable".

Aguilar[7] started by the use of a communication tool considered as a medium that provides us with an insight into students' emotional experiences related to the teaching and learning of mathematics. Aguilar took advantage of the familiarity of Twitter in students' everyday life to engage mathematics students in and out of the classroom. In his article, he argued that this social network could serve as a medium that provides us with an insight into students' emotional experiences related to the teaching and learning of mathematics. To illustrate this, a selection and categorization of tweets about mathematics was presented.

The discussion following the papers presentation focused on the theoretical background that supports the study of these interactions. More precisely, what can be the role of digital technologies in the development of educational actors' interactions? Examples given in all the presentations show that digital technology provides a powerful media helping for communication between actors, but also with knowledge at stake. It could also be the place of interactions encouraging actors to look at knowledge from a different perspective. The discussion that followed the presentations returned on the concept of boundary object (Prodromou et al.[5]), showing at a micro level how two communities (researchers and teachers) can approach each other by crossing the boundary. In this case of crossing, technology played the role of a shared place where different points of view from two communities could crystallize on a common one then adopted by both communities. At a meso level, interactions between two communities (teachers and students) could profit from social media (Aguilar[7]) and emotional experiences that students face during their mathematics studies were revealed. More investigation should be done for recognising and studying in different educational contexts objects at the boundary between communities, and their evolution over time, along with the causes of this evolution process, including not only cognitive reasons, but also meta-cognitive aspects that may have propulsor roles in these processes.

2.2.2. Session 2 part 2

The second part of the session was dedicated to different uses of Digital Geometry Software in the teaching of Geometry.

Shao[8] reported on orchestration of lessons using 3D geometry software in contrasted cases in France and China. Drawing on the instrumental orchestration (Trouche, 2004) and the instrumental genesis frameworks, Shao's paper contrasted the case of a Chinese mathematics teacher with a French one, investigating how they have managed to integrate GeoGebra in their class on 3D geometry. The results opened some perspectives for further investigation, including the teacher's documentation work before the class, and the factors that could influence their choices of instrumental orchestration.

Ramatlapana[9] explored geometry technological content knowledge of mathematics prospective teachers within a GeoGebra environment. The presenter explored geometry technological content knowledge displayed by mathematics prospective teachers when working on a high school circle geometry task within a GeoGebra-based environment. The investigation analyses six prospective mathematics teachers' thinking as displayed in their solutions to the technological content knowledge-based task, particularly on what the GeoGebra constructions revealed about teachers' competence with geometry diagrams within a GeoGebra environment. The narratives and constructions were expected to reflect the teachers' ability to transform the statements from a static environment to a dynamic construction, employing GeoGebra as a construction tool. The affordances and constraints of GeoGebra when making connections between the construction and geometric principles emerged.

Guerrero-Ortiz[10] dealt with task design, highlighting the possibilities offered by a technological environment, according to the TPACK (Koehler & Mishra, 2009) framework and following a qualitative perspective. The paper presents different tasks designed by pre-service mathematics teachers, analyzing their peculiar elements related to the domains of TPACK. In the study aspects related to modeling, simulation, visualization and the use of the tools of a Dynamic Geometrical System (DGS) are highlighted. This work allows us to know how in the tasks design the pre-service teachers' knowledge can be evidenced, the results also show how they conceptualize modeling in a technological environment.

The discussion started with the three following questions:

- How is it possible to go beyond the identification of teachers' knowledge in the use of DGS?
- Is it possible to think through a networking of theories both the use of DGS and the design of resources?
- How can we contrast and compare different contributions of these theories?

The first question refers to works such as the one by Guerrero-Ortiz[10] and Ramatlapana[9], who paid attention to the specialized knowledge manifested by prospective teachers when facing the resolution or design of mathematical tasks based on the use of dynamic geometry systems. These studies confirmed how the analysis of the use of digital tools by mathematics teachers could serve as a window to teachers' specialized knowledge. However, we ask ourselves how we can use this identified specialized knowledge to promote an adequate and rational implementation of technological tools in the teaching of mathematics.

The second and third questions are theory-oriented investigations whose purpose was to promote the discussion of the potential that theory networking could offer for the study of situations involving the use of DGS as a teaching tool, but also as a means to design and implement mathematical tasks. Likewise, the particular affordances that different theories could offer for the study of the role and the use of technology in the teaching and learning of mathematics at upper secondary level were put at the center of the discussion. These presentations, as well as the plenary discussion, lead us to consider DGS through different theoretical filters which each of them brings a particular highlighting on the relationships between the artefact and its use.

2.3. Session 3

Session 3 was also split into 2 parts with 7 papers presented (Tab. 3).

Tab. 3 Papers presented in Session 3

Paper and author(s)
Part 1
[11] Mathematics VR teaching design mode and its practice at upper secondary level: based on VR All-in-one Computer. ***Jijian Lu****, Xiaoyuan Shen, and Yi Lv* (China).
[12] Mobilizing mathematics: how technology enhances embodied learning. ***Stefan Rothschuh*** (Canada).
[13] The reading and the comprehension of mathematics text: an eye-tracking study with primary pre-service teachers. *Roberto Capone, Federica Ferretti, Alessandro Gambini, and* ***Camilla Spagnolo*** (Italy).
[14] Computational thinking for mathematical learning. ***Yahya Tabesh*** (USA).
Part 2
[15] Students' understanding of the notion of collinear vectors in dynamic geometry environment. ***Jose Orozco-Santiago*** (Mexico).
[16] Enhancing metacognition by using flipping classroom with GeoGebra. ***Chak-Him Fung****, Kin-Keung Poon, and Michael Besser* (Hong Kong SAR, China).
[17] Students coping with a post-16 mathematics course: flipped learning, self-regulation and technology. ***Sofya Lyakhova****, Marie Joubert, and Dominic Richard Oakes* (UK).

2.3.1. Session 3 part 1

The first part of the third session was dedicated to discuss the research on interactive technologies, particularly Augmented and Virtual Reality, embodiment and reasoning.

Lu et al.[11] addressed the creation and use of VR all-in-one virtual simulation software and hardware platform as an alternative to overcome the limitations of traditional online technologies. They used VR head display and VR glasses in students' collaborative interaction and integration in mathematics teaching at upper secondary level, building a mathematics VR teaching design mode, including: resource selection, interaction design, development and innovation.

Their team constructed the high school mathematics teaching design mode assisted by VR all-in-one computer, and would address more studies towards the extended practice of mathematical maker education, teachers and students could carry out interdisciplinary inquiry learning practice.

Rothschuh[12] discussed the theoretical considerations and the practical implementation of a research on technologically enhanced embodied mathematics learning. It sought to study and improve the practice of learning and teaching mathematical functions at the secondary level by incorporating embodied learning designs. It drew on established theories of how individuals learn mathematics, recent developments that aimed to incorporate embodiment and technology in mathematics learning processes, and the desire to study learning where it naturally occurs, as the long paper by Alberto et al.[3]. Using a design-based research approach, the researcher and partnering teachers developed and implemented a set of technologically enhanced embodied lesson designs. Over the course of three iterations, the lesson designs were continuously revised and improved, to promote embodied learning of the function concept, and harmonize technology-integration in these learning environments. It showed how calculus became meaningful for modeling everyday experiences in technologically augmented classroom inquiry, rather than being a domain that is merely focused on number and calculation.

Capone et al.[13] dealt with the type of text — by using eye-tracking tools — to investigate students' attention during reading tasks. It pointed out that, being recognized in literature, the central role of argumentation in the teaching-learning process and the type of text affecting students' reading, the use of eye-tracking might render it possible to understand students' reading of a mathematical text. In particular, we may understand if selective readings with a focus on some textual elements considered essential may lead students to a lack of understanding of the problematic situation. The research, carried out with the innovative tool of eye-tracking, shows a first exploratory study conducted with primary pre-service teachers while dealing with mathematics texts.

Tabesh[14] presented an intuitive digital learning model, focused on problem-solving through computational thinking and is targeted to empower teenagers. The proposed model is a hands-on interactive web platform for mathematical problem-solving that enables creative engagement, develops mathematical skills, and supports a growth mathematical mindset. It illustrated the benefits arising from engaging youth with progressively more complex tasks and giving them increased ownership of their learning. As a theoretical foundation, the development teams consider the use-modify-create framework to offer a helpful progression for developing mathematical thinking. It presented a computational thinking playground and a functional programming paradigm in a platform for creative problem-solving. In the platform one can use models and simulations to represent phenomena which, by playing with a mathematical framework, will be learned through creative and innovative thinking. The gained knowledge and skills of this cognitive learning both empower learners and enhance creativity.

After all the presentations, the discussion started by the following three questions:

- What advances can the use of interactive technologies provide to research in Mathematics Education?
- How can interactive technology be used in mathematics education? How to use or combine embodiment, virtual reality, augmented reality, particularly when solving mathematical problems?
- What is the role of these environments in the development of computational thinking itself?

Motivated by the third presentation — that highlighted innovative tools for research methodology advances — the first discussion question supported reflections on different lines of innovation in the studies. It used eye-tracking contact with the text as a tool to collect data for the study about argumentation. It led us to question the processes shown by tracks. The first presentation focused on the advances regarding cognitive development by using VR devices. The second and fourth presentations focused on developing tools. The second regarded a model of lesson design with embodiment technology. The fourth was a playing platform to support students' problem-solving. The second discussion question was motivated by the different uses and the variety of interactive technologies such as Virtual Reality all in one computer and computing thinking, creativity on embodied cognition with the integration of technology and graphs generation through movements linked to the classification of gestures. However, the uses are more isolated for a while. The fourth presentation led to our third discussion question. The TSG-26 discussion to reach the importance of computational thinking incentive through a platform to solve problems; using interactive games. Nonetheless, we need more research on tasks and students' work associated with these innovative tools to improve mathematical learning.

2.3.2. Session 3 part 2

The second part of the session was focused on the ways (i.e. flipped classroom) of using dynamic geometry software to promote students' understanding and teaching effect. Orozco-Santiago[15] proposed a contribution to the current/new frameworks for developing and analyzing new technologies integration in mathematics teaching and learning from didactical, cognitive and epistemological perspectives. They designed the tasks considering the potential of the dragging tool in a dynamic geometry environment. The tasks were assigned in a linear algebra course in engineering, and the work of one student in France was analyzed, as a case study, to examine the actions instrumented under the framework of instrumental genesis. They showed that the suggested design supported the student to explore what remains invariant under dragging, and to conjecture about the meaning of collinearity.

Fung et al.[16] followed a quasi-experiment design to study the Flippeding Classroom (FC) assisted by GeoGebra to increase Chinese college students' mathematical metacognition in comparison to FC assisted by video and/ or direct instruction. The result revealed that the main effect of the metacognition was significant, while the interaction between the metacognition and the teaching methods was not significant. It suggested that significant improvement could be observed among students but no significant difference could be observed among the teaching methods in terms of the metacognition. In other words, the FC supported by GeoGebra is an effective teaching method in terms of students' metacognition development.

Lyakhova et al.[17] interviewed sixteen students from two research projects in the UK which employed technology to compensate for the lack of in-school resources (advanced post-16 mathematics course). The study showed the evidence that technology could create new learning situations as well as new learning materials that students perceive as beneficial, although perhaps an effort is required from students (i.e., self-regulation skill) to successfully adapt to these.

Inspired by the three reports mentioned above, our panel discussion focused on the following two areas:

- What is the theory behind dynamic geometry software to promote students' understanding of geometric concepts?
- How to effectively improve the effectiveness of flipped classroom teaching?

The discussion of the first question revealed the challenges that iterative updates in technology pose on how to study the effectiveness of classroom instruction that incorporates information technology, and that research on this issue needs to be supported by further theoretical exploration. The discussion of this problem focused on how to develop theoretically relevant assessment frameworks to track students' development in mathematical thinking at a higher level. The second discussion topic focused on the popular flipped classroom model, in which participants were very concerned about how to improve students' self-discipline and initiative in the flipped classroom? The effective use of flipped classrooms requires a change in students' attitudes toward learning. On the one hand, the content of the flipped classroom needs to be carefully designed, and on the other hand, the improvement of students' self-learning ability, an issue that involves further exploration of meta-cognition such as self-monitoring and self-evaluation.

3. Poster Presentations

In a special session, ten posters also composed TSG-26 studies with researchers from Belgium, Cambodia, China, Morocco, Nepal, and Peru (Tab. 4).

Tab. 4 Posters with TSG-26

Poster and author(s)
[18] The study of mathematic classroom teaching integrated information technology and mathematic multirepresentations. ***Hua Wu*** (China).
[19] A blended approach to support aspiring engineering students. ***Paul Georges Igodt*** (The Netherlands).
[20] The study of mathematical multi-representations and teaching scaffolding in the smart-classroom environment. ***Na Han*** (China).
[21] Integration of ICT in modeling and experimentation of interdisciplinary problems. ***My-Lhassan Riouch*** (Morocco).
[22] Instrumentation of the symbolic artifact quadratic function. *Daysi Julissa Garcia Cuellar*.
[23] Application of GeoGebra based on AR/VR technology in high school solid geometry teaching. ***Xue Huang*** (China).
[24] Students self-regulated learning strategies, perceptions and mathematics performance in a mobile technology-integrated mathematics classroom. ***Gerald Cristobal Apostol*** (Nepal).
[25] The effective strategies of teaching trigonometry function using ICT applications: GeoGebra and Wolfram. ***Leangsim Im*** (Cambodia).
[26] Practical research on the application of information technology in function review lectures. ***Xiayan Shao*** (China).
[27] A case study on TPACK performance of Chinese middle school mathematics teachers. ***Huishui Ye*** (China).

These posters also discussed Interactive technologies with a study applying GeoGebra based on AR/VR technology in high school solid geometry teaching by Huang[23]. Multirepresentation was the focus of two posters. Wu[18] discussed IT and its integration into the classroom. Han's study[20] focused on approaching it in a smart-classroom environment with the lens of teaching scaffolding. As for different teaching methodologies, Igodt[19] presented a blended approach specially built to support students aspiring to study engineering, and Riouch[21] discussed interdisciplinarity as modelling and experimentation of interdisciplinary problems. Regarding the instrumental approach, Cuellar[22] presented an analysis of the instrumentation of the symbolic artifact quadratic function. Mathematics functions were also highlighted by Im[25], while discussing the effectiveness of a teaching trigonometry function approach using GeoGebra and Wolfram. As for the literature on ICT integration, Shao[26] showed a review of practical research on the application of IT in Function. Apostol's paper[24] about self-regulated learning strategies, perceptions, and mathematics performance in a mobile technology-integrated mathematics classroom represented students' learning study. Finally, on the side of teachers' knowledge, Ye[27] presented a case study on the TPACK performance of Chinese middle school mathematics teachers.

4. From the Past to the Future: Challenging Themes of Discussion

The TSG-26 has been the occasion for contrasting, discussing, and comparing different themes with arguments in relation to theories, to teachers' education and engagement, and to students, in relation to the use of technologies in educational settings.

What emerged in relation to theories is the use of TPACK to investigate teachers' knowledge, when they are engaged in design cycles, or the new emerging approach of boundary crossing between communities, when speaking of a methodological tool to favor deep understanding: MERLO item. Commognition theory seems to have a fundamental role in analyzing the learning present in virtual reality context. Likewise, the notion of praxeology — including the particular case of meta-didactical praxeologies — was the key element to analyse the involvement of students in designing digital curriculum resources, and to outline the evolution of teachers' interactions when designing learning objects. More than in the past studies, recently the importance of explicitly defining the role of the teacher in multiple didactical tasks, and their engagement in design/use/orchestrate mathematical activity emerges in the papers and posters presented. And, moreover, the students' role has been addressed with a higher level of engagement and sharing of responsibility: not only in solving tasks, but also both in co-designing tasks and in being engaged in the assessment process with more awareness. Here the notion of instrumental orchestration (Trouche, 2004) becomes relevant to establish and organize the conditions that favor the involvement of students and the sharing of responsibility for their learning. This gaining in responsibility of teachers and students is possible also thanks to the technology used, which mediates the participation in common activities, and gives immediate feedback of the actions done. Technologies such as dynamic software, or

Web 2.0 environments give the educational support to learning and teaching mathematics in many different kinds of activities: inquiry tasks from simple variation to co-variation situations. Also technologies for embodied approaches that detouch motions/eye movements/virtual exploration are providing us with empirical data that support new ways of looking at the construction of knowledge.

These tools and the data that they produce have the potential to offer a fresh look at the study of the complexities of mathematics teaching and learning. Future research in this area could address how these technologies can be integrated into the rapidly changing instructional landscape.

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